# Examples
# Exponential Equations in One Variable

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

# Learning Objectives

1. Use logarithms to solve exponential equations.
2. Use a short-cut to to solve certain exponential equations.

# Example 1: Solve Exponential Equations

Solve $3^x = 243$. Show the solution in logarithmic form and without logarithms (round to two decimal places, if necessary).

Let's use the logarithm with base ***e***.

$\ln 3^x = \ln 243$

Using the Power Rule, we get

$x \ln 3 = \ln 243$

$x = \frac{\ln 243}{\ln 3}$

The solution in logarithmic form is $\frac{\ln 243}{\ln 3}$. It is considered the "exact" solution. Using the calculator, we find that the solution is **5**.

# Example 2: Solve Exponential Equations

Solve $3^x = 243$ again. Show the solution in logarithmic form and without logarithms (round to two decimal places, if necessary).

This time, let's use the logarithm with base 10.

$\log 3^x = \log 243$

Using the Power Rule, we get

$x \log 3 = \log 243$

$x = \dfrac{\log 243}{\log 3}$

The solution in logarithmic form is $\dfrac{\log 243}{\log 3}$. It is considered the "exact" solution. Using the calculator, we find that the solution is **5**.

# Example 3: Solve Exponential Equations

Solve $3^x = 243$ again.

This time we notice that we can write 243 as $3^5$. Therefore, we can solve for $x$ without using logarithms.

$3^x = 243$

$3^x = 3^5$

Given that the bases on both sides of the equal signs are the same, the exponents MUST be equal as well. Therefore, we can set the exponents equal to get

$x = 5$ which is the solution we already found in Examples 1 and 2.

# Example 4: Solve Exponential Equations

Solve $10^x = 8000$. Show the solution in logarithmic form and without logarithms (round to two decimal places, if necessary).

Let's use the logarithm with base ***e***.

$\ln 10^x = \ln 8000$

Using the Power Rule, we get

$x \ln 10 = \ln 8000$

$x = \frac{\ln 8000}{\ln 10}$

The solution in logarithmic form is $\frac{\ln 8000}{\ln 10}$. It is considered the "exact" solution. Using the calculator, we find that the solution is **3.90**.

# Example 5: Solve Exponential Equations

Solve $10^x = 8000$ again. Show the solution in logarithmic form and also without logarithms. Round to two decimal places, if necessary.

this time, let's use the logarithm with base 10.

$\log 10^x = \log 8000$

Using the Power Rule, we get

$x \log 10 = \log 8000$

Here we note that $\log 10 = \log_{10} 10 = 1$ according to a basic logarithm principle.

Therefore, we do not need to divide by log 10, but simply state that $x = \log 8000$ which is the "exact" solution. Using the calculator, we find that the solution without logarithms is approximately **3.90**.

# Example 6: Solve Exponential Equations 

Solve $3^{2x-1} = 7^{x+1}$. Show the solution in logarithmic form and also without logarithms. Round to two decimal places, if necessary.

$\ln 3^{2x-1} = \ln 7^{x+1}$ used the natural logarithm

$(2x - 1) \ln 3 = (x + 1) \ln 7$ used the power rule ( exponents MUST be enclosed in parentheses!0

Finally, we will apply algebraic rules and carefully solve for *x*. The first thing we will do is distribute the logarithms on both sides of the equal sign to the terms in parentheses.

$2x \ln 3 - \ln 3 = x \ln 7 + \ln 7$

# Example 6: Solve Exponential Equations (2 of 3)

Next, we will move all terms containing *x* to the left of the equal sign and all constants to the right.

$2x \ln 3 - x \ln 7 = \ln 3 + \ln 7$

Now, we will factor *x* out of the two terms on the left.

$x\,(2 \ln 3 - \ln 7) = \ln 3 + \ln 7$

Lastly, we will solve for *x* by dividing both sides by (2 ln3 – ln 7)

$x = \dfrac{\ln 3 + \ln 7}{2 \ln 3 - \ln 7}$ This is the "exact" solution in logarithmic form.

# Example 6: Solve Exponential Equations (3 of 3)

We will use a calculator to find the solution without logarithms. We will input the entire logarithmic expression into the calculator. However, we MUST enclose both the numerator and denominator in parentheses!

Calculator Input: $(\ln(3) + \ln(7)) \div (2 \times \ln(3) - \ln(7))$ ENTER

We find that the solution without logarithms is approximately 12.11.

# Example 7: Solve Exponential Equations (1 of 3)

Solve $6^{3x} = 5^{x+2}$. Show the solution in logarithmic form and also without logarithms. Round to two decimal places, if necessary.

Let’s place log in front of the right side and the left side of the equal sign.

$\log 6^{3x} = \log 5^{x+2}$

Next, we will use the Power Rule to get

$3x \log 6 = (x + 2) \log 5.$

We will now apply algebraic rules and carefully solve for $x$. The first thing we will do is distribute the logarithm on the right side of the equal sign to the terms in parentheses.

$3x \log 6 = x \log 5 + 2 \log 5$

# Example 7: Solve Exponential Equations (2 of 3)

Next, we will move all terms containing $x$ to the left of the equal sign and all constants to the right.

$3x \log 6 - x \log 5 = 2 \log 5$

Now, we will factor $x$ out of the two terms on the left.

$x\,(3 \log 6 - \log 5) = 2 \log 5$

Lastly, we will solve for $x$ by dividing both sides by $(3 \log 6 - \log 5)$.

$$x = \frac{2 \log 5}{2 \log 6 - \log 5}$$

This is the “exact” solution in logarithmic form.

# Example 7: Solve Exponential Equations (3 of 3)

We will use a calculator to find the solution without logarithms. We will input the entire logarithmic expression into the calculator. However, we MUST enclose both the numerator and denominator in parentheses!

Calculator Input: (2log(5))÷ (3 × log(6) – log(5)) ENTER

We find that the solution without logarithms is approximately 0.855.

# Example 8: Solve Exponential Equations (1 of 2)

Solve $8^{x+2} = 4^{x-3}$.

Please note that we could take the logarithm of both sides and solve for *x*. However, we just so notice that we can write 8 as $2^3$ and 4 as $2^2$ . Therefore, we can solve for *x* as follows:

$(2^3)^{x+2} = (2^2)^{x-3}$

$2^{3(x+2)} = 2^{2(x-3)}$

Given that the bases on both sides of the equal signs are the same, the exponents MUST be equal as well. Therefore, we can set the exponents equal to get

$3(x + 2) = 2(x - 3)$

# Example 8: Solve Exponential Equations 

$$3(x+2) = 2(x-3)$$

$$3x + 6 = 2x - 6$$

$$x + 6 = -6$$

$$x = -12$$

which is the solution of the given equation.

# Example 9: Solve Exponential Equations

Solve $5^x = 125$.

Please note that we could take the logarithm of both sides and solve for *x*. However, we just so notice that we can write 125 as $5^3$. Therefore, we can solve for *x* as follows:

$5^x = 125$

$5^x = 5^3$

Given that the bases on both sides of the equal signs are the same, the exponents MUST be equal as well. Therefore, we can set the exponents equal to get

$x = 3$ which is the solution of the given equation.