Examples
Exponential Equations in One Variable

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Use logarithms to solve exponential equations.
2. Use like bases to solve exponential equations.

Example 1: Use Logarithms to Solve Exponential Equations

Solve $5^x = 134$.

$$5^x = 134$$

$$\ln 5^x = \ln 134 \qquad \text{used the natural logarithm}$$

$$x \ln 5 = \ln 134 \qquad \text{used } log_b M^p = p log_b M$$

$$x = \frac{\ln 134}{\ln 5} \approx 3.04$$

The exact solution is $\frac{\ln 134}{\ln 5}$. This is approximately equal to 3.04.

If required, do not change the logarithms to decimals until the variable is completely isolated!!!

Example 2: Use Logarithms to Solve Exponential Equations

Solve $10^x = 8000$ exactly and rounded to two decimal places.

$$10^x = 8000$$

$$\log 10^x = \log 8000$$ used the common logarithm

$$x \log 10 = \log 8000$$ used $log_b M^p = plog_b M$ and remembering that $\log_{10} 10 = 1$

$$x = \log 8000 \approx 3.90$$

The exact solution is **log 8000**. This is approximately equal to 3.90.

Do not change the logarithms to decimals until the variable is completely isolated!!!

Example 3: Use Logarithms to Solve Exponential Equations

Solve $3^{2x-1} = 7^{x+1}$ exactly and rounded to two decimal places.

$$3^{2x-1} = 7^{x+1}$$

$$\ln 3^{2x-1} = \ln 7^{x+1}$$ used the natural logarithm

$$(2x-1)\ln 3 = (x+1)\ln 7$$ used *$\log_b M^p = p\log_b$* M – exponents MUST be enclosed in parentheses!

Finally, we will apply algebraic rules and carefully solve for *x*. The first thing we will do is distribute the logarithms on both sides to the terms in parentheses.

$$2x\ln 3 - \ln 3 = x\ln 7 + \ln 7$$

Example 3: Use Logarithms to Solve Exponential Equations
(2 of 3)

Next, we will move all terms containing *x* to the left of the equal sign and all constants to the right. Yes, **ln3** and **ln7** are constants because we can find a decimal equivalent for them.

$$2x\ln 3 - x\ln 7 = \ln 3 + \ln 7$$

Now, we will factor *x* out of the two terms on the left.

$$x(2\ln 3 - \ln 7) = \ln 3 + \ln 7$$

Lastly, we will solve for *x* by dividing both sides by (2ln3 – ln 7)

$$x = \frac{\ln 3 + \ln 7}{2\ln 3 - \ln 7}$$

This is considered the EXACT value of *x*.

Example 3: Use Logarithms to Solve Exponential Equations

Lastly, we will use a calculator to find the decimal approximation of *x*. We will input the entire logarithmic expression into the calculator. However, we MUST enclose both the numerator and denominator in parentheses!

Calculator Input: (ln(3) + ln(7))÷ (2× ln(3) – ln(7)) ENTER

We find $x \approx 12.11$.

Example 4: Use Like Bases to Solve Exponential Equations

Solve $5^x = 125$.

Please note that we could take the logarithm of both sides and solve for x. However, we just so notice that we can write 125 as 5^3. Therefore, we can solve for *x* as follows:

$5^x = 125$

$5^x = 5^3$

Given that the bases on both sides of the equal signs are the same, the exponents MUST be equal as well. Therefore, we can set the exponents equal to get

$x = 3$ which is the solution of the given equation.

Example 4: Use Like Bases to Solve Exponential Equations
(1 of 2)

Solve $8^{x+2} = 4^{x-3}$.

Please note that we could take the logarithm of both sides and solve for *x*. However, we just so notice that we can write 8 as 2^3 and 4 as 2^2 . Therefore, we can solve for *x* as follows:

$(2^3)^{x+2} = (2^2)^{x-3}$

$2^{3(x+2)} = 2^{2(x-3)}$

Given that the bases on both sides of the equal signs are the same, the exponents MUST be equal as well. Therefore, we can set the exponents equal to get

$3(x + 2) = 2(x - 3)$

Example 4: Use Like Bases to Solve Exponential Equations

$$3(x+2)=2(x-3)$$

$$3x+6=2x-6$$

$$x+6=-6$$

$$x=-12$$

which is the solution of the given equation.