Examples
Arithmetic Sequences and Series

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Memorize the definition of an arithmetic sequence and find the common difference.
2. Find the value of a term of an arithmetic sequence.
3. Memorize the definition of a finite arithmetic series and evaluate its sum.

Example 1: Find the Common Difference of an Arithmetic Sequence

a. Given the arithmetic sequence 3, 5, 7, 9, 11, 13, … find the common difference *d*.

   Subtracting the first term from the second term: d = 5 – 3 = 2

   We check this by observing that every term after the first one is 2 more than the preceding term.

b. Given the arithmetic sequence 70, 40, 10, –20 , – 50, … find the common difference *d*.

   Subtracting the first term from the second term: d = 40 – 70 = – 30

   We check this by observing that every term after the first one is 30 less than the preceding term.

Example 2: Find the Value of a Term of an Arithmetic Sequence

Find the value of the 6^{th} term of the arithmetic sequence whose first term is 12 and whose common difference *d* is 4.

To find the 6^{th} term a_6 , we will write out the previous 5 terms first.

$a_1 = 12$

$a_2 = 12 + 4 = 16$

$a_3 = 16 + 4 = 20$

$a_4 = 20 + 4 = 24$

$a_5 = 24 + 4 = 28$

$a_6 = 28 + 4 = 32$ This is the value of the 6^{th} term.

Example 3: Find the Value of a Term of an Arithmetic Sequence

In Example 2, we found the value of the 6^{th} term of an arithmetic sequence by writing out the 1^{st} through the 6^{th} term.

Now, we will use the formula $a_n = a_1 + (n - 1)d$ where $a_1 = 12$, $n = 6$, and $d = 4$ to find a_6.

$$a_6 = 12 + (6 - 1)(4)$$
$$= 12 + (5)(4)$$
$$= 12 + (5)(4)$$
$$= 32$$

We also find that the value of the 6^{th} term is 32.

Example 4: Find the Value of a Term of an Arithmetic Sequence

Find the value of the 90th term of the arithmetic sequence whose first term a_1 is 6 and whose common difference d is −5.

We will use the formula $a_n = a_1 + (n - 1)d$ instead of writing out the previous 89 terms. We let $a_1 = 6$, $n = 9$, and $d = -5$ to find a_{90}.

$$a_{90} = 6 + (90 - 1)(-5)$$
$$= 6 + (89)(-5)$$
$$= 6 - 445$$
$$= -439$$

The value of the 90th term is – 439.

Example 5: Evaluate the Sum of an Arithmetic Series (1 of 2)

Evaluate the sum of the arithmetic series $\sum_{k=1}^{25}(5k-9)$.

This series has lots of terms. Instead of writing out all terms and then adding them, we will use the Summation Formula $S_n = \frac{n}{2}\left(a_1 + a_n\right)$.

Let's find the values we need for this formula.

- The number of terms n in the sum is 25.
- We find the first term a_1 by evaluating $(5k - 9)$ for $k = 1$
  Specifically, $a_1 = 5(1) - 9 = -4$
- We find the last term a_{25} by evaluating $(5k - 9)$ for $k = 25$.
  Specifically, $a_{25} = 5(25) - 9 = 116$

Example 5: Evaluate the Sum of an Arithmetic Series (2 of 2)

Finally, given $a_1 = -4$ and $a_{25} = 116$, we can find S_{25}. Specifically,

$$S_{25} = \frac{25}{2}(-4 + 116)$$

$$= \frac{25}{2}(112)$$

$$= 1400$$

Example 6: Evaluate the Sum of an Arithmetic Series (1 of 2)

Find the sum of the first 15 terms of the arithmetic series 3 + 6 + 9 + 12 + ...

We will use the Summation Formula $S_n = \frac{n}{2}(a_1 + a_n)$. Let's find the values we need for it.

- The number of terms *n* in the sum is 15.
- We know that the first term a_1 is 3.
- We find the last term a_{15} by using the formula $a_n = a_1 + (n - 1)d$.

  However, first we need to find the common difference *d*. We study the terms of the arithmetic series and notice that each term is 3 larger than the preceding term. That is, $d = 3$.

Example 6: Evaluate the Sum of an Arithmetic Series (2 of 2)

Given $d = 3$, we can now find the value of a_{15}.

That is, $a_{15} = 3 + (15 – 1)(3)$

$= 3 + 14(3)$

$= 45$

Finally, given $a_1 = 3$ and $a_{15} = 45$, we can find S_{15}. Specifically,

$$S_{15} = \frac{15}{2}(3+45)$$

$$= \frac{15}{2}(48)$$

$$= 360$$