Concepts
Transformations of Some Functions

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Recognize vertical shifts of graphs of some functions.
2. Recognize horizontal shifts of graphs of some functions.
3. Recognize reflections of graphs of some functions.
4. Recognize vertical stretches and compressions of graphs of some functions.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

What are "Transformations"?

We will apply "transformations" to the following functions. In this lesson, we will refer to them as "basic" functions.

$$f(x) = |x| \qquad f(x) = x^3$$

$$f(x) = x^2 \qquad f(x) = \sqrt[3]{x}$$

$$f(x) = \sqrt{x}$$

Transformations allow us to move and resize the graphs of these functions by shifting them vertically and horizontally; by reflecting them in the *x*- and *y*-axis; and by vertically and horizontally stretching and compressing them.

1. Vertical Shifts (1 of 2)

Let f be one of the "basic" functions and c a positive real number.

The graph of $g(x) = f(x) + c$ is the graph of f shifted c units vertically upward.

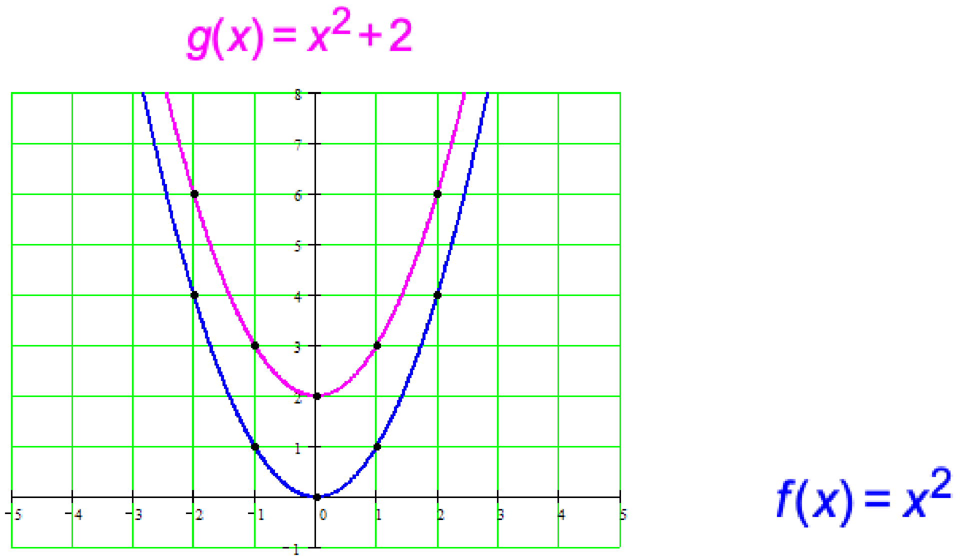
The graph of $h(x) = f(x) - c$ is the graph of f shifted c units vertically downward.

Note: Vertical Shifts affect ONLY the y-coordinates of points on graphs of "basic" functions to be shifted.

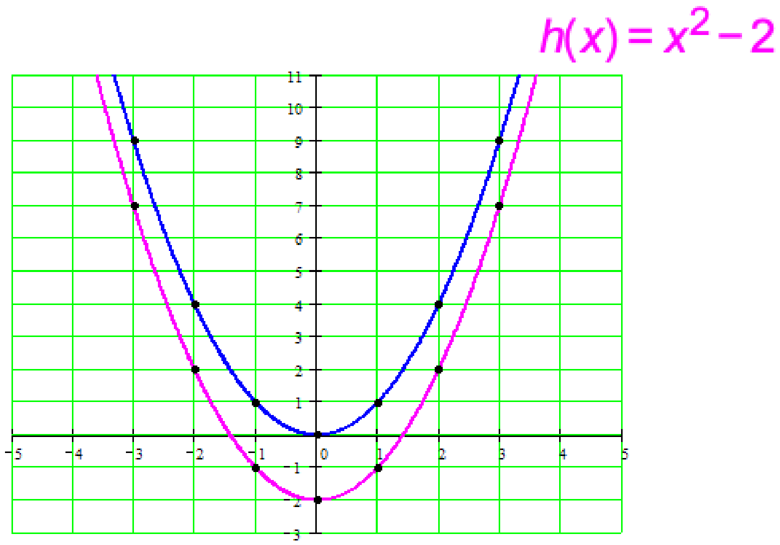
Vertical Shifts (2 of 2)

Example 1:

Following is the graph of $f(x) = x^2$ shifted vertically up by 2 units to become the graph of $g(x) = x^2 + 2$.



Following is the graph of $f(x) = x^2$ shifted vertically down by 2 units to become the graph of $h(x) = x^2 - 2$.



2. Horizontal Shifts (1 of 2)

Let f be one of the "basic" functions and c a positive real number.

The graph of $g(x) = f(x + c)$ is the graph of f shifted c units to the left.

The graph of $h(x) = f(x - c)$ is the graph of f shifted c units to the right.

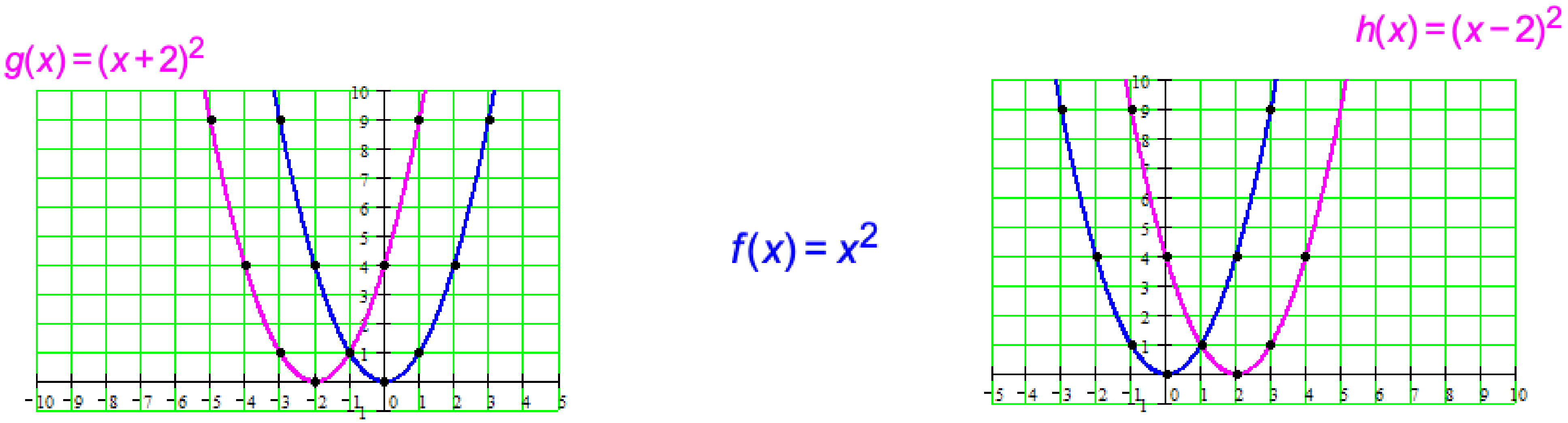
Note: Horizontal Shifts affect ONLY the x-coordinates of points on graphs of "basic" functions to be shifted.

Horizontal Shifts (2 of 2)

Example 2:

Following is the graph of $f(x) = x^2$ shifted horizontally to the left by 2 units to become the graph of $g(x) = (x + 2)^2$.

Following is the graph of $f(x) = x^2$ shifted horizontally to the right by 2 units to become the graph of $h(x) = (x - 2)^2$.



3. Reflections (1 of 2)

Reflection across the y-Axis

The graph of $h(x) = f(-x)$ is the graph of one of the ”basic” functions f reflected across the y-axis.

Note: Reflections across the y-axis affect ONLY the x-coordinates of points on graphs of “basic” functions to be shifted.

Reflection across the x-Axis

The graph of $g(x) = -f(x)$ is the graph of one of the ”basic” functions f reflected across the x-axis.

Note: Reflections across the x-axis affect ONLY the y-coordinates of points on graphs of “basic” functions to be shifted.

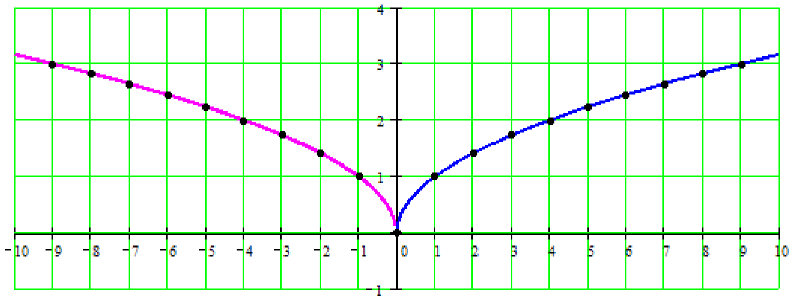
Reflections (2 of 2)

Example 3:

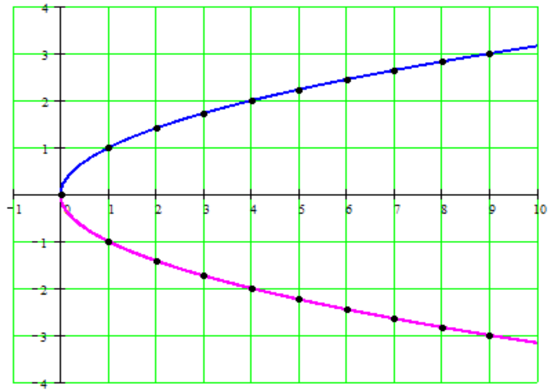
Following is the graph of $f(x) = \sqrt{x}$ reflected across the y-axis to become the graph of $h(x) = \sqrt{-x}$.

Following is the graph of $f(x) = \sqrt{x}$ reflected across the x-axis to become the graph of $g(x) = -\sqrt{x}$.

$h(x) = \sqrt{-x}$



$f(x) = \sqrt{x}$



$g(x) = -\sqrt{x}$

4. Vertically Stretching and Compressing (1 of 2)

Let f be one of the "basic" functions and c a positive real number.

If $0 < c < 1$, the graph of $h(x) = cf(x)$ is the graph of a "basic" function $f(x)$ vertically stretched by a factor of c.

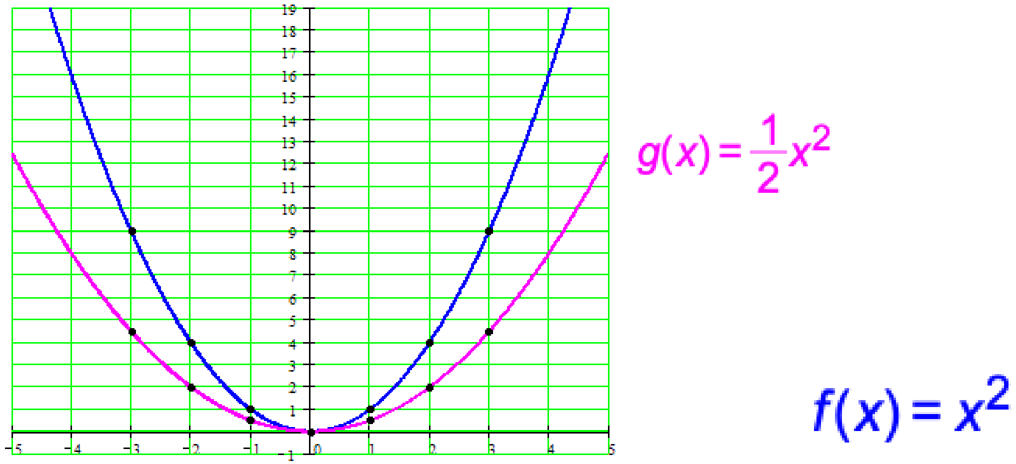
If $c > 1$, the graph of $g(x) = cf(x)$ is the graph of a "basic" function f vertically compressed by a factor of c.

Note: Vertical stretching and compressing affects ONLY the y-coordinates of points on graphs of "basic" functions to be shifted.

Vertically Stretching and Compressing (2 of 2)

Example 4:

Following is the graph of $f(x) = x^2$ vertically stretched to become the graph of $g(x) = \frac{1}{2}x^2$.



Following is the graph of $f(x) = x^2$ vertically compressed to become the graph of $h(x) = 2x^2$.