Concepts
Transformations of Common Functions

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Recognize vertical shifts of graphs of common functions.
2. Recognize horizontal shifts of graphs of common functions.
3. Recognize reflections of graphs of common functions.
4. Recognize vertical stretches and compressions of graphs of common functions.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

What are “Transformations”?

Transformations allow us to move and resize the graphs of common functions by shifting them vertically and horizontally; by reflecting them in the x- and y-axis; and by vertically and horizontally stretching and compressing them.

1. Vertical Shifts (1 of 2)

Let f be a common function and c a positive real number.

The graph of $g(x) = f(x) + c$ is the graph of f shifted c units vertically upward.

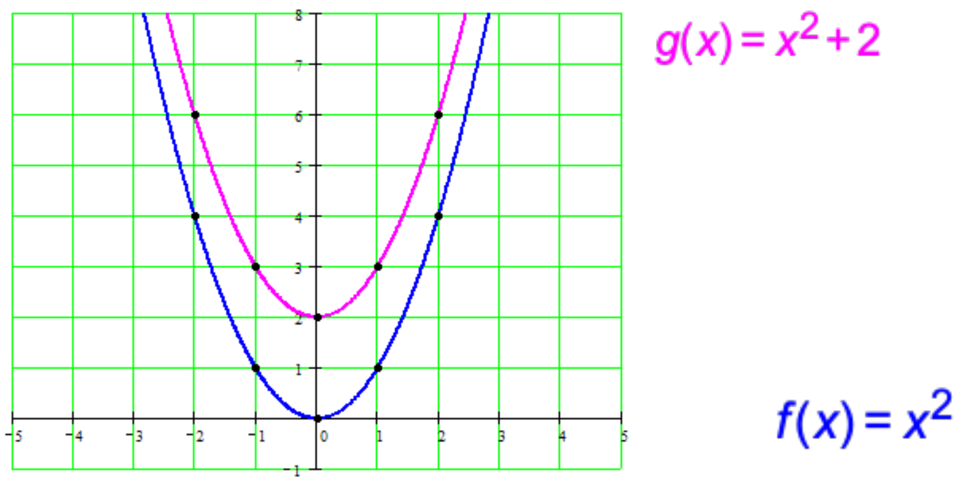
The graph of $h(x) = f(x) - c$ is the graph of f shifted c units vertically downward.

Note: Vertical Shifts affect ONLY the y-coordinates of points on graphs of common functions to be shifted.

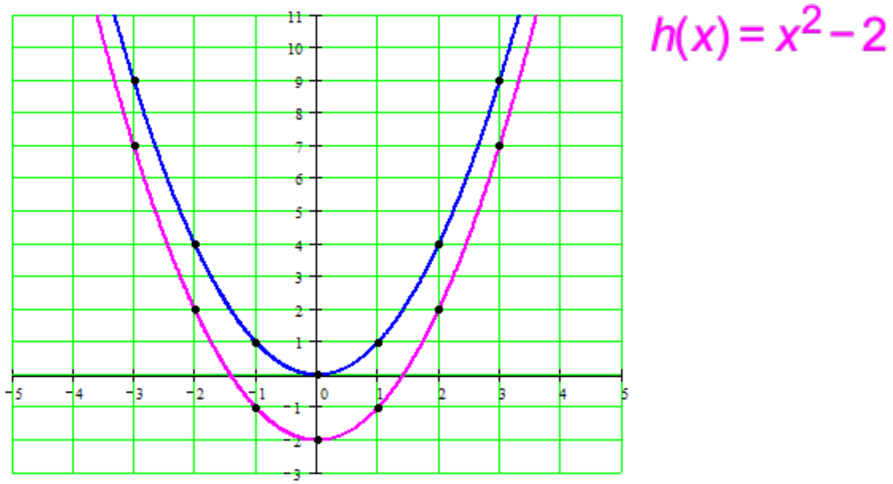
Vertical Shifts (2 of 2)

Example 1:

Following is the graph of the common function *f(x)* shifted vertically up by 2 units and becomes the graph of $g(x) = f(x) + 2$.



Following is the graph of the common function *f(x)* shifted vertically down by 2 units and becomes the graph of $h(x) = f(x) - 2$.



2. Horizontal Shifts (1 of 2)

Let f be a common function and c a positive real number.

The graph of $g(x) = f(x + c)$ is the graph of f shifted c units to the left.

The graph of $h(x) = f(x - c)$ is the graph of f shifted c units to the right.

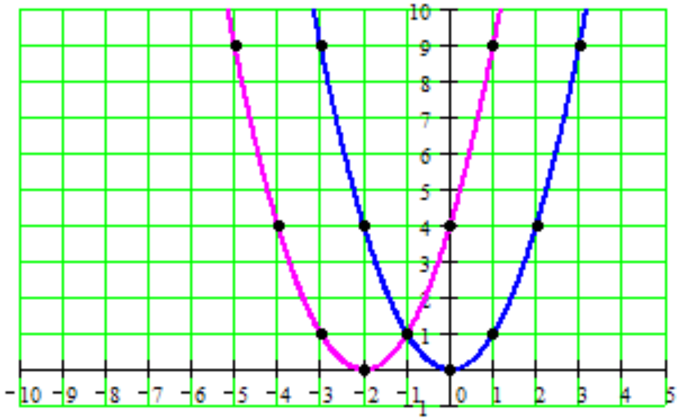
Note: Horizontal Shifts affect ONLY the x-coordinates of points on graphs of common functions to be shifted.

Horizontal Shifts (2 of 2)

Example 2:

Following is the graph of the common function *f(x)* shifted horizontally to the left by 2 units and becomes the graph of *g(x) = f(x + 2).*

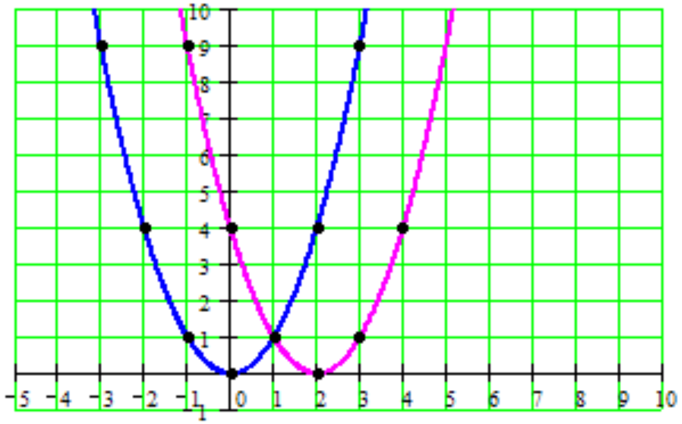
$g(x) = (x + 2)^2$



$f(x) = x^2$

Following is the graph of the common function *f(x)* shifted horizontally to the right by 2 units and becomes the graph of *h(x) = f(x – 2).*

$h(x) = (x - 2)^2$



3. Reflections (1 of 2)

Reflection across the y-Axis

The graph of $h(x) = f(-x)$ is the graph of the common function f reflected across the y-axis.

Note: Reflections across the y-axis affect ONLY the x-coordinates of points on graphs of common functions to be shifted.

Reflection across the x-Axis

The graph of $g(x) = -f(x)$ is the graph of the common function f reflected across the x-axis.

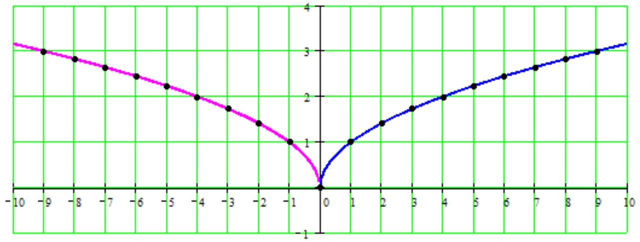
Note: Reflections across the x-axis affect ONLY the y-coordinates of points on graphs of common functions to be shifted.

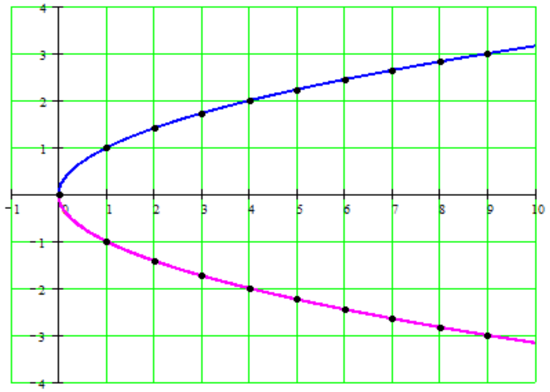
Reflections (2 of 2)

Example 3:

Following is the graph of the common function *f(x)* reflected across the *y*-axis and becomes the graph of $h(x) = f(-x)$.

Following is the graph of the common function *f(x)* reflected across the *x*-axis and becomes the graph of $g(x) = -f(x)$.





4. Vertically Stretching and Compressing (1 of 2)

Let f be a common function and c a positive real number.

If $0 < c < 1$, the graph of $h(x) = cf(x)$ is the graph of a common function $f(x)$ vertically stretched by a factor of c.

If $c > 1$, the graph of $g(x) = cf(x)$ is the graph of a common function f vertically compressed by a factor of c.

Note: Vertical stretching and compressing affects ONLY the *y*-coordinates of points on graphs of common functions to be shifted.

Vertically Stretching and Compressing (2 of 2)

Example 4:

Following is the graph of the common function *f(x)* vertically stretched and becomes the graph of $g(x) = \frac{1}{2}f(x)$

Following is the graph of the common function *f(x)* vertically compressed and becomes the graph of $h(x) = 2f(x)$.