Concepts
Systems of Linear Inequalities in Two Variables

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Recognize linear inequalities in two variables.
2. Graph linear inequalities in two variables by hand.
3. Solve systems of two linear inequalities in two variables.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

1. Definition of Linear Inequalities in Two Variables

Linear inequalities in TWO variables are much like linear equations in TWO variables, however, they differ from linear equalities because they don't contain an equal sign. Instead, they contain inequality symbols.

Examples:

Linear Equation in Two Variables:

$6x - 15y = 3$ (not in general form)

Four different Linear Inequalities in Two Variables:

$6x - 15y \geq 3$

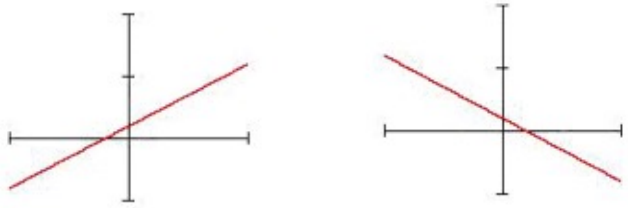
$6x - 15y > 3$

$6x - 15y \leq 3$

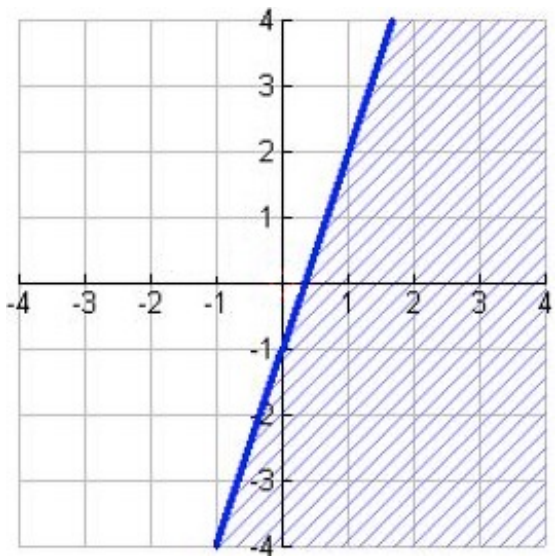
$6x - 15y < 3$

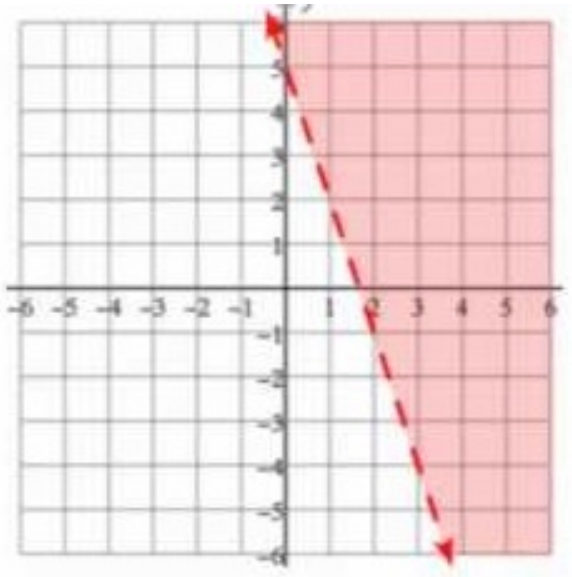
2. Graphs of Linear Inequalities in Two Variables (1 of 6)

The graphs of linear EQUATIONS in two variables consist of straight lines. For example,



On the other hand, the graphs of linear INEQUALITIES in two variables consist of plane regions. For example,





The solid dark blue line and the dashed red line are called **boundary lines**. The graphs of linear inequalities consist of the boundary lines and the shaded plane regions.

Graphs of Linear Inequalities in Two Variables (2 of 6)

Graphing Linear Inequalities in two variables by hand:

Step 1 - Replace the inequality symbol with an equal sign.

Example 1:

Given the linear inequality $2x - y \geq 4$, we change it to $2x - y = 4$.

Step 2 – Graph the linear equation from Step 1. We consider this the "boundary" line of the graph of the linear inequality.

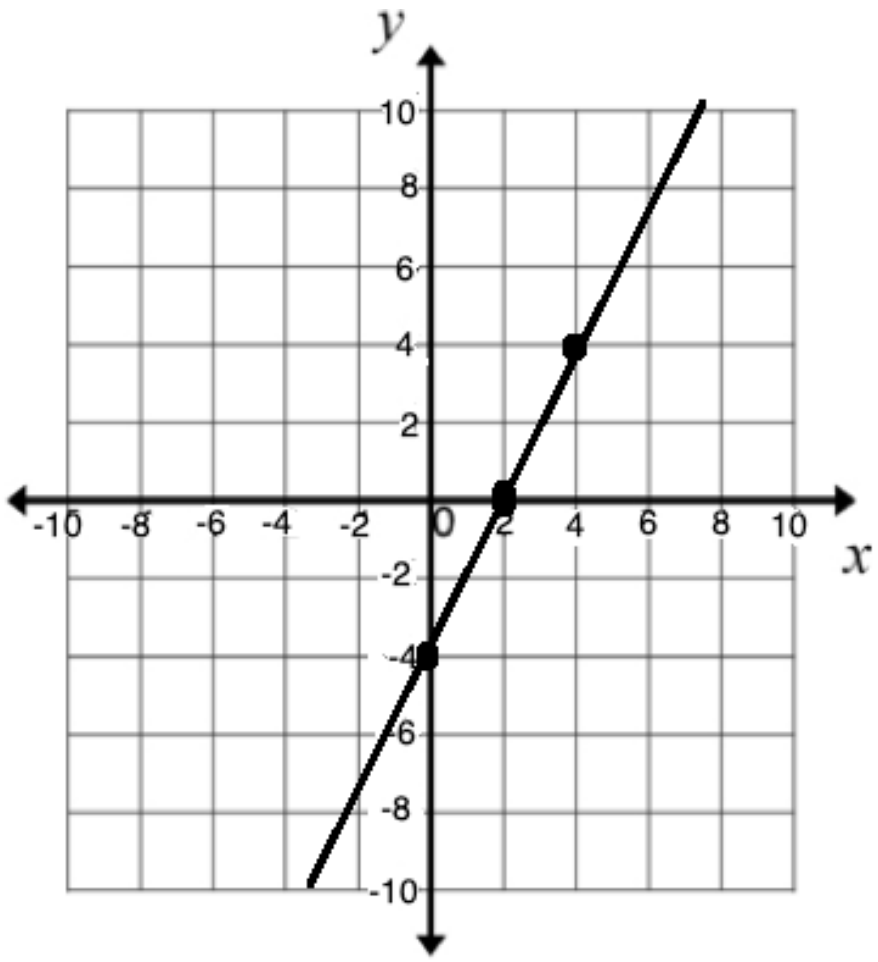
- Draw a solid line if the original inequality contains a ≤ or ≥ symbol.
- Draw a dashed line if the original inequality contains a < or > symbol

Example 1 continued:

Given ≥ (greater than or equal) in $2x - y \geq 4$, we have a solid boundary line.

Graphs of Linear Inequalities in Two Variables (3 of 6)

Example 1 continued



Graphs of Linear Inequalities in Two Variables (4 of 6)

Step 3 – Create a "True" or "False" Statement. Choose any point that does NOT lie on the boundary line. We call it a "test point". Substitute the coordinates of the test point into the inequality.

Example 1 continued:

We use the point created by the ordered pair (0, 0) since it does not lie on the boundary line $2x - y = 4$. See graph in previous slide.

Therefore, we place (0, 0) into $2x - y \geq 4$ to get

$2(0) - 0 \overset{?}{\geq} 4$

$0 \overset{?}{\geq} 4$ FALSE!

We created a "False" statement.

Graphs of Linear Inequalities in Two Variables (5 of 6)

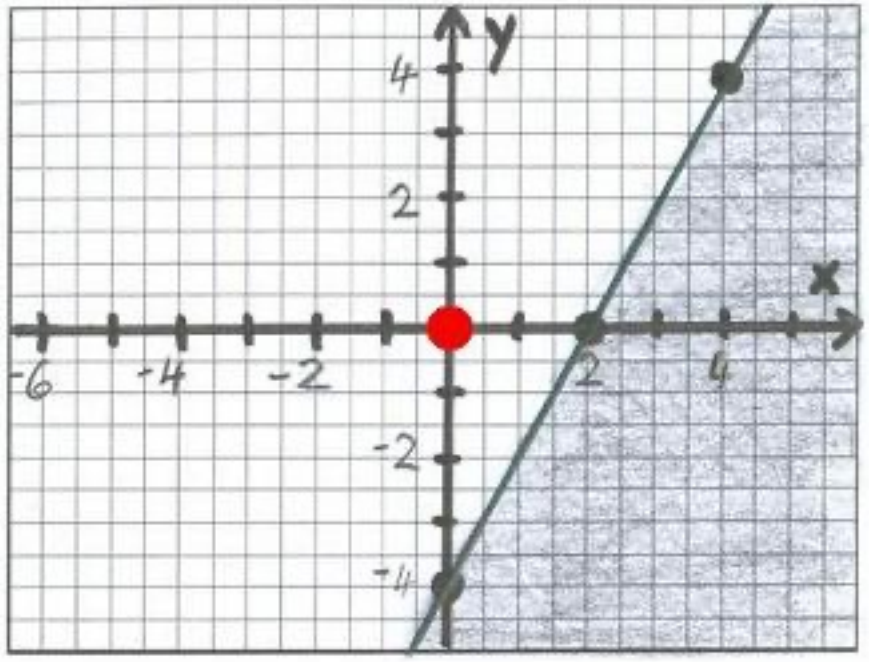
Step 4 – Find the portion of the coordinate plane to be included in the graph.

- If the calculations in Step 3 result in a “False” statement, shade the side of the boundary line NOT containing the test point. This is the graph of the linear inequality in two variables.
- If the calculations in Step 3 result in a “True” statement, shade the side of the boundary line containing the test point. This is the graph of the linear inequality in two variables.

Graphs of Linear Inequalities in Two Variables (6 of 6)

Example 1 continued:

Following is the graph of $2x - y \geq 4$. Please observe the solid boundary line, the ”test point”, and the subsequent shading of the plane region.



3. Solve Systems of Two Linear Inequalities in Two Variables

(1 of 5)

So far, we learned how to solve systems of two linear equations in two variables. Namely, we used the Substitution, Addition, or Matrix Method. We either found one solution, infinitely many solutions, or no solution. We agreed that such systems can also be solved graphically by examining the intersections of the two lines. However, we usually stayed away from this method because it can give inexact results.

On the other hand, systems of two linear inequalities in two variables can only be solved graphically. Specifically, we graph each inequality in the same *Rectangular Coordinate System*.

The solution set of the system will then be the overlapping region of the individual solution sets of the two inequalities.

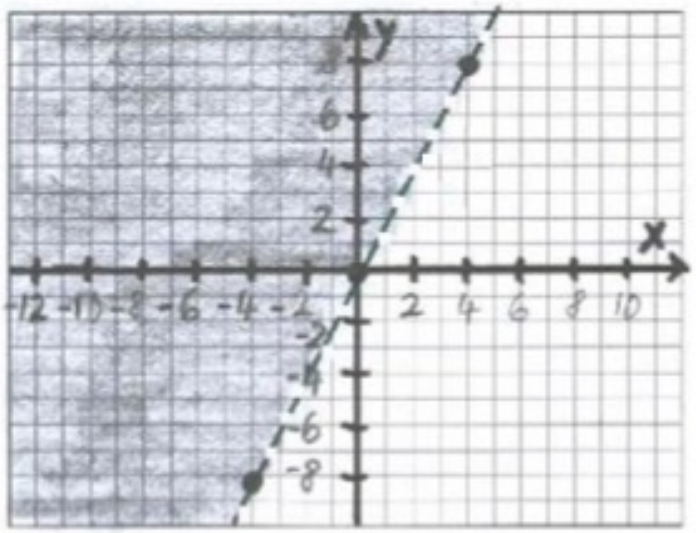
Solve of Systems of Two Linear Inequalities in Two Variables
(2 of 5)

Example 2:

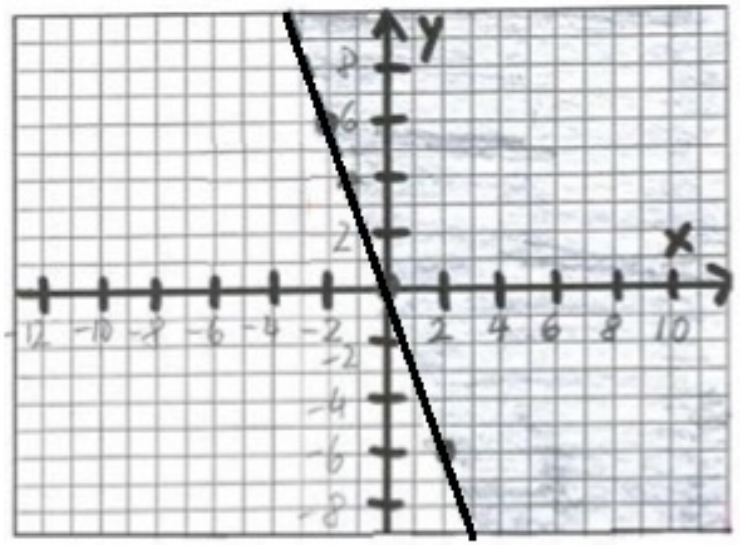
Given the following system

$$\begin{cases} y > 2x \\ 3y \geq -9x \end{cases}$$

The graph of $y > 2x$:



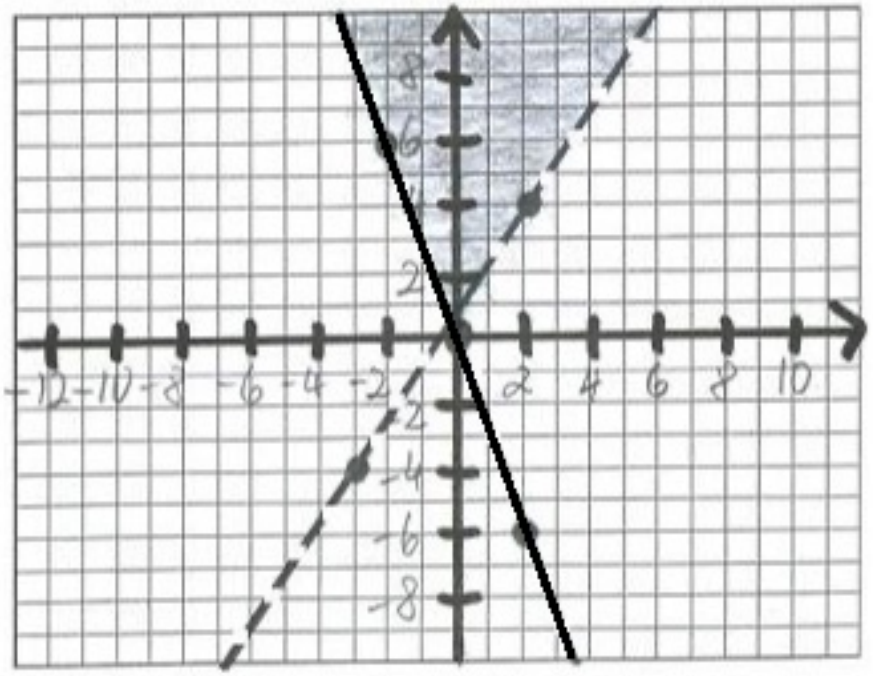
The graph of $3y \geq -9x$:



Solve of Systems of Two Linear Inequalities in Two Variables
(3 of 5)

Example 2 continued:

Given the individual graphs of the linear inequalities in the system, we can then determine the solution of the system by observing where the shaded regions overlap. See graph below.



It might take some imagination to determine the overlapping region! We can always print out the two graphs and then place them on top of each other!

Solve of Systems of Two Linear Inequalities in Two Variables

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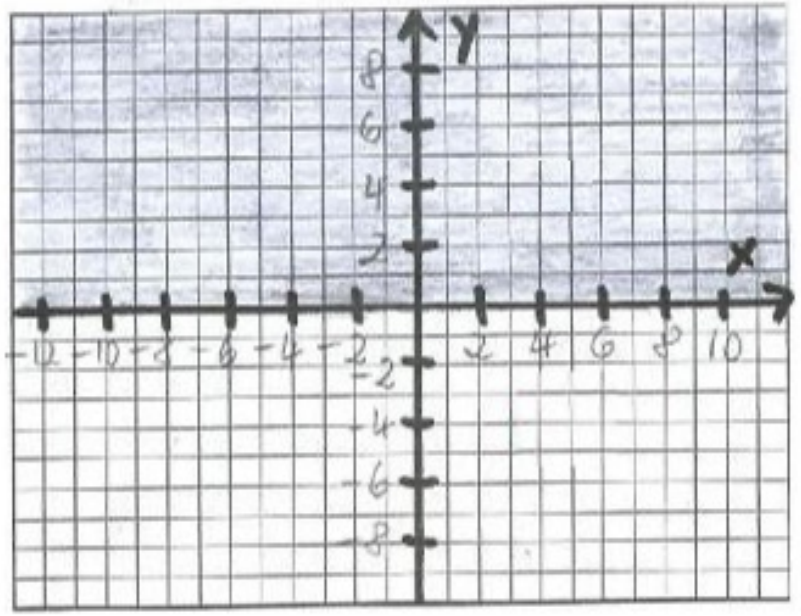
Example 3:

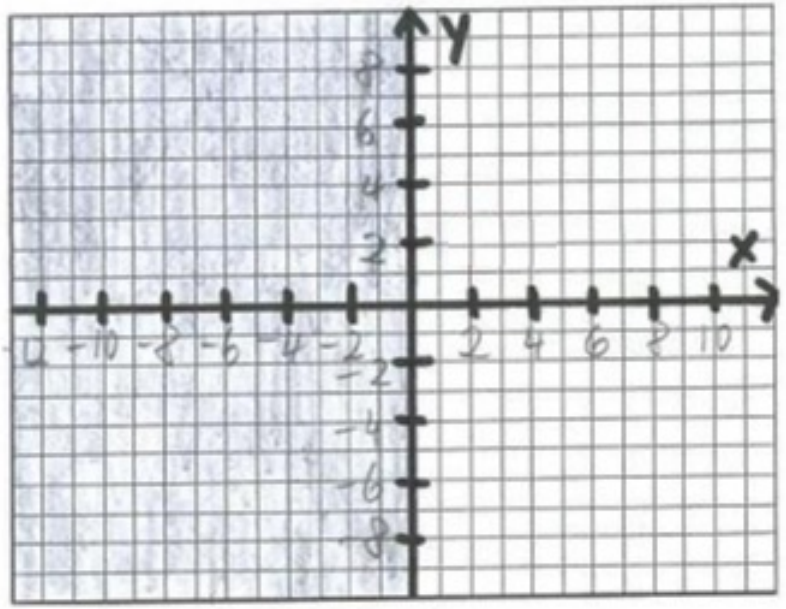
Given the following system

$$\begin{cases} y \geq 0 \\ x \leq 0 \end{cases}$$

The graph of $y \geq 0$:



The graph of $x \leq 0$:



Solve of Systems of Two Linear Inequalities in Two Variables
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Example 3 continued:

Given the individual graphs of the linear inequalities in the system, we can then determine the solution of the system by observing where the shaded regions overlap. See graph below.