Concepts
Uses of the Slope of a Line

Based on power point presentations by Pearson Education, Inc.

Revised by Ingrid Stewart, Ph.D.

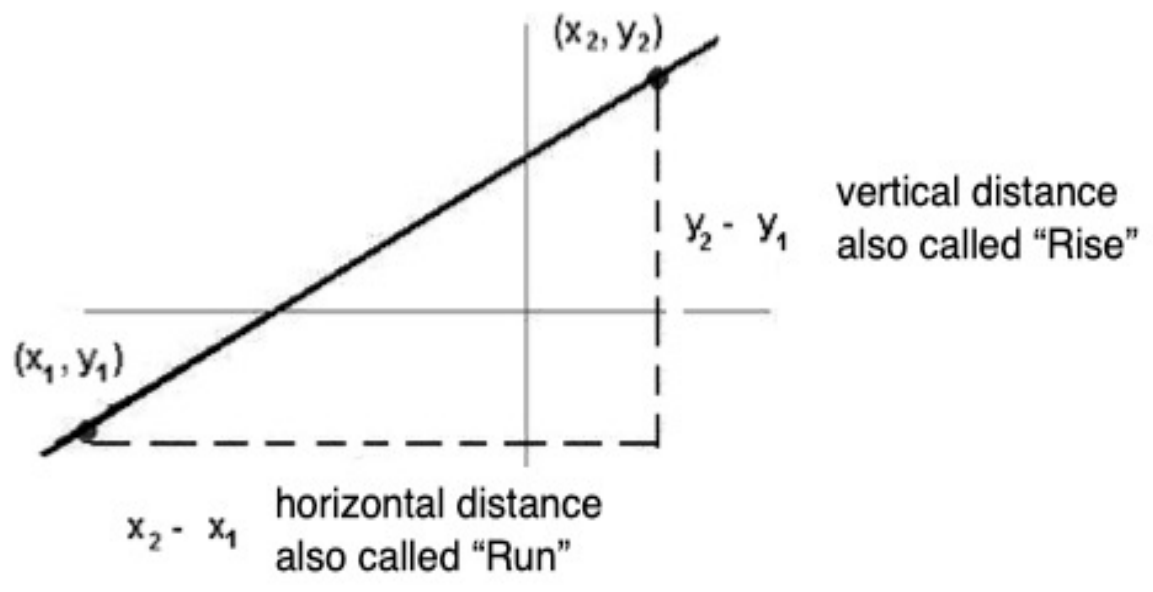
Learning Objectives

1. Define and calculate the slope of a line.
2. Identify the slopes of increasing, decreasing, vertical, and horizontal lines.
3. Identify the slope and the *y*-intercept in the equation of a line.
4. Write the slope-intercept equation of a line, if possible.
5. Identify and use slopes of parallel and perpendicular lines.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

1. Definition of the Slope of a Line (1 of 3)

In layman's terms, the slope of a line is a measure of its **steepness**. It is defined to be the change in vertical distance divided by the change in horizontal distance as we "travel" from one point determined by the ordered pair (x_1, y_1), to another point determined by the ordered pair (x_2, y_2) in a rectangular coordinate system.



Definition of the Slope of a Line (2 of 3)

In mathematics, the slope of a line is indicated by using the lower-case letter *m*. Why *m*? No one knows for sure. Some mathematicians claim the *m* comes from the French word “monter” which means “to climb”.

The slope of the line through two distinct points determined by the ordered pairs (x_1, y_1) and (x_2, y_2) is formally defined as

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

NOTE: Regardless of the sign of the *x*-coordinates or the *y*-coordinates, the minus sign between the *y*-values and the *x*-values in the slope calculation must always be there.

We can also say $m = \frac{\text{Rise}}{\text{Run}}$ or $m = \frac{\textit{change in y}}{\textit{change in x}}$

Definition of the Slope of a Line

Example 1:

Find the slope of the line passing through the points determined by the ordered pairs (4, −2) and (−1, 5).

We will let (4, −2) equal (x_1, y_1) and (−1, 5) equal (x_2, y_2). However, you can also let (−1, 5) equal (x_1, y_1) and (4, −2) equal (x_2, y_2). In either case, you will get the same answer.

Let's say that (4, −2) equals (x_1, y_1) and (−1, 5) equals (x_2, y_2). Be sure not to get confused!

Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-2)}{-1-4} = \frac{5+2}{-5} = \frac{7}{-5} = -\frac{7}{5}$

2. Identify the Slopes of Lines (1 of 2)

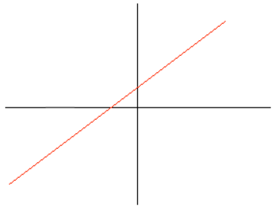
Given a rectangular coordinate system, we found out that the slope of the line through two distinct points determined by the ordered pairs (x_1, y_1) and (x_2, y_2) is formally defined as

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Now, we will investigate how the slope affects the characteristics of a line.

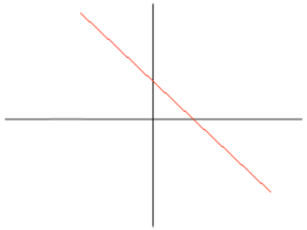
SLOPE OF INCREASING (RISING) LINES

An increasing line has a positive slope.



SLOPE OF DECREASING (FALLING) LINES

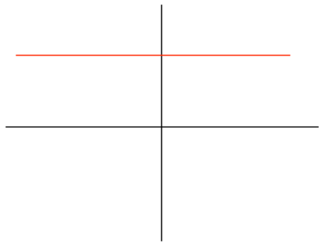
A decreasing line has a negative slope.



Identify the Slopes of Lines (2 of 2)

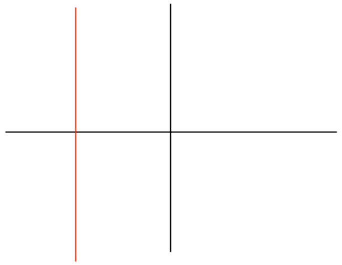
SLOPE OF HORIZONTAL LINES

All horizontal lines have a slope of 0.



SLOPE AND VERTICAL LINES

All vertical lines have an undefined slope.



3. The Slope-Intercept Equation of a Line (1 of 2)

We have already discussed the *general form* of the equation of a line. It is $\boldsymbol{Ax + By + C = 0}$, where $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ are real numbers, but $\boldsymbol{A}$ and $\boldsymbol{B}$ cannot both be 0. Now we are going to discuss a different form of the equation of a line.

It is called the **slope-intercept form** and is defined as

$\boldsymbol{y = mx + b}$ where $\boldsymbol{m}$ is the slope and $\boldsymbol{b}$ is the y-intercept

Note, the y-variable is "isolated" on the left!

Examples of equations of lines in *slope-intercept form*:

$y = -18x + 11$ (here $b = 11$)
$y = 5x - 7$ (here $b = -7$)
$y = 2x$ (here $b = 0$)

The Slope-Intercept Form of the Equations of a Line (2 of 2)

Example 2:

Given the *general form* of the line $-18x - y + 11 = 0$, change it to *slope-intercept form*. Let's do the following manipulations:

$-y + 11 = 18x$ (added $18x$ to both sides)

$-y = 18x - 11$ (subtracted 11 from both sides)

$y = -18x + 11$ (multiplied both sides by -1)

As you can see, y is now isolated on one side. The given equation is now in slope-intercept form with $m = -18$ and $b = 11$,

4. Write the Slope-Intercept Equation of a Line

Often, we are asked to find the *slope-intercept equation* of a line either (1) given the slope m and a point created by the ordered pair (x_1, y_1) or (2) given two points created by the ordered pairs (x_1, y_1) and (x_2, y_2).

(1) Find the *slope-intercept equation* of a line given the slope *m* and a point created by the ordered pair (x_1, y_1)

Step 1 – Place m and (x_1, y_1) into $y = mx + b$, IF possible, and find the value of b!

Example 3:

If possible, find the *slope intercept equation* of a line whose graph has slope $m = -4$ and passes through the point created by the ordered pair $(-2, -3)$.

We will use $m = -4$ and $(-2, -3)$ and place them into $y = mx + b$ to find b.

Write the Slope-Intercept Form of a Linear Equation

Example 3 continued:

$-3 = -4(-2) + b$

$-3 = 8 + b$

$-11 = b$

Step 2 – Using *m* and b, write the slope-intercept form of the equation of the line.

Example 3 continued:

We can now write the *slope-intercept equation* of a line with slope -4 passing through the point created by the ordered pair $(-2, -3)$, namely $\boldsymbol{y = -4x - 11}$.

Write the Slope-Intercept Form of a Linear Equation

(2) Find the *slope-intercept equation* of a line given two points created by the ordered pairs (x_1, y_1) and (x_2, y_2).

Step 1 – Use (x_1, y_1) and (x_2, y_2) to find m.

Example 4:

If possible, find the *slope intercept equation* of a line whose graph passes through the points determined by the ordered pairs (− 4, − 6) and (− 1, 3).

We need to find m. Let (− 4, − 6) equal (x_1, y_1) and (− 1, 3) equal (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-6)}{-1-(-4)} = \frac{3+6}{-1+4} = \frac{9}{3} = 3$$

Write the Slope–Intercept Form of a Linear Equation (4 of 5)

Step 2 – Place m and one of the ordered pairs into $y = mx + b$, IF possible, and find the value of b!

Example 4 continued:

We will use $m = 3$ and one of the given ordered pairs, say $(-4, -6)$, and place all into $y = mx + b$ to find the value of b:

$-6 = 3(-4) + b$

$-6 = -12 + b$

$-6 + 12 = b$

$b = 6$

Write the Slope-Intercept Form of a Linear Equation (5 of 5)

Step 3 – Using *m* and b, write the slope-intercept form of the equation of the line.

Example 4 continued:

Given $m = 3$ and $b = 6$, we can now write the *slope-intercept equation* of a line passing through the points created by the ordered pairs $(-4, -6)$ and $(-1, 3)$, namely $\boldsymbol{y = 3x + 6}$.

5. The Slopes of Parallel and Perpendicular Lines (1 of 2)

1. If two lines are parallel, then they have the same slope.
2. If two lines are perpendicular (intersect at 90^o angles), then their slopes are negative reciprocals.

Example 5:

Are the lines $y = 3x - 6$ and $y = 3x + 11$ parallel or perpendicular or neither?

We notice that both lines have the same slope, namely $m = 3$. Therefore, the lines are parallel.

The Slopes of Parallel and Perpendicular Lines (2 of 2)

Example 6:

Are the lines $y = -2x + 5$ and $y = \frac{1}{2}x - 4$ parallel or perpendicular or neither?

The lines are certainly not parallel because they have different slopes. Let's check if they are perpendicular.

We notice that the slope of the first line is -2 and the slope of the second line is $\frac{1}{2}$.

Now, the reciprocal of $-2 = -\frac{2}{1}$ is $-\frac{1}{2}$. Then the negative reciprocal is $-\left(-\frac{1}{2}\right) = \frac{1}{2}$.

Since -2 and $\frac{1}{2}$ are negative reciprocals, we can say that the given lines are perpendicular.