Concepts
Uses of the Slope of a Line

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Revised by Ingrid Stewart, Ph.D.

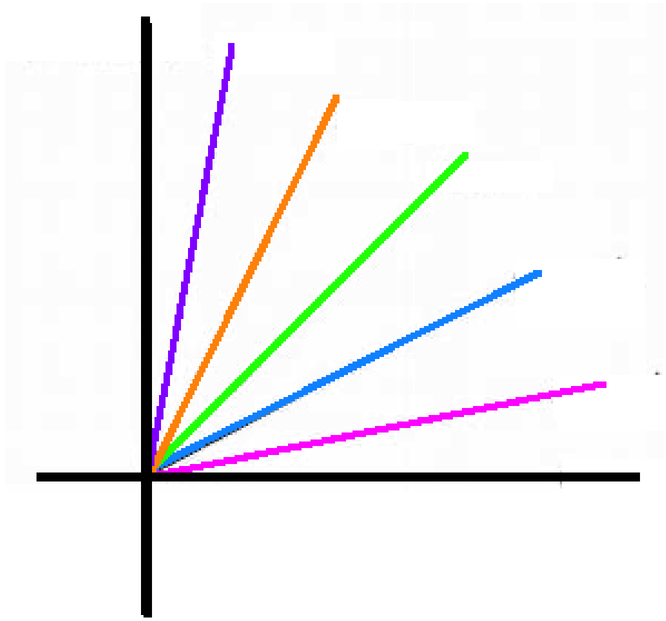
Learning Objectives

1. Define and find the slope of a line.
2. Define the slope-intercept equation of a line.
3. Write the slope-intercept equation of increasing or decreasing lines given certain information.
4. Identify and use slopes of parallel and perpendicular lines.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

1. Definition of the Slope of a Line

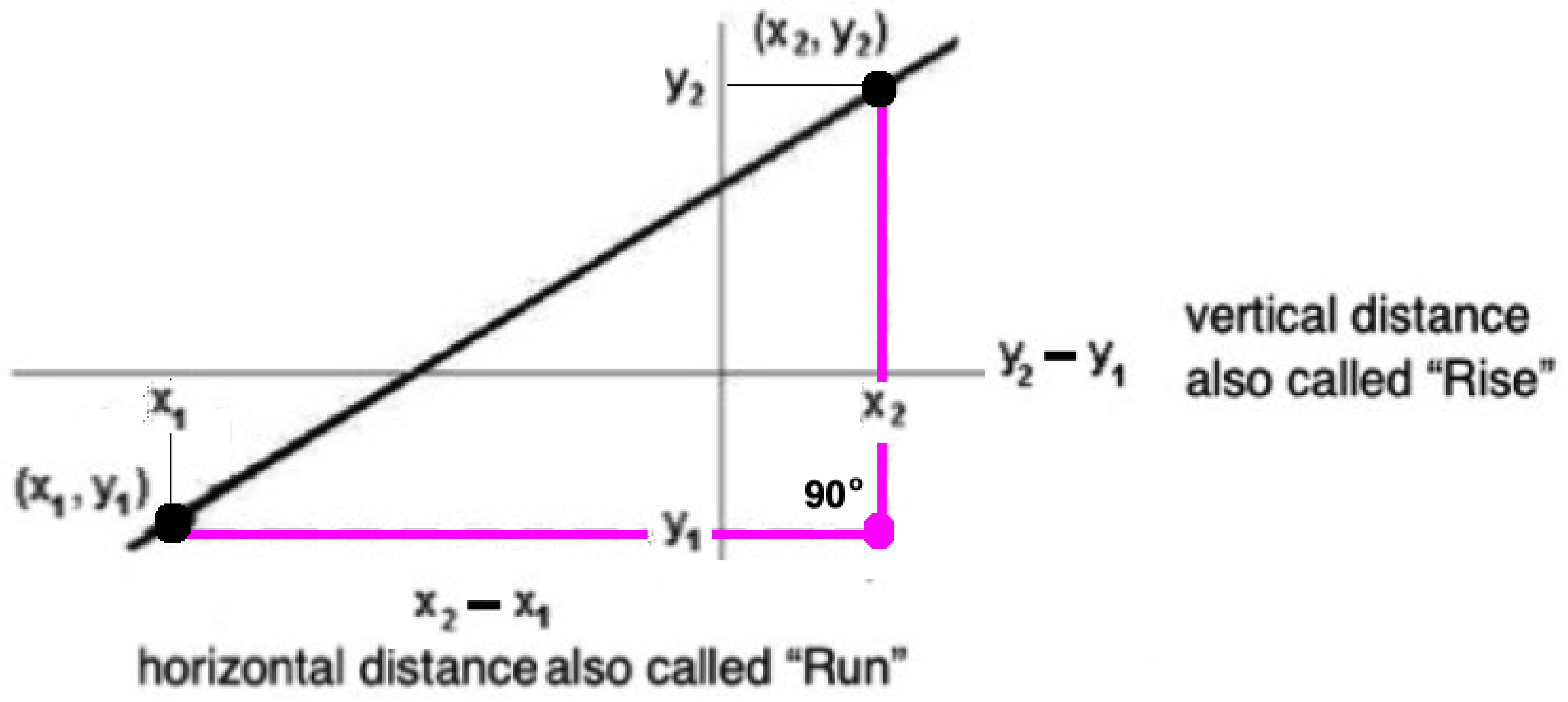
In the last lesson, we discussed the graphs of linear equations in two variables. We found out that their graphs are either increasing or decreasing lines. Let's look at a picture of several increasing lines.



We notice that each line has a different steepness. For example, the purple line is much steeper than the pink line.

Definition of the Slope of a Line (2 of 6)

In mathematics, the steepness of a line is measured by an entirely man-made **Slope Formula**. Specifically, at one point it was defined to be the change in vertical distance divided by the change in horizontal distance as we "travel" from one point on a line in a rectangular coordinate system to another point. Usually, these two points are defined by the ordered pairs (x_1, y_1) and (x_2, y_2).



Definition of the Slope of a Line (3 of 6)

The slope of a line is indicated by the lower-case letter *m*. Why *m*? No one knows for sure. Some mathematicians claim the *m* comes from the French word "monter" which means "to climb".

The slope of the line through two distinct ordered pairs (x_1, y_1) and (x_2, y_2) is defined by the following formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

NOTE: Regardless of the sign of the *x*-coordinates or the *y*-coordinates, the minus sign between the *y*-values and the *x*-values in the slope calculation must always be there.

We can also say $m = \frac{\text{Rise}}{\text{Run}}$ or $m = \frac{\textit{change in y}}{\textit{change in x}}$

Definition of the Slope of a Line (4 of 6)

Example 1:

Find the slope of the line passing through the points determined by the ordered pairs (4, −2) and (−1, 5).

We will let (4, −2) equal (x_1, y_1) and (−1, 5) equal (x_2, y_2). However, you can also let (−1, 5) equal (x_1, y_1) and (4, −2) equal (x_2, y_2). In either case, you will get the same answer.

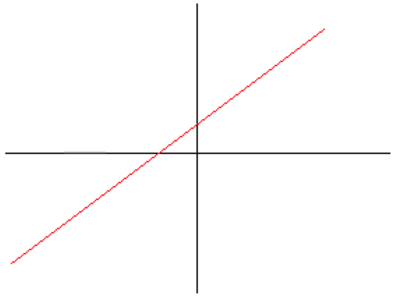
Let's say that (4, −2) equals (x_1, y_1) and (−1, 5) equals (x_2, y_2). Be sure not to get confused!

Then $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-(-2)}{-1-(+4)} = \dfrac{5+2}{-5} = \dfrac{7}{-5} = -\dfrac{7}{5}$

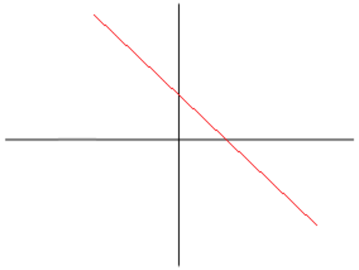
2. Definition of the Slope of a Line (5 of 6)

We will not state how certain slope properties affect the characteristics of a line.

POSITIVE SLOPE
All increasing lines have a positive slope.



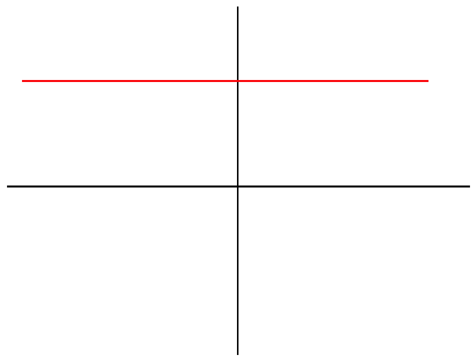
NEGATIVE SLOPE
ALL decreasing lines have a negative slope.



Definition of the Slope of a Line

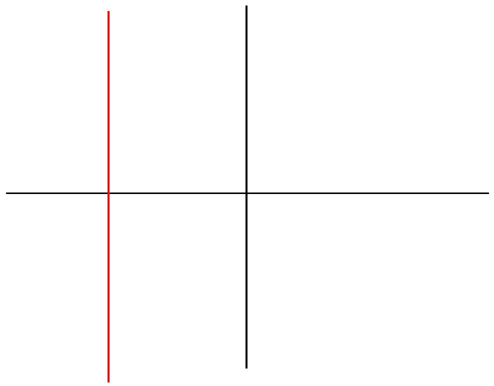
SLOPE OF 0

All horizontal lines have a slope of 0.
Slopes of 0 occur when the numerator in the slope formula is 0.



UNDEFINED SLOPE

All vertical lines have an undefined slope.
Undefined slopes occur when the denominator in the slope formula is 0.



2. The Slope-Intercept Equations of Lines and their Graphs

We have already discussed the **general equation** of a line. It is $ax + by + c = 0$, where a, b, and c are real numbers, but a and b cannot be 0.

We are now going to discuss the **slope-intercept equation** of a line which is

$y = mx + b$ where m is the slope of the line and b is the y-intercept.

NOTE: The slope-intercept equation of a line is simply a mathematical "manipulation" of the general equation with the introduction of the letter m.

Please study Example 1 on the next slides.

The Slope-Intercept Equations of Lines and their Graphs

Example 1:

Write the equation of the general form of the line $-18x - 2y + 11 = 0$ in slope-intercept form.

Let's do the following manipulations:

$-2y + 11 = 18x$ (added $18x$ to both sides)

$-2y = 18x - 11$ (subtracted 11 from both sides)

$y = \dfrac{18x - 11}{-2}$ (divided both sides by -2)

As you can see, y is now isolated on one side. But we are not quite done!

The Slope-Intercept Equations of Lines and their Graphs

Example 1 continued:

We will now distribute – 2 to every term in the numerator on the right side of the equal sign as follows:

$y = \frac{18x}{-2} - \frac{11}{-2}$

and $y = -9x + \frac{11}{2}$. This is $-18x - 2y + 11 = 0$ written in slope-intercept form.

The Slope-Intercept Equations of Lines and their Graphs

Examples of equations of lines in *slope-intercept form*:

$y = -18x + 11$ (where $m = -18$ is the slope of the line and $b = 11$ is the y-intercept)

$y = 2x$ (where $m = 2$ is the slope of the line and $b = 0$ is the y-intercept)

$y = -x - 1$ (this is still considered slope-intercept form) because the equation can be written as $y = -x + (-1)$. Therefore, $m = -1$ is the slope of the line and $b = -1$ is the y-intercept.

The Slope-Intercept Equations of Lines and their Graphs

Example 2:

Graph the linear equation $y = -3x - 6$ by hand. This linear equation is in *slope-intercept form*!

NOTE:

In a previous algebra courses, you may have learned a method of graphing called the "Slope-Intercept" Method. We will not use this method in our course. All linear equations in two variables can be graphed using the Point-by-Point Plotting Method or the Intercept Method or a combination of both.

The Slope-Intercept Equations of Lines and their Graphs

Example 2 continued:

Since we are not told which graphing method to use, let’s use the *Intercept Method*.

Find the ordered pair associated with the *y*-intercept.

Since the linear equation is in slope- intercept form, we know that b is the *y*-intercept, therefore, the *y*-intercept is -6

The ordered pair associated with the *y*-intercept is $(0, -6)$.

The Slope-Intercept Equations of Lines and their Graphs

Example 2 continued:

Find the ordered pair associated with the x-intercept.

Let $y = 0$ and solve for x.

$0 = -3x - 6$ (this is a linear equation in one variable)

$3x = -6$

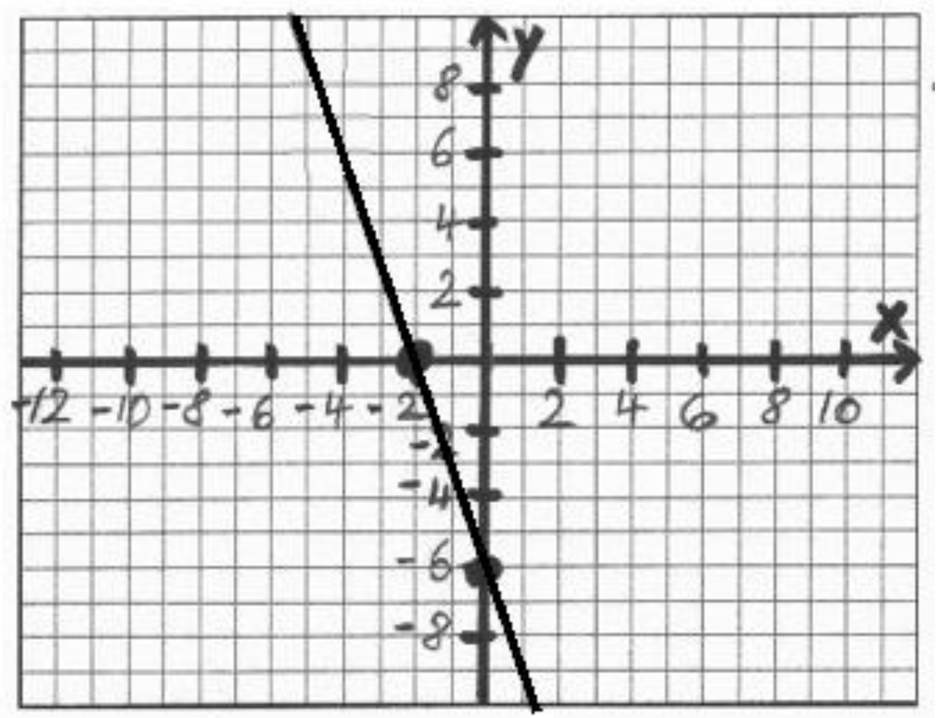
$x = -2$

The x-intercept is -2, so the ordered pair associated with it is $(-2, 0)$.

The Slope-Intercept Equations of Lines and their Graphs

Example 2 continued:

Graph the linear equation by drawing a line through the points created by the ordered pairs associated with the *y*- and *x*-intercepts.



3. Write the Slope-Intercept Equation of Increasing and Decreasing Lines (1 of 6)

Often, we are asked to find the *slope-intercept equation* of increasing or decreasing lines either (1) given the slope m and a point on the line created by the ordered pair (x_1, y_1) or (2) given two points on the line created by the ordered pairs (x_1, y_1) and (x_2, y_2).

(1) Write the *slope-intercept equation* of increasing or decreasing lines given the slope m and a point on the line created by the ordered pair (x_1, y_1)

Step 1 – Place m and (x_1, y_1) into $y = mx + b$ and find the value of b!

Write the Slope-Intercept Equation of Increasing and Decreasing Lines

Example 4:

Write the *slope-intercept equation* of a line whose graph has slope $m = -4$ and passes through the point created by the ordered pair $(-2, -3)$.

We will use $m = -4$ and $(-2, -3)$ and place them into $y = mx + b$ to find *b.*

$-3 = -4(-2) + b$

$-3 = 8 + b$

$-11 = b$

Write the Slope-Intercept Equation of Increasing and Decreasing Lines (3 of 6)

Step 2 – Using m and b, write the *slope-intercept equation* of the line.

Example 4 continued:

Given $m = -4$ and $b = -11$ and knowing $y = mx + b$, we can now write the *slope-intercept equation* of a line passing through the points created by the ordered pairs $(-2, -3)$.

That is, we get $\boldsymbol{y = -4x + (-11)}$ **or** $\boldsymbol{y = -4x - 11}$.

Write the Slope-Intercept Equation of Increasing and Decreasing Lines (4 of 6)

(2) Write the *slope-intercept equation* of increasing or decreasing lines given two points on the line created by the ordered pairs (x_1, y_1) and (x_2, y_2).

Step 1 – Use (x_1, y_1) and (x_2, y_2) to find m.

Example 5:

Write the *slope-intercept equation* of a line whose graph passes through the points determined by the ordered pairs (− 4, − 6) and (− 1, 3).

We need to find m. Let (− 4, − 6) equal (x_1, y_1) and (− 1, 3) equal (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-6)}{-1-(-4)} = \frac{3+6}{-1+4} = \frac{9}{3} = 3$$

Write the Slope-Intercept Equation of Increasing and Decreasing Lines (5 of 6)

Step 2 – Place *m* and one of the ordered pairs into $y = mx + b$ and find the value of *b*!

Example 5 continued:

We will use $m = 3$ and one of the given ordered pairs, say $(-4, -6)$, and place all into $y = mx + b$ to find the value of *b*:

$-6 = 3(-4) + b$

$-6 = -12 + b$

$-6 + 12 = b$

$6 = b$

Write the Slope-Intercept Equation of Increasing and Decreasing Lines (6 of 6)

Step 3 – Using *m* and *b*, write the *slope-intercept equation* of the line.

Example 5 continued:

Given $m = 3$ and $b = 6$ and knowing $y = mx + b$, we can now write the *slope-intercept equation* of a line passing through the points created by the ordered pairs $(-4, -6)$ and $(-1, 3)$.

That is, $\boldsymbol{y = 3x + 6}$.

4. The Slopes of Parallel and Perpendicular Lines (1 of 3)

1. If two lines are parallel, then they have the same slope.
2. If two lines are perpendicular (intersect at 90^o angles), then their slopes are negative reciprocals**.

The **reciprocal**** of a number is a new number in which the numerator and denominator of the given number is exchanged.

For example, the reciprocal of $\frac{7}{11}$ is $\frac{11}{7}$ and the negative reciprocal of $\frac{7}{11}$ is $-\frac{11}{7}$.

The Slopes of Parallel and Perpendicular Lines (2 of 3)

Example 6:

Are the lines $y = 3x - 6$ and $y = 3x + 11$ parallel or perpendicular or neither?

We notice that both lines have the same slope, namely $m = 3$. Therefore, the lines are parallel.

The Slopes of Parallel and Perpendicular Lines (3 of 3)

Example 7:

Are the lines $y = -2x + 5$ and $y = \frac{1}{2}x - 4$ parallel or perpendicular or neither?

The lines are certainly not parallel because they have different slopes. Let's check if they are perpendicular.

We notice that the slope of the first line is -2 and the slope of the second line is $\frac{1}{2}$.

Now, the reciprocal of $-2 = -\frac{2}{1}$ is $-\frac{1}{2}$. Then the negative reciprocal is $-\left(-\frac{1}{2}\right) = \frac{1}{2}$.

Since -2 and $\frac{1}{2}$ are negative reciprocals, we can say that the given lines are perpendicular.