Concepts
Rational Equations in One Variable

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Memorize the definition of a rational equation.
2. Know how to solve rational equations.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

1. Definition of a Rational Equation (1 of 2)

A rational equation is of the form $\frac{p}{q} = 0$, where ***p*** and ***q*** are polynomial expressions and $q \neq 0$.

For example, $\frac{x^2 + x - 6}{x^2 - 8x + 12} = 0$ or $\frac{x - 6}{7} = 0$

The second equation is actually a linear equation involving fractions

(see Lesson 1) because it can be written as $\frac{1}{7}x - \frac{6}{7} = 0$, which is slope-intercept form.

NOTE: In this lesson, the rational equations we will be most interested in are the ones with variables in the denominator.

2. Solve Rational Equations (1 of 4)

Strategy for Solving Rational Equations with Variables in the Denominator:

Step 1: Find a mathematical expression divisible by all denominators.

Note that the least complex expression is preferred because subsequent calculations are not as cumbersome.

Step 2: Multiply both sides of the rational equation by the expression found in Step 1 and solve. This gives us proposed solutions. Factoring, the *Quadratic For*mula, or the *Square Root Property* might have to be used.

Step 3: Check the proposed solutions in the denominators of the original equation, rejecting any that produce 0, which would lead to an undefined condition.

Be sure to study the problems located in the "Examples" document!

Solve Rational Equations (2 of 4)

How to find the **<u>least complex</u>** mathematical expression divisible by all denominators.

Step 1: Write each denominator as a product of **prime factors**.

For example, the expression $\mathbf{(x + 2)(x + 3)}$ consist of prime factors because the terms in all factors are only divisible by 1.

On the other hand, the expression $\mathbf{(4x + 2)(12x + 3)}$ does NOT consist of prime factors because the terms in the first factor are both divisible by 2 and the terms in the second factor are both divisible by 3.

Solve Rational Equations (3 of 4)

Step 2: Form a product of all prime factors found in Step 1. This is the **<u>least complex</u>** expression divisible by all denominators.

NOTE: Use all factors only once EXCEPT when any one prime factor shows up more than once within one or more denominators. Then we use the greatest occurrence.

For example, say we have two denominators, one consists of the prime factors $\mathbf{2(x + 2)(x + 3)}$ and the other one $\mathbf{(x + 2)(x + 2)(x + 3)}$. To find the least complex expression divisible by both denominators, we would form the product $\mathbf{2(x + 2)(x + 2)(x + 3)}$.