Concepts
Quadratic Functions – Part 2

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Define the *Standard Form* of a quadratic function.
2. Given a quadratic function in standard form, find the coordinates of the vertex and the equation of the axis of symmetry of its graph.
3. Graph quadratic functions by hand when given in standard form.

1. The Standard Form of a Quadratic Function (1 of 3)

The *general form* of a quadratic function is defined to be $\boldsymbol{f(x) = ax^2 + bx + c}$. However, there is also a *standard form*.

Specifically, the **standard form** of the quadratic function in x is

$$\boldsymbol{f(x) = a(x - h)^2 + k}, \text{ where } a \neq 0$$

minus is part of the form!

Examples of quadratic functions in standard form:

$g(x) = 4(x - 2)^2 + 1$ (standard form with $a = 4$, $h = +2$ and $k = +1$)

$p(x) = (x - (-3))^2 + (-5)$ (standard form with $a = 1$, $h = -3$ and $k = -5$)

Please note that this function is usually written as $p(x) = (x + 3)^2 - 5$. We eliminate the double signs!

The Standard Form of a Quadratic Function (2 of 3)

Example 1:

Change the quadratic function $g(x) = 4(x - 2)^2 + 1$ to *general form*:

Given the standard form of a quadratic function, let's first eliminate the parentheses using FOIL and the we will combine like terms.

$$\begin{aligned} g(x) &= 4(x - 2)(x - 2) + 1 \\ &= 4(x^2 - 4x + 4) + 1 \\ &= 4x^2 - 16x + 16 + 1 \\ &= 4x^2 - 16x + 17 \text{ which is the general form.} \end{aligned}$$

The Standard Form of a Quadratic Function (3 of 3)

Example 2:

Is the graph of the quadratic function $g(x) = 4(x - 2)^2 + 1$ a parabola open up or open down?

Since $a = 4$ which is greater than 0, the graph of the quadratic function is a parabola open up.

2. Coordinates of the Vertex and Equation of the Axis of Symmetry (1 of 2)

Given the standard form $f(x) = a(x - h)^2 + k$,

- the coordinates of the vertex are ***(h, k)***
- the equation of the axis of symmetry is $x = h$

There is a proof in the learning materials showing that *(h, k)* are indeed the coordinates of the vertex.

It also shows that *(h, k)* is equivalent to $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ **.**

Coordinates of the Vertex and Equation of the Axis of Symmetry (2 of 2)

Example 3:

Given the quadratic function $p(x) = (x + 3)^2 - 5$, find the coordinates of the vertex of its graph and the equation of the axis of symmetry

This function is almost in *standard form*. If we make a few sign changes, we can use h and k for the coordinates of the vertex.

Specifically, if we let $p(x) = (x - (-3))^2 + (-5)$ we have *standard form*.

We see that $h = -3$ and $k = -5$ and the coordinates of the vertex are $(-3, -5)$.

The equation of the axis of symmetry is $x = h$. Then $x = -3$.

3. Graph Quadratic Functions by Hand Given Standard Form

The strategy for graphing quadratic functions in standard form is equal to the strategy for graphing quadratic functions in general form.