Concepts
Quadratic Functions – Part 1

Based on power point presentations by Pearson Education, Inc.

Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Define the *General Form* of a quadratic function.
2. Recognize characteristics of parabolas.
3. Given a quadratic function in general form, find the coordinates of the vertex and the equation of the axis of symmetry of its graph.
4. Graph quadratic functions by hand when given in general form.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

1. The General Form of a Quadratic Function (1 of 2)

We have already been exposed to quadratic equations in one variable, for example, $x^2 - 5x - 6 = 0$.

Now, we will discuss quadratic equations in two variables, for example, $y = x^2 - 5x - 6$, where x is the independent variable.

Quadratic equations in two variables are functions, therefore, we can replace the dependent variable with function notation, for example, $g(x) = x^2 - 5x - 6$.

The **general form** of the quadratic function in x is

$$\boldsymbol{f(x) = ax^2 + bx + c}, \text{ where } \boldsymbol{a, b,} \text{ and } \boldsymbol{c} \text{ are real numbers and } \boldsymbol{a} \neq 0$$

Domain: *All Real Numbers* or $(-\infty, \infty)$ in *Interval Notation*.

The General Form of a Quadratic Function (2 of 2)

Examples of quadratic functions:

$g(x) = x^2 + 5x + 6$ ($a = 1$, $b = 5$, and $c = 6$)

$p(x) = -4x^2 + (-2x)$ ($a = -4$, $b = -2$, and $c = 0$)

Please note that this function is usually written as $p(x) = -2x^2 - 2x$. We eliminate the double signs!

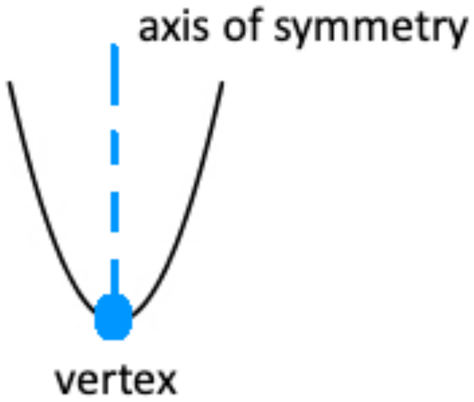
$k(x) = 3x^2 + 21$ ($a = 3$, $b = 0$, and $c = 21$)

$h(x) = x^2$ ($a = 1$, $b = 0$, and $c = 0$)

Special case of the quadratic function. It is often called the *Square Function*!

2. Characteristics of the Graphs of Quadratic Functions (1 of 2)

The graph of a quadratic function is called **parabola**. We already encountered this graph when we discussed the square function $h(x) = x^2$.



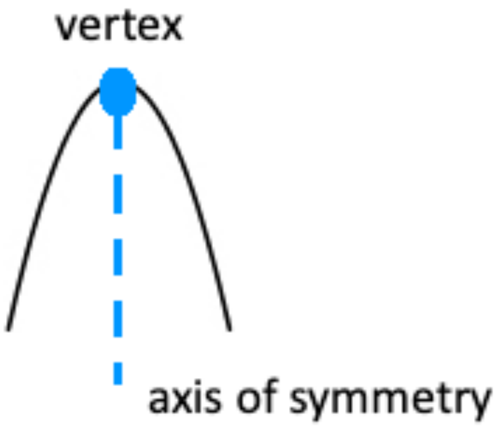
The **axis of symmetry** is an invisible vertical line parallel to the *y*-axis that divides the parabola into two identical halves!

We say that the parabola is open up. This happens when the coefficient a in $f(x) = ax^2 + bx + c$ is greater than 0, that is $a > 0$.

Please notice that the graph has a smooth curve around the vertex!

Characteristics of the Graphs of Quadratic Functions

However, we can also encounter a parabola that us open down. This happens when the coefficient a in $f(x) = ax^2 + bx + c$ is less than 0, that is $a < 0$.



NOTE: Depending on whether the parabola opens up or down, the **vertex** is either the lowest point or the highest point, respectively on the graph of the parabola.

3. Coordinates of the Vertex and Equation of the Axis of Symmetry (1 of 2)

Given the *general form* ***f(x) = ax² + bx + c,***

- the coordinates of the vertex are $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$
- the equation of the axis of symmetry is $x = -\frac{b}{2a}$

NOTE: There is a proof in the learning materials showing that $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ **are indeed the coordinates of the vertex.**

Coordinates of the Vertex and Equation of the Axis of Symmetry (2 of 2)

Example:

Given the quadratic function $g(x) = x^2 - 4x - 5$, find the coordinates of the vertex of its graph and the equation of the axis of symmetry.

The x-coordinate of the vertex:

In the given function, $a = 1$, $b = -4$, and $c = 5$. Then $x = -\frac{-4}{2(1)} = 2$.

The y-coordinate of the vertex:

$g(2) = (2)^2 - 4(2) - 5 = -9$

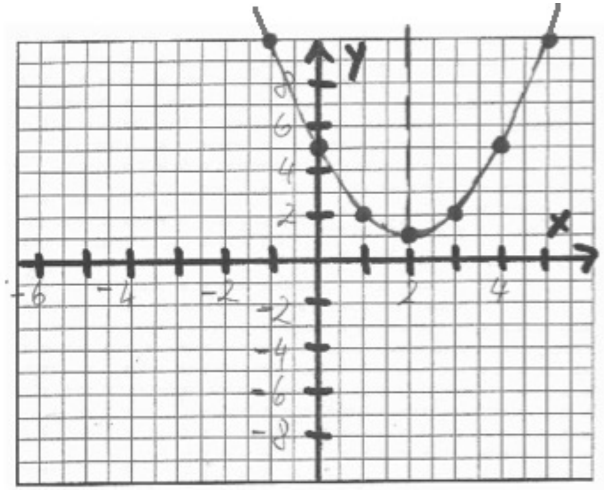
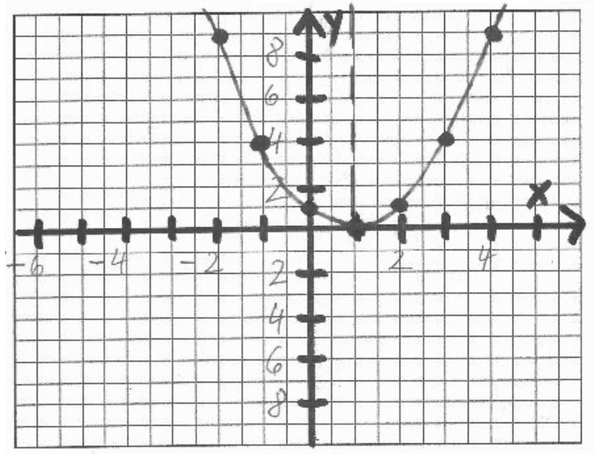
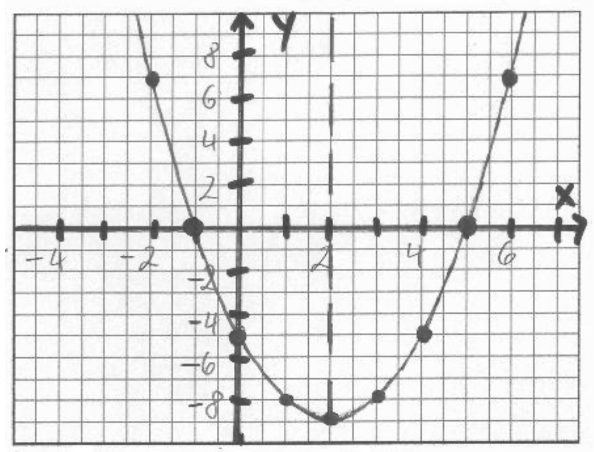
The coordinates of the vertex are (2, – 9).

The equation of the axis of symmetry is $x = -\frac{-4}{2(1)} = 2$.

4. Graph Quadratic Functions by Hand Given General Form

Graphing Strategy:

1. Determine whether the parabola opens up or down. If $a > 0$, it opens upward. If $a < 0$, it opens downward.
2. Find and plot the coordinates of the vertex of the parabola.
3. If possible, find and plot the coordinates of point(s) associated with the x-intercept(s). There could be 0, 1, or 2. See examples below.

Graph Quadratic Functions by Hand Given General Form
(2 of 2)

4. Find and plot the coordinates of the point associated with the *y*-intercept. There is always exactly 1.
5. Find and graph the axis of symmetry as a dashed line.
6. Plot additional points close to the vertex, as necessary. Connect all points with a smooth curve that is shaped like a bowl $\smile$ or an inverted bowl $\frown$.