Concepts
Logarithm Rules

Based on power point presentations by Pearson Education, Inc.
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Learning Objective

Memorize and apply the logarithm rules.

The Logarithm Rules (1 of 4)

We are now going to discuss three rules that allow us to expand and condense logarithm. This becomes important when we solve exponential and logarithmic equations. You can find proof of these rules in *MyOpenMath* in the appropriate *Learning Materials* folder.

Product Rule

Let b, M, and N be positive real numbers with $b \neq 1$.

$$\log_b MN = \log_b M + \log_b N$$

When the logarithm argument is a product, we can write the logarithm as the sum of two logarithms.

Example 1: $\log_2 xy = \log_2 x + \log_2 y$

Example 2: $\log x + \log (x + 3) = \log (x(x + 3))$

The Logarithm Rules (2 of 4)

Quotient Rule

Let b, M, and N be positive real numbers with $b \neq 1$.

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

When the logarithm argument is a quotient, we can write the logarithm as the difference of two logarithms.

Example 3: $\log_8\left(\frac{y+2}{x-1}\right) = \log_8(y+2) - \log_8(x-1)$

Example 4: $\ln x - \ln(x+2) = \ln\left(\frac{x}{x+2}\right)$

The Logarithm Rules (3 of 4)

Power Rule

Let b and M be positive real numbers with $b \neq 1$, and let p be any real number.

$\log_b M^p = p \log_b M = p \cdot \log_b M$

NOTE: There is multiplication between the p and $\log_b M$!!!

When the logarithm argument has an exponent, we can write the logarithm as a product of factors consisting of the exponent and the logarithm without the exponent on the argument.

Example 5: $\log_2 x^3 = 3\log_2 x$

NOTE: There is multiplication between the 3 and $\log_2 x$!!!

The Logarithm Rules (4 of 4)

Example 6: $ln(4x)^2 = 2\ ln(4x)$

Example 7: $ln\ 4x^2 \neq 2\ ln\ 4x$

The exponent only pertains to the x. Here we must use the Product Rule before we can use the Power Rule!

$$ln\ 4x^2 = ln\ 4 + ln\ x^2$$
$$= ln\ 4 + 2\ ln\ x$$

Example 8: $ln\ 5^{x+2} \neq x + 2\ ln\ 5$

The quantity $(x + 2)$ is in the exponent. Therefore, the p is $(x + 2)$. This must be indicated with parentheses as follows:

$$ln\ 5^{x+2} = (x + 2)\ ln\ 5$$