Concepts
Linear and Constant Functions

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Define linear functions and graph them by hand.
2. Define constant functions and graph them by hand.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

1. Linear Functions and their Graphs (1 of 9)

When we discussed the *Vertical Line Test* earlier, we determined that linear equations in two variables are functions! Specifically, we can call them **linear functions**.

The linear function is defined, say in *x,* as ***f(x) = mx + b.***

Please note that the definition of a linear function is nothing but the *slope-intercept form* of a linear equation in two variables, which is $y = mx + b$ where m is the slope and b the y-intercept of the graph of the function.

Domain: *All Real Numbers* or $(-\infty, \infty)$ in *Interval Notation*.

Linear Functions and their Graphs (2 of 9)

Example 1:

Graph the linear function $\boldsymbol{g}(x) = -3x - 6$ by hand.

NOTE:

In a previous algebra courses, you may have learned a method of graphing called the "Slope-Intercept" Method. We will not use this method in our course. All linear functions can be graphed using the Point-by-Point Plotting Method or the Intercept Method or a combination of both.

Linear Functions and their Graphs (3 of 9)

Example 1 continued with $\boldsymbol{g}(x) = -3x - 6$:

Since we are not told which graphing method to use, let's use the *Intercept Method*.

Find the ordered pair associated with the *y*-intercept.

Since the linear function is in slope- intercept form, we know that *b* is the *y*-intercept, therefore, the *y*-intercept is – 6.

The ordered pair associated with the *y*-intercept is (0, – 6).

Linear Functions and their Graphs (4 of 9)

Example 1 continued with $\boldsymbol{g}(x) = -3x - 6$:

Find the ordered pair associated with the x-intercept.

Let $\boldsymbol{g}(x) = y = 0$ and solve for x.

$0 = -3x - 6$ (this is a linear equation in one variable)

$3x = -6$

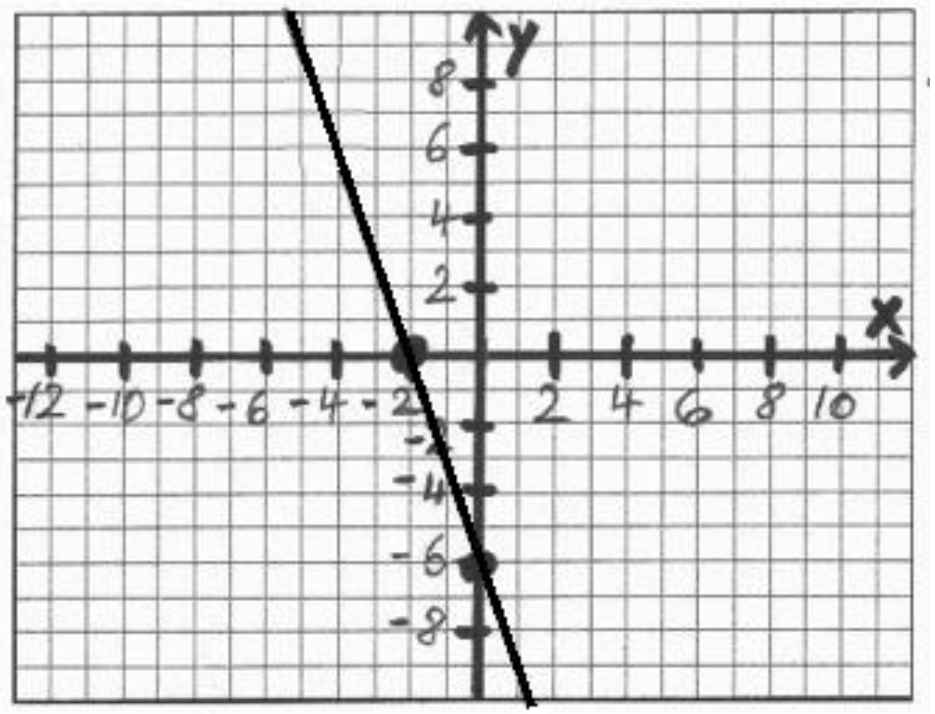
$x = -2$

The x-intercept is -2, so the ordered pair associated with it is $(-2, 0)$.

Linear Functions and their Graphs (5 of 9)

Example 1 continued:

Graph the linear function $\boldsymbol{g}(x) = -3x - 6$ by drawing a line through the points created by the ordered pairs associated with the *y*- and *x*-intercepts (– 2, 0) and (0, – 6).



Linear Functions and their Graphs (6 of 9)

Example 2:

Graph the linear function $\boldsymbol{h}(x) = x$ by hand.

NOTE: This particular linear function is also called **Identity Function**!

Since we are not told which graphing method to use, let's use the *Intercept Method*.

Find the ordered pair associated with the *y*-intercept.

Since the linear function is in slope- intercept form, we know that *b* is the *y*-intercept, therefore, the *y*-intercept is 0.

The ordered pair associated with the *y*-intercept is (0, 0).

Linear Functions and their Graphs (7 of 9)

Example 2 continued with $\boldsymbol{h}(x) = x$:

Find the ordered pair associated with the x-intercept.

Let $\boldsymbol{h}(x) = y = 0$ and solve for x.

$0 = x$

The x-intercept is 0, so the ordered pair associated with it is (0, 0).

Linear Functions and their Graphs (8 of 9)

Example 2 continued with $\boldsymbol{h}(x) = x$:

When the *Intercept Method* only produces one ordered pair, we use the *Point-by-Point Plotting Method* to find one or more other ordered pairs.

We want to pick values for x that are far enough away from 0 to create an accurate graph. How about we let x equal -4 and 4?

Then, given the function $\boldsymbol{h}(x) = x$, we get the following:

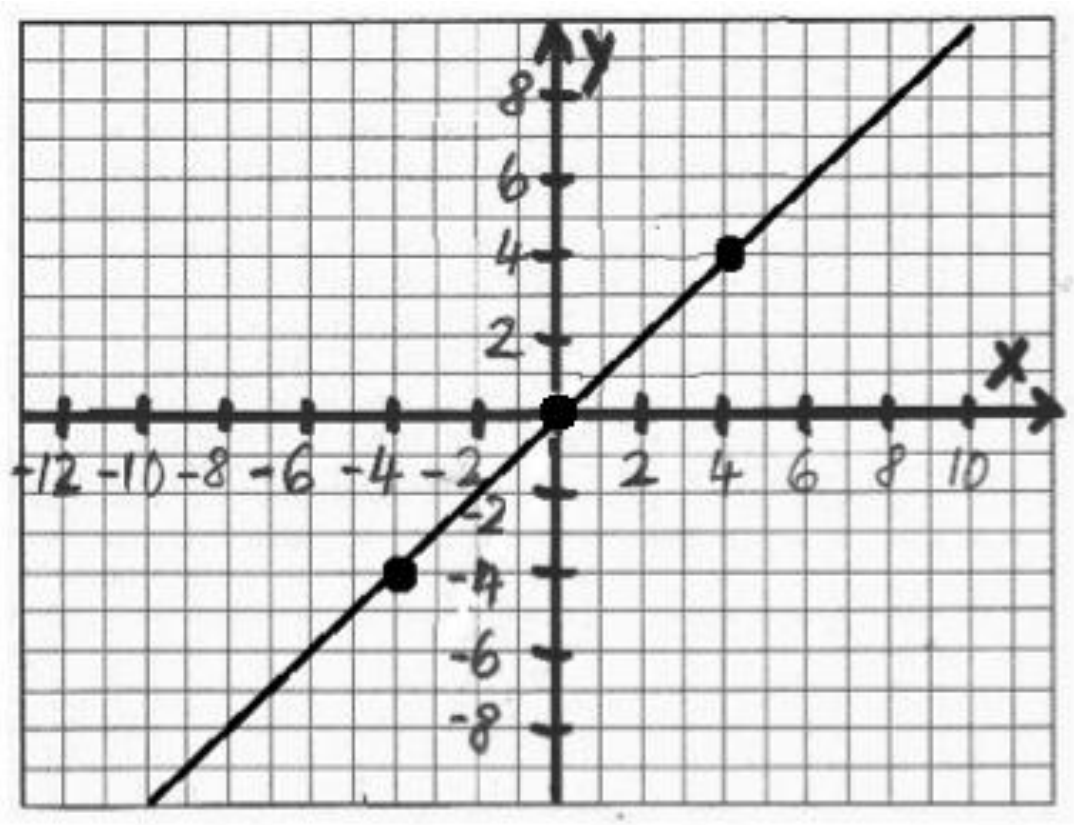
$\boldsymbol{h}(-4) = -4$ and $\boldsymbol{h}(4) = 4$

Therefore, the coordinates of the additional ordered pairs are $(-4, -4)$ and $(4, 4)$.

Linear Functions and their Graphs (9 of 9)

Example 2 continued:

Graph the linear function $\boldsymbol{h}(x) = x$ by drawing a line through the points created by the ordered pairs $(-4, -4)$, $(0, 0)$, and $(4, 4)$.



Observe that in an *Identity Function* *all* y-coordinates are equal to their respective x-coordinates.

2. Constant Functions and their Graphs (1 of 2)

The *Constant Function* is defined as ***f(x) = b***.

Note that the definition of a constant function is that of a horizontal line. It is nothing but $y = b$ where $m = 0$ is the slope of the graph of the function and b is the y-intercept.

Domain: *All Real Numbers* or $(-\infty, \infty)$ in *Interval Notation*.

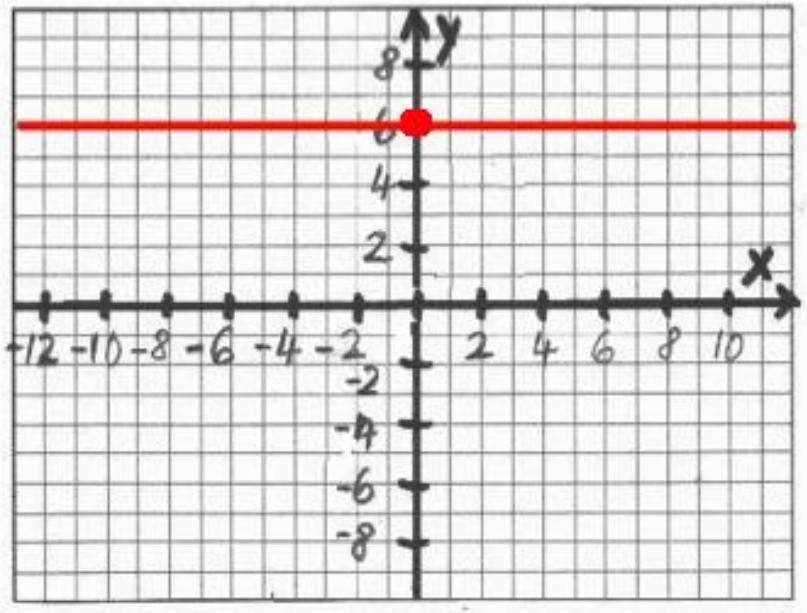
Please note that vertical lines are not a functions. Do the *Vertical Line Test* to convince yourself!!!

Constant Functions and their Graphs (2 of 2)

Example 3:

Graph the function $\boldsymbol{g}(x) = 6$ by hand.

From the function we know that we are dealing with a *Constant Function* whose graph is a horizontal line parallel to the *x*-axis. We note that $b = 6$ which is the *y*-intercept. Therefore, the ordered pair associated with this intercept is (0, 6). Let's plot this ordered pair and then simply draw a horizontal line through it that is parallel to the *x*-axis.



You MUST memorize that a *Constant Function* is $\boldsymbol{f(x) = b}$ where $\boldsymbol{b}$ can be any real number.