Concepts
Linear Equations in Two Variables

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Memorize the definition of independent and dependent variables.
2. Memorize the general form of linear equations in two variables.
3. Memorize the characteristics of the graphs of linear equations in two variables.
4. Graph linear equations in two variables by hand in the rectangular coordinate system using the *Point-by-Point Plotting Method* and the *Intercept Method*.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

1. Equations in Two Variables (1 of 2)

Up to this point, we have only been exposed to equations in one variable, for example, $\mathbf{6x + 3 = 0}$.

Now, we will discuss equations in which two variables are used, for example, $\mathbf{6x + 3y = 0}$.

NOTE: In mathematics we usually use the variables ***x*** and ***y***. We then call ***x*** the **independent variable** and ***y*** the **dependent variable**.

We usually assign real numbers to the independent variable. The value of the dependent variable depends on the number we pick for the independent variable.

Equations in Two Variables (2 of 2)

Example 1:

Given $6x + 3y = 0$, let the independent variable x equal 1 and find the value of the dependent variable y.

Given $x = 1$, we calculate $6(1) + 3y = 0$

then $6 + 3y = 0$

$3y = -6$

and $y = -2$

The value of y is -2. It **depends** on our pick of $x = 1$!

2. Linear Equations in Two Variables (1 of 2)

The **general form** of a linear equation in TWO variables, say ***x*** and ***y***, is

Ax + By + C = 0, where ***A***, ***B***, and ***C*** are real numbers, but ***A*** and ***B*** cannot both be 0

Incidentally, linear equations in two variables do not necessarily have to appear in *general form*. Mathematics just likes to define them that way!

Linear Equations in Two Variables (2 of 2)

Examples of linear equations in two variables:

$2x + 5y + 6 = 0$ (general form with $A = 2$, $B = 5$, and $C = 6$)

$-18x + y + 11 = 0$ (general form with $A = -18$, $B = 1$, and $C = 11$)

$9x - y = 0$ (general form with $A = 9$, $B = -1$, and $C = 0$)

$x + y + 1 = 0$ where $A = 1$, $B = 1$, and $C = 1$

$-4x + (-6y) + (-7) = 0$ (general form with $A = -4$ and $B = -6$ and $C = -7$)

Please note that this equation is usually written as $-4x - 6y - 7 = 0$. We eliminate the double signs!

$-x - 7y = 4$ (not in general form, but still a linear equation in two variables)

3. Graphs of Linear Equations in Two Variables (1 of 4)

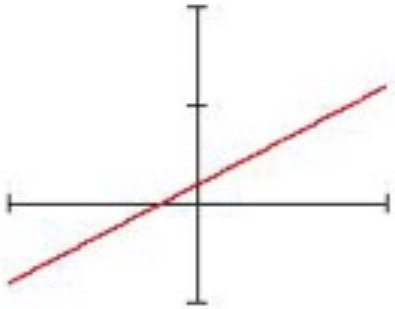
We can make graphs (pictures) of any equation in two variables. We often refer to this as ”graphing.” We do this with the help of the *Rectangular Coordinate System* in which the independent variable is assigned to the horizontal axis and the dependent variable to the vertical axis. We then label the axes appropriately.

For example, given $\mathbf{6x + 3y = 0}$ … and knowing that in mathematics we consider $\boldsymbol{x}$ to be the independent variable … we would call the horizontal axis the ***x*-axis** and the vertical axis the ***y*-axis**. Any point in this coordinate system would be of the form $\mathbf{(x, y)}$.

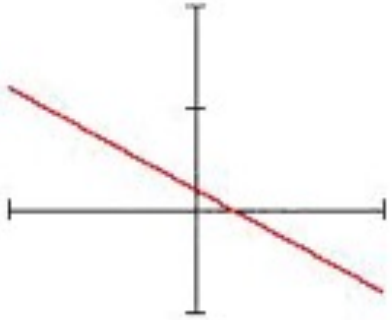
Graphs of Linear Equations in Two Variables (2 of 4)

Graphs of equations in two variables often give us a better understanding of some of the characteristics of the equation. All equations in two variables have their own unique graphs.

The graphs of linear equations in two variables are straight lines. These lines can be ”increasing” or “decreasing” as long as A and B in $Ax + By + C = 0$ are not 0.



Increasing Line



Decreasing Line

Graphs of Linear Equations in Two Variables (3 of 4)

The graph of any equation in two variables, say ***x*** and ***y***, can have ***x*-intercepts** and ***y*-intercepts**.

The ***x*-intercept** is the *x*-coordinate of a point where the graph intersects with the *x*-axis. The *y*-coordinate of this point is always 0.

The ***y*-intercept** is the *y*-coordinate of a point where the graph intersects with the *y*-axis. The *x*-coordinate of this point is always 0.

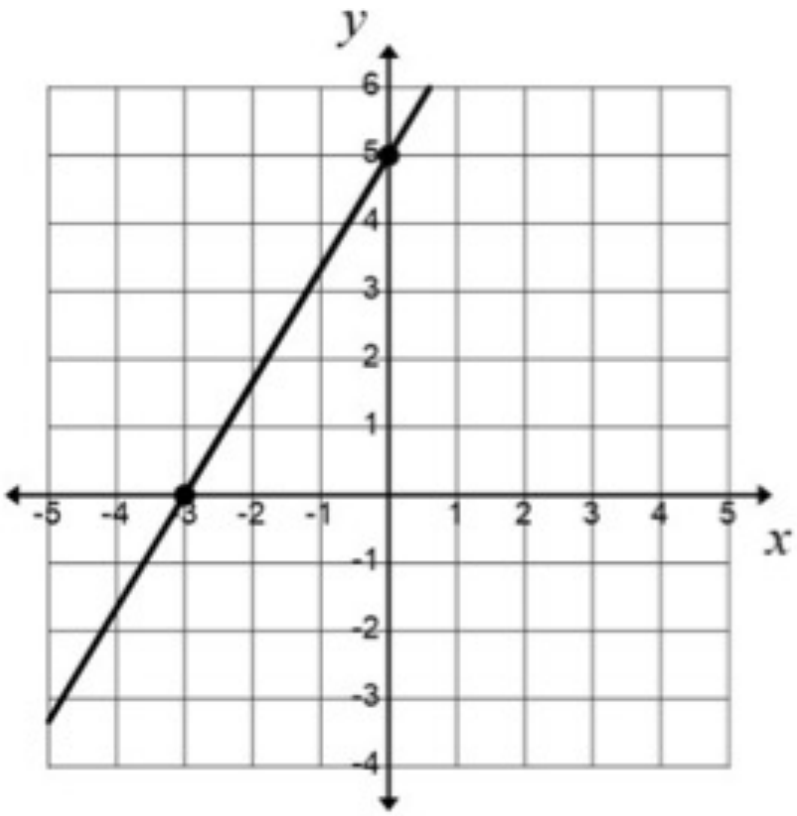
Graphs of Linear Equations in Two Variables (4 of 4)

Example 2:

Given the graph of some linear equation $Ax + By + C = 0$, find the x- and y-intercepts.

We observe that the line crosses the x-axis at the point determined by the ordered pair $(-3, 0)$. On the other hand, the line crosses the y-axis at the point determined by the ordered pair $(0, 5)$.

In this case, the x-intercept is -3 and the y-intercept is 5.



Note that we labeled the horizontal axis with an x and the vertical axis with a y because the given equation contains the variables x and y.

4. Methods for Graphing Equations in Two Variables by Hand

(1 of 6)

When we graph equations in two variables by hand, we either use the *Point-by-Point Plotting Method* or the *Intercept Method* to help us get started.

The Point-by-Point Plotting Method

Step 1 – Pick a "sufficient" number of "appropriate" values for the independent variable. ”Sufficient" and "appropriate" depends on certain characteristics of an equation in two variables.

Example 3:

Graph the linear equation $3x + y = -6$ by hand using the *Point-by-Point Plotting Method*. Assume x is the independent variable.

“Appropriate” and ”sufficient” values for x **given any linear equation** are 1-2 integers to the right and left of 0 as well as 0.

Let’s pick $x = -4, -2, 0, 1$

Methods for Graphing Equations in Two Variables by Hand
(2 of 6)

Step 2 - Use the values for the independent variable from Step 1 and calculate corresponding values for the dependent variable using the given equation in two variables. Then create ordered pairs.

Example 3 continued with the linear equation $3x + y = -6$:

Let $x = -4$, then $3(-4) + y = -6$ and $y = 6$
Let $x = -2$, then $3(-2) + y = -6$ and $y = 0$
Let $x = 0$, then $3(0) + y = -6$ and $y = -6$
Let $x = 1$, then $3(1) + y = -6$ and $y = -9$

Each x-value created the following ordered pairs:

$(-4, 6)$ and $(-2, 0)$ and $(0, -6)$ and $(1, -9)$

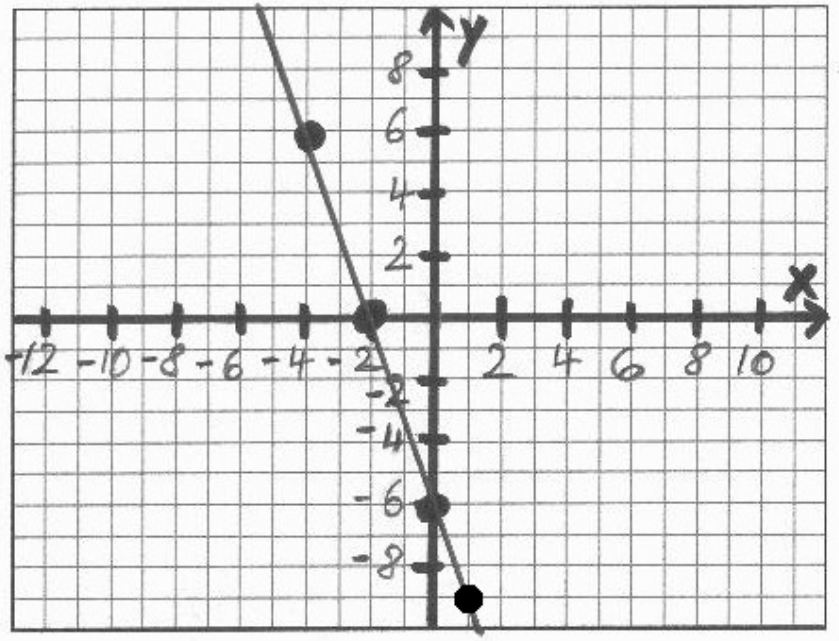
Methods for Graphing Equations in Two Variables by Hand

(3 of 6)

Step 3 – Plot the ordered pairs from Step 2 in the *Rectangular Coordinate System* and connect them keeping in mind the general shape of the graph of the given equation in two variables.

Example 3 continued with the linear equation $3x + y = -6$:

Since we are dealing with a linear equation whose unique graph is a straight line, we will plot the ordered pairs from Step 2 and connect them with a straight line.



Note that we labeled the horizontal axis with an *x* and the vertical axis with a *y* because the given equation contains the variables *x* and *y*.

Methods for Graphing Equations in Two Variables by Hand

(4 of 6)

The Intercept Method

NOTE: Sometimes the *Intercept Method* will not give us two distinct ordered pairs. When this happens, we must use the *Point-by-Point Plotting Method* to find more ordered pairs.

Step 1 – Find the *x*-intercept and the ordered pair associated with it. Given an equation in two variables, **let the dependent variable equal 0** and solve for the independent variable.

Example 4:

Graph the linear equation $3x + y = -6$ by hand using the *Intercept Method*. Assume *x* is the independent variable!

Let $y = 0$ and solve for *x*. Then $3x - 0 = -6$ and $x = -2$. This is the *x*-intercept! The ordered pair associated with the *x*-intercept is $(-2, 0)$.

Methods for Graphing Equations in Two Variables by Hand (5 of 6)

Step 2 – Find the *y*-intercept and the ordered pair associated with it. Given an equation in two variables, **let the independent variable equal 0** and solve for the dependent variable.

Example 4 continued with the linear equation $3x + y = -6$:

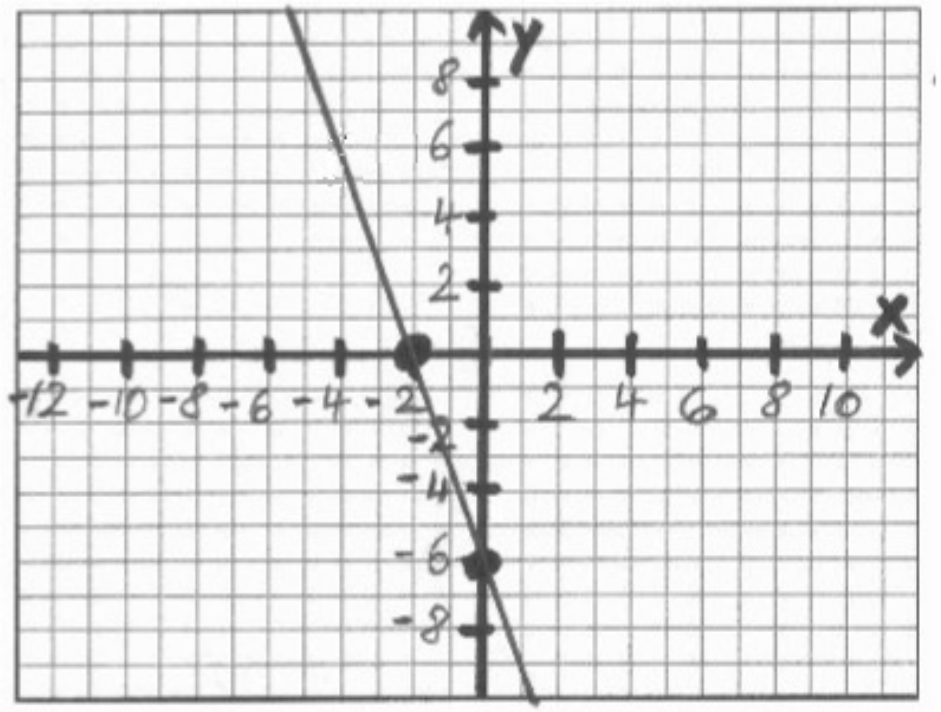
Let $x = 0$ and solve for y. Then $3(0) + y = -6$ and $y = -6$. This is the *y*-intercept. The ordered pair associated with the *y*-intercept is $(0, -6)$.

Step 3 – Plot the ordered pairs from Steps 1 and 2 in the *Rectangular Coordinate System* and connect them keeping in mind the general shape of the graph of a given equation in two variables.

Methods for Graphing Equations in Two Variables by Hand (6 of 6)

Example 4 continued with the linear equation $3x + y = -6$:

Since we are dealing with a linear equation whose unique picture is a straight line, we will plot the ordered pairs from Step 2 and connect them with a straight line.



Note that we labeled the horizontal axis with an *x* and the vertical axis with a *y* because the given equation contains the variables *x* and *y*.