Concepts
Linear Equations in Two Variables

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Memorize the definition of independent and dependent variables.
2. Memorize the general form of linear equations in two variables.
3. Memorize the characteristics of the graphs of linear equations in two variables.
4. Graph linear equations in two variables by hand in the rectangular coordinate system using the *Point-by-Point Plotting Method* and the *Intercept Method*.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

1. Equations in Two Variables (1 of 2)

Up to this point, we have only been exposed to equations in one variable, for example, $\mathbf{6x + 3 = 0}$.

Now, we will discuss equations in which two variables are used, for example, $\mathbf{6x + 3y = 0}$.

> NOTE: In mathematics we usually use the variables ***x*** and ***y***. We then call ***x*** the **independent variable** and ***y*** the **dependent variable**.

We usually (there are some exceptions) assign real numbers to the independent variable. The value of the dependent variable depends on the number we pick for the independent variable.

Equations in Two Variables (2 of 2)

Example 1:

Given $6x + 3y = 0$, let the independent variable x equal 1 and find the value of the dependent variable y.

Given $x = 1$, we calculate $6(1) + 3y = 0$

then $6 + 3y = 0$. This is a linear equation in one variable.

Using the Subtraction Axiom, we get

$3y = -6$

and using the Division Axiom we end up with $y = -2$.

The value of y is -2. It **depends** on our pick of $x = 1$!

2. Graphs of Equations in Two Variables (1 of 4)

We can make pictures called “graphs” of any equation in two variables. We often refer to this as ”graphing.” We do this with the help of the *Rectangular Coordinate System* in which the independent variable is assigned to the horizontal axis and the dependent variable to the vertical axis. We then label the axes appropriately.

For example, given **6*x* + 3*y* = 0** and letting ***x*** be the independent variable, we would call the horizontal axis in a *Rectangular Coordinate System* the ***x*-axis** and the vertical axis the ***y*-axis**. Any ordered pair in this coordinate system would be of the form **(*x*, *y*)** where ***x*** is usually called the ***x*-coordinate** and ***y*** the ***y*-coordinate**.

Graphs of equations in two variables often give us a better understanding of some of the characteristics of the equation. All equations in two variables have their own unique graphs.

Graphs of Equations in Two Variables (2 of 4)

The graph of any equation in two variables, say ***x*** and ***y***, can have ***x*-intercepts** and ***y*-intercepts**.

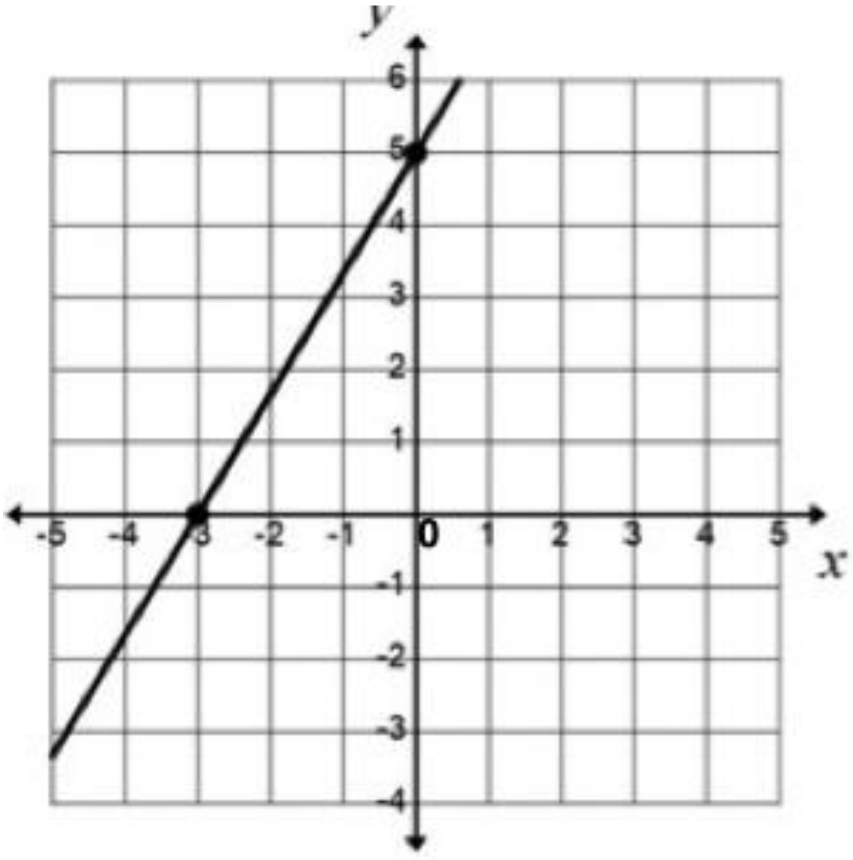
The ***x*-intercept** is the *x*-coordinate of a point where the graph intersects with the *x*-axis. The *y*-coordinate of this point is always 0.

The ***y*-intercept** is the *y*-coordinate of a point where the graph intersects with the *y*-axis. The *x*-coordinate of this point is always 0.

Graphs of Equations in Two Variables (3 of 4)

Example 2:

Examine the graph of an equation in two variables, say *x* and *y*. Find its *x*-intercept and its *y*-intercept.



We observe that the graph crosses the *x*-axis at the point determined by the ordered pair (− 3, 0). On the other hand, it crosses the *y*-axis at the point determined by the ordered pair (0, 5).

In this case, the *x*-intercept is − 3 and the *y*-intercept is 5.

Graphs of Equations in Two Variables (4 of 4)

We try to graph equations in two variables by hand with the help of the **Point-by-Point Plotting Method** or the **Intercept Method** or a combination of both. Later on, we will learn other method specific to certain equations in two variables.

When using the **Point-by-Point Plotting Method**, we create a "sufficient" number of ordered pairs by picking values for the independent variable. Using them and the given equation, we then calculate corresponding values for the dependent variable. Note: "Sufficient" depends on the type of equation we are working with.

When using the **Intercept Method**, we find the ordered pairs associated with the *x*-intercepts and the *y*-intercepts.

NOTE: Sometimes the *Intercept Method* will not give use enough ordered pairs to create a representative graph. When this happens, we must use the *Point-by-Point Plotting Method* to find more ordered pairs.

3. Linear Equations in Two Variables and their Graphs (1 of 3)

The **general form** of a linear equation in TWO variables, say ***x*** and ***y***, is

> $\boldsymbol{ax + by + C = 0}$, where x it the independent variable and ***a***, ***b***, and ***c*** are real numbers, but ***a*** and ***b*** are never 0.

Incidentally, linear equations in two variables do not necessarily have to appear in *general form*. Mathematics just likes to define them that way!

Linear Equations in Two Variables and their Graphs

Examples of linear equations in two variables:

$2x + 5y + 6 = 0$ (general form with $a = 2$, $b = 5$, and $c = 6$)

$-18x + y + 11 = 0$ (general form with $a = -18$, $b = 1$, and $c = 11$)

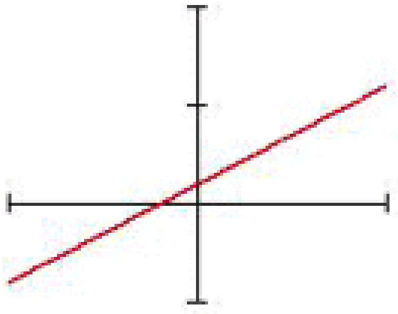
$9x + y = 0$ (general form with $a = 9$, $b = 1$, and $c = 0$)

$x + y + 1 = 0$ (general form where $a = 1$, $b = 1$, and $c = 1$)

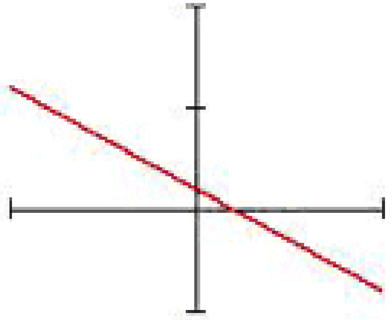
$-4x - 6y - 7 = 0$ (this is still considered general form) because the equation can be written as $-4x + (-6y) + (-7) = 0$. Therefore, $a = -4$, $b = -6$, and $c = -7$.

Linear Equations in Two Variables and their Graphs (3 of 3)

The graphs of linear equations in two variables are straight lines. These lines can be ”increasing” or “decreasing” as long as *a* and *b* in $ax + by + c = 0$ are not 0.



Increasing Line



Decreasing Line

4. Graphing Linear Equations in Two Variables by Hand (1 of 6)

When we graph linear equations in two variables by hand, we either use the *Point-by-Point Plotting Method* or the *Intercept Method* or a combination of both, to help us get started.

The Point-by-Point Plotting Method Given a Linear Equation:

Step 1 – Given a linear equation in x and y, where x is the independent variable, pick at least two reasonable integers to the right and left of 0 for the independent variable. Use these values and calculate corresponding values for the dependent variable using the given equation. Then create ordered pairs.

Note: Be sure to pick integer values for the independent variable that also create integer values for the dependent variable! This way we do not have to plot fractions.

Graphing Linear Equations in Two Variables by Hand

Example 3:

Graph the linear equation $3x + y + 6 = 0$ by hand using the *Point-by-Point Plotting Method*. Assume x is the independent variable and y the dependent variable.

Let's pick $x = -4, -2, 1$, and 2.

Let $x = -4$, then $3(-4) + y + 6 = 0$ and $y = 6$
Let $x = -2$, then $3(-2) + y + 6 = 0$ and $y = 0$
Let $x = 1$, then $3(1) + y + 6 = 0$ and $y = -9$
Let $x = 2$, then $3(2) + y + 6 = 0$ and $y = -12$

Each x-value created the following ordered pairs:

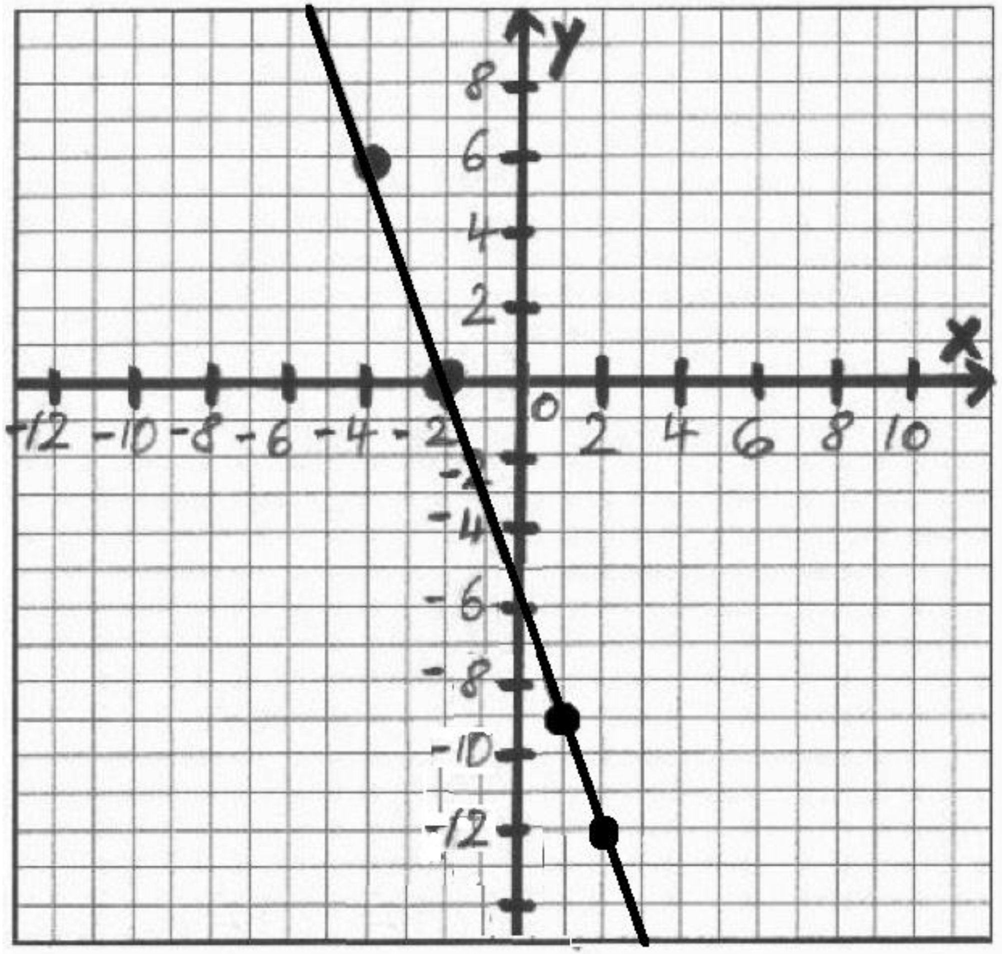
$(-4, 6)$ and $(-2, 0)$ and $(1, -9)$ and $(2, -12)$

Graphing Linear Equations in Two Variables by Hand (3 of 6)

Step 2 – Plot the ordered pairs from Step 2 in the *Rectangular Coordinate System* and connect them keeping in mind the shape of the graph of a linear equation in two variables.

Example 3 continued with the linear equation $3x + y = -6$:

Since we are dealing with a linear equation whose unique graph is a straight line, we will plot the ordered pairs from Step 2 and connect them with a straight line.



Graphing Linear Equations in Two Variables by Hand (4 of 6)

The Intercept Method Given a Linear Equation:

Step 1 – Given a linear equation in x and y, where x is the independent variable, **find the y-intercept** and the ordered pair associated with it. We do this by **letting the independent variable equal 0** and solving for the dependent variable.

Example 4:

Graph the linear equation $3x + y + 6 = 0$ by hand using the *Intercept Method*. Assume x is the independent variable and y the dependent variable!

Let $x = 0$ and solve for y. Then $3(0) + y + 6 = 0$ and $y = -6$. This is the y-intercept. The ordered pair associated with the y-intercept is $(0, -6)$.

Graphing Linear Equations in Two Variables by Hand

Step 2 – Given a linear equation in *x* and *y*, where *x* is the independent variable, **find the *x*-intercept** and the ordered pair associated with it. We do this by **letting the dependent variable equal 0** and solving for the independent variable.

Example 4 continued:

Let $y = 0$ and solve for x. Then $3x + 0 + 6 = 0$ and $x = -2$. This is the *x*-intercept! The ordered pair associated with the *x*-intercept is $(-2, 0)$.

Graphing Linear Equations in Two Variables by Hand (6 of 6)

Step 3 – Plot the ordered pairs from Steps 1 and 2 in the *Rectangular Coordinate System* and connect them keeping in mind the shape of the graph of a linear equation in two variables.

Example 4 continued with the linear equation $3x + y + 6 = 0$:

Since we are dealing with a linear equation whose unique picture is a straight line, we will plot the ordered pairs from Steps 1 and 2 and connect them with a straight line.