Concepts
Introduction to Algebra
Linear Equations in One Variable

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Learning Objectives

1. Memorize and use vocabulary of algebra.
2. Memorize and use the *Distributive Property*.
3. Memorize the basic principles of equations.
4. Memorize the definition of linear equations.
5. Know how to solve linear equations involving integers and decimals.
6. Know how to solve linear equations containing rational numbers a/b.
7. Memorize and use the cross-multiplication principle.

What is Algebra? (1 of 2)

Just like arithmetic, algebra is another important branch of mathematics. The word is derived from the Arabic word” al-jabr”, which is an ancient medical term meaning the “reunion of broken parts”.

Unlike arithmetic, algebra deals with unknown quantities in combination with numbers and many different mathematical operations.

1. Some Vocabulary of Algebra (1 of 4)

Variable: It is a letter of the alphabet that takes on different values in different situations. In mathematics, we most often use the letters ***x*** and ***y***. However, we can use any other letter.

Expression: A combination of variables and numbers linked by addition, subtraction, multiplication, and division.

Example: $7x + 9y - 8$

Please note that there is an implied multiplication sign between the numbers and the variables. $7x$ actually means $7 \cdot x$ and $9y$ means $9 \cdot y$. The multiplication sign is usually not written.

Equation: An expression that is equal to another expression. In mathematics we use the = (equal) symbol.

Example: $7x + 9y - 8 = 5$

Some Vocabulary of Algebra (2 of 4)

Evaluating an Expression: Find the value of the expression when the variables are replaced with numbers.

Example: Find the value of the expression $7x + 9y - 8$ when $x = 1$ and $y = 2$.

$$7(1) + 9(2) - 8 = 7 + 18 - 8 = 17$$

Please note that there is an implied multiplication sign between the numbers and the variables. When a variable is replaced by a number, we use parentheses ( ) to indicate multiplication.

Term: Those parts of an expression separated by addition or subtraction.

Example: In $7x + 9y - 8$, the terms are $7x$, $9y$, and -8

Constant: A term that consists of just a number.

Example: In $7x + 9y - 8$, the constant is -8

Some Vocabulary of Algebra (3 of 4)

Coefficient: The numerical part of a term (excluding the constant).

Example: In $7x + 9y - 8$, the coefficients are 7 and 9

Factor: A factor is a number that divides into another number evenly without leaving a remainder.

Example: 5 is a factor of 10 because $10 \div 5 = 2$ Remainder 0. On the other hand, 6 is not a factor of 10 because $10 \div 6 = 1$ Remainder 4.

Like Term: Terms having the exact same variable factors.

Example: In $5x - 3y + 2x$, the like terms are $5x$ and $2x$

Combining Like Term: To combine like terms, we add/subtract the coefficients of like terms.

Example: In $5x - 3y + 2x$, we get $(5 + 2)x - 3y$ or $7x - 3y$

Some Vocabulary of Algebra (4 of 4)

Conjugate – Given an expression with exactly two terms being added or subtracted, its conjugate is an expression with the same terms but the plus or minus in the middle of these terms is changed to the opposite one.

Example: Given $-5x - 9$, its conjugate is $-5x + 9$.

Simplify: The word "simplify" takes on many meanings in mathematics. The word could mean to add and subtract; to multiply and divide; to combine like terms; etc. We must figure out its meaning from the make-up of the given mathematical expression.

Example: "simplify $2x + 3x$" means the same as "add $2x + 3x$".

2. The Distributive Property (Memorize!)

The *Distributive Property* states the following:

Given any terms a, b, and c, then $a(b + c) = ab + ac$.

Please note that there is an assumed multiplication sign between ***a*** and the parentheses ( ) as well as between ***a*** and ***b*** and ***a*** and ***c***. Incidentally, the singular is one parenthesis!

Example:

Utilize the *Distributive Property* given $3(2x + 5)$.

$3(2x + 5) = 3(2x) + 3(5)$

Please note that under multiplication, say $3(2x)$, we multiply constants/coefficients with constants/coefficients and variables with variables. Therefore, when we carry out the multiplication $3(2x)$ as follows $3 \cdot 2 \cdot x = 6x$.

We find that $3(2x + 5)$ equals $6x + 15$.

3. Basic Principles of Equations

Any equation can be transformed into an equivalent equation by using one or more of the following axioms (statements that are self-evident and require no proof):

Addition Axiom - Add the same quantity to **BOTH** sides of the equal sign.

Subtraction Axiom - Subtract the same quantity from **BOTH** sides of the equal sign.

Multiplication Axiom – Multiply **BOTH** sides of the equal sign by the same **nonzero** quantity.

Division Axiom – Divide **BOTH** sides of the equal sign by the same **nonzero** quantity.

4. Definition of a Linear Equation in One Variable (Memorize)

Given the real numbers $\boldsymbol{A}$ and $\boldsymbol{B}$ with $\boldsymbol{A \neq 0}$, the *general form* of a linear equation in one variable, say x, is

$\boldsymbol{Ax + B = 0}$

Examples of Linear Equations in One Variable in General Form:

$5x + 9 = 0$ where $A = 5$ and $B = 9$

$3x = 0$ where $A = 3$ and $B = 0$

$-x - 7 = 0$ where $A = -1$ and $B = -7$

Please note that $-x$ means $-1x$ or $-1 \cdot x$, but in mathematics a coefficient of 1 is usually not stated!

5. Solve Linear Equations involving Integers or Decimals

When we are asked to "solve" a linear equation, say in *x,* we are actually asked to determine all values of *x* that result in a true statement when substituted into the equation. Such values are called **solutions**.

Solution Strategy:

1. Simplify the algebraic expression on each side of the equal sign by removing grouping symbols and combining like terms.
2. Using the Addition, Subtraction, Multiplication, and/or Division Axioms (often several times) to completely isolate the variable on one side of the equal sign. Its coefficient MUST be 1. It does not matter on which side you isolate it, right or left. Although, in mathematics the left side is preferred. This is the proposed solution of the linear equation.
3. Check the proposed solution in the original equation. If it results in a true statement, then the proposed solution is an actual solution.

6. Solve Linear Equations Involving Rational Numbers a/b

Easiest Strategy:

- Eliminate all fractions by multiplying **BOTH** sides of the equation (preferably) by the smallest number that all denominators divide into evenly.
- Then proceed with the strategy for solving linear equations involving integers or decimals.

7. Cross-Multiplication Principle (Memorize!)

If two numbers $\frac{a}{b}$ and $\frac{c}{d}$ are equal, that is $\frac{a}{b} = \frac{c}{d}$, then ***a·d = b·c*** where $b \neq 0$ and $d \neq 0$. We call ***a·d*** and ***b·c*** the cross products.

Please note that this cross-multiplication principle does not have to be used, but it often makes the calculations easier.

We could simply eliminate all denominators by multiplying both sides of the equation by the smallest number that all denominators divide into evenly.