Concepts
Inverses of Functions

Based on power point presentations by Pearson Education, Inc.
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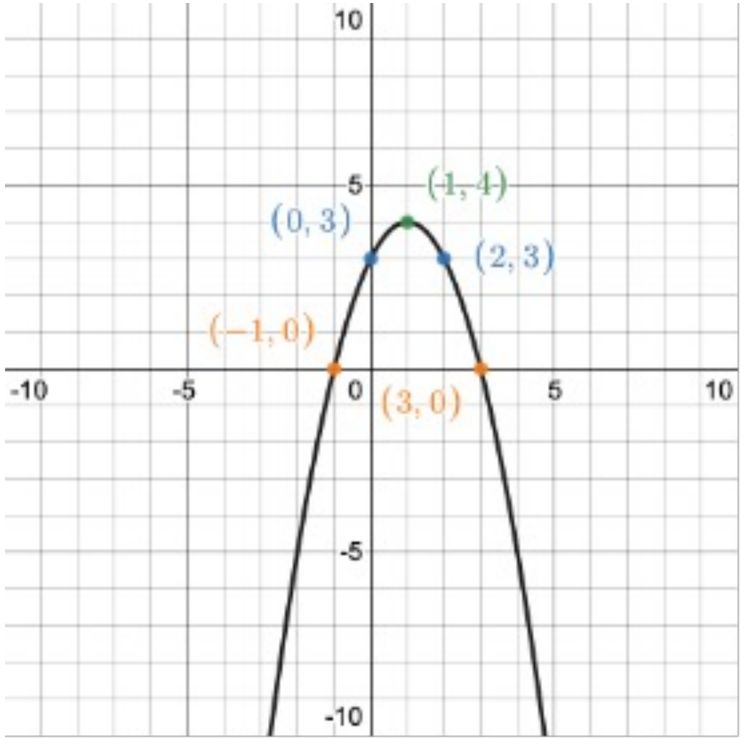
Learning Objectives

1. Define one-to-one functions.
2. Find inverse functions.
3. Verify inverses.

1. Definition of One-to-One Functions (1 of 2)

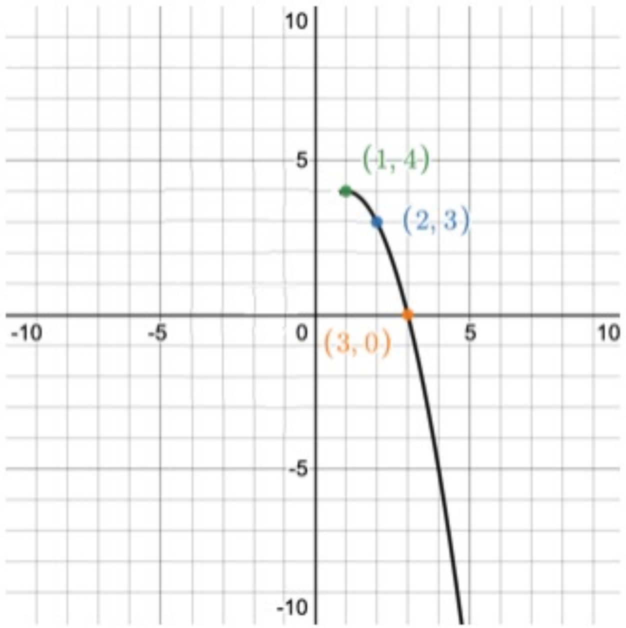
One-to-one function are special function where the range values never repeat.

For instance, $f(x) = -(x-1)^2 + 4$, whose domain consists of *all real numbers,* is a function, but it is NOT one-to-one. For example, both f(0) and f(2) equal 3. The range value 3 repeats! Let's look at its graph.



Definition of One-to-One Functions (2 of 2)

Now, if we restrict the domain of $f(x) = -(x-1)^2 + 4$ to say $x \geq 1$ then we can say the function is one-to-one! Let's look at its graph.



We can make any function one-to-one by using domain restrictions.

2. Find the Inverse Function of a Given Function (1 of 2)

Only one-to-one functions have inverse functions!

The inverse function of a one-to-one function, say in x and y, is found by exchanging the x and y variables.

The final step involves solving the newly found inverse function so that y is isolated on one side of the equal sign.

During this exchange, the domain of a given function becomes the range of its inverse function. Likewise, the range of a given function becomes the domain of its inverse function.

Find the Inverse Function of a Given Function (2 of 2)

Optional Inverse Function Notation

Sometimes the following notation is used to refer to the inverse function of a function *f:*

$$f^{-1}$$

which is read as ”*f inverse*”

NOTE: f^{-1} does not mean $\frac{1}{f}$.

3. Verify Inverse Functions

A function *f* and its inverse *g* satisfy the following:

$(f \circ g)(x) = x$ function composition!

and

$(g \circ f)(x) = x$ function composition!

Please note that the results of $(f \circ g)(x)$ and $(g \circ f)(x)$ are equal. This only happens when the two functions are inverses!