# Concepts Introduction to Algebra

Based on power point presentations by Pearson Education, Inc. Revised by Ingrid Stewart, Ph.D.

## Learning Objectives

#### 1. Use the vocabulary of algebra.

#### 2. Use the Distributive Property.

# NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

#### What is Algebra?

Just like arithmetic, **algebra** is another important branch of mathematics. Arithmetic investigates how numbers are combined and transformed using mathematical operations like addition, subtraction, multiplication, and division.

Algebra relies on the same operations while allowing "unknown" numbers. We call these "unknown" numbers **variables**. They consist of letters of the alphabet that take on different values in different situations. In mathematics, we most often use the letters *x* and *y*. However, we can use any other letter.

For example, in arithmetic we might state 7 + 2 = 9. In algebra, on the other hand, we might state x + 2 = 9. (It should be quite obvious that the variable x equals 7.)

### Why Study Algebra?

Algebra is an essential tool that has real-life uses and plays a significant role in different fields such as science, engineering, economics, and statistics. Furthermore, by learning basic algebra, we develop critical thinking and problem-solving skills as well as logical reasoning. All three skills are essential for success in any field and for making informed decisions in everyday life.

#### 1. Some Vocabulary of Algebra (1 of 7)

**Mathematical Expression:** A combination of variables and/or numbers linked by addition, subtraction, multiplication, and division.

For example, 7x + 2y + 8.

NOTE: There is an implied multiplication sign between the numbers and the variables. 7x actually means  $7 \cdot x$  and 2y means  $2 \cdot y$ . The multiplication sign is usually not written.

**Equation:** A mathematical expression that is equal to another mathematical expression. In algebra we use the = (equal) sign.

For example, 7x + 2y + 8 = 5x - 3.

#### Some Vocabulary of Algebra (2 of 7)

**Term:** Those parts of a mathematical expression separated by addition or subtraction.

For example, in 7x + 2y + 8, the terms are 7x, 2y, and 8.

**Like Terms:** Terms that have the exact same variables. For example, in 5x - 3y + 2x, the like terms are 5x and 2x.

**Constant:** A term that consists of just a number. For example, in 7x + 2y + 8, the constant is 8.

**Coefficient:** Any number in front of a variable.

For example, in 3x + y + 9, the coefficients are 3 and 1. NOTE: In mathematics, we usually do not write a coefficient of 1. That is, when

we don't see a coefficient, it is assumed to be 1!

#### Some Vocabulary of Algebra (3 of 7)

**Combining Like Terms:** The act of simplifying a mathematical expression by adding/subtracting any constants and the coefficients of any like terms.

For example, given 5x - 3y + 2x with like terms 5x and 2x, we add their coefficients 5 and 2 to get 7x. We end up with 7x - 3y. Note that we cannot subtract 3 from 7 because they are not coefficients of like terms.

**Evaluating a Mathematical Expression:** Find the value of the expression when the variables are replaced with numbers.

#### Some Vocabulary of Algebra (4 of 7)

Example 1:

Find the value of the expression 7x + 9y + 8 when x = 1 and y = 2.

7(1) + 9(2) + 8 then 7 + 18 + 8 and the value of this is 33.

NOTE: There is implied multiplication between the numbers and the variables. When a variable is replaced by a number, we use the grouping symbols called parentheses () to indicate that multiplication must be done.

#### Some Vocabulary of Algebra (5 of 7)

**Conjugate:** Given an expression with exactly two terms being added or subtracted, then its conjugate is an expression with the same terms but the operation changes to the opposite one.

For example, given -5x - 9, its conjugate is -5x + 9.

**Simplify:** The word "simplify" takes on many meanings in mathematics. The word could mean to add, subtract, multiply, divide, etc. Usually, we must figure out its meaning from the make-up of the given mathematical expression.

For example, "simplify 2 + 3" means the same as "add 2 + 3".

#### Some Vocabulary of Algebra (6 of 7)

Example 2:

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Simplify xy + 3 + 2x + 6xy - 3x + 9y - 6.
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Here "simplify" means to add and subtract. However, we can only add/subtract like terms. Let's give the individual like terms different colors.

$$xy + 3 + 2x + 6xy - 3x + 9y - 6$$

Notice that we can combine *xy*-terms and *x*-terms by adding/subtracting their coefficients. We can also add/subtract constants.

Let's write this as (1 + 6)xy + (2 - 3)x + 9y + 3 - 6NOTE: There is an implied coefficient of 1 in front of xy!

#### Some Vocabulary of Algebra (7 of 7)

Example 2 continued:

Now we must add some integers. Namely, (2 – 3) for the *x*-terms and + 3 – 6 for the constants. We will use the "gambling gains/losses" method. Here, in both cases, our "gains" (+) are less than our "losses" (–). Therefore, we still end up with losses.

That is, 2 - 3 = -1 and 3 - 6 = -3

In summary, we find that (1 + 6)xy + (2 - 3)x + 9y + 3 - 6 can be simplified to

7xy - x + 9y - 3.

NOTE: In mathematics, we usually do not write a coefficient of 1. This is why the second term is -x and not -1x !!!

#### 2. The Distributive Property (1 of 3)

Say, you are asked to evaluate 3(5 + 2x). How would you go about doing this? NOTE: There is implied multiplication between 3 and the parentheses ().

For this calculation, mathematicians use the **Distributive Property**. It directs how to multiply a sum or difference of two or more numbers within parentheses by a number outside the parentheses.

Specifically, it states that we must multiply each number in the parentheses individually by the number outside the parentheses and then add the products. For example, 3(5 + 2x) = 3(5) + 3(2x).

NOTE: There is also an implied multiplication between 3(5) and 3(2x).

The Distributive Property (2 of 3)

Example 3:

Evaluate 3(5 + 2*x*).

Using the Distributive Property, we get 3(5) + 3(2x).

It is obvious that 3(5) equals 15. But what about 3(2x)?

The multiplication rule is that we multiply numbers with numbers and variables with variables. That is,  $3(2x) = (3 \cdot 2) \cdot x = 6 \cdot x = 6x$ .

In summary, we find 3(5 + 2x) can be simplified to 15 + 6x.

#### The Distributive Property (3 of 3)

Example 4:

Simplify -2(-4 - 7x + 9y).

We are going to use the *Distributive Property* to simplify. Please note that now we have three terms in the parentheses. However, this will not be an issue.

According the the *Distributive Property*, we simply multiply the number on the outside of the parentheses with every term in the parentheses.

$$-2(-4 - 7x + 9y) = -2(-4) + (-2)(-7x) + (-2)(9y)$$
$$= 8 + 14x + (-18y)$$
$$= 8 + 14x - 18y$$