Concepts
Operations on Functions

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Add, subtract, multiply and divide functions.
2. Compose functions.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

1. Add, Subtract, Multiply, and Divide Functions (1 of 3)

Let **f** and ***g*** be two functions. Their sum, difference, product, and quotient are also functions where the domains are the intersection of the domain of **f** with the domain of ***g***.

Sum: $(f + g)$

Difference: $(f - g)$ or $(g - f)$

Product: $(f \cdot g)$ or $(f\,g)$

Quotient: $\left(\frac{f}{g}\right)$ or $\left(\frac{g}{f}\right)$ The denominator cannot be equal to 0!

Add, Subtract, Multiply, and Divide Functions (2 of 3)

Example 1:

Let $f(x) = -2x$ and $g(x) = 5x + 3$. Find the functions $f + g$, $f - g$, fg, and $\frac{f}{g}$.

$$(f + g)(x) = (-2x) + (5x + 3)$$
$$= -2x + 5x + 3$$
$$= 3x + 3$$

$$(f - g)(x) = (-2x) - (5x + 3)$$
$$= -2x - 5x - 3$$
$$= -7x - 3$$

Add, Subtract, Multiply, and Divide Functions (3 of 3)

Example 1 continued:

$(f\,g)(x) = (-2x)(5x + 3) = -10x^2 - 6x$ (here we used the *Distributive Property* and the *power rule of exponents*!)

$$\left(\frac{f}{g}\right)(x) = \frac{-2x}{5x+3}$$

where the denominator $5x + 3$ cannot be equal to 0!

2. The Composition of Functions (1 of 4)

Given the functions **f** and ***g***, we can form their composites in two different ways. We can find the composition of the function **f** with ***g*** and the composition of the function ***g*** with **f**.

The notation for the composition of the function **f** with ***g*** is **f** ∘ ***g***. We read this as "f composed of *g*".

The notation for the composition of the function ***g*** with **f** is ***g*** ∘ **f**. We read this as "*g* composed of f".

NOTE: The symbol for function composition is a circle ∘ and NOT A DOT! Please do not confuse it with a multiplication symbol!

The Composition of Functions (2 of 4)

The notation $\mathbf{f} \circ \boldsymbol{g}$ means $\mathbf{f}\,(\,\boldsymbol{g}(\boldsymbol{x})\,)$ which is read as "f of *g* of *x*".

We find $\mathbf{f}\,(\,\boldsymbol{g}(\boldsymbol{x})\,)$ by replacing every occurrence of *x* in the function **f** with the function ***g***.

The notation $\boldsymbol{g} \circ \mathbf{f}$ means $\boldsymbol{g}\,(\,\mathbf{f}(\boldsymbol{x})\,)$ which is read as "*g* of f of *x*".

We find $\boldsymbol{g}\,(\,\mathbf{f}(\boldsymbol{x})\,)$ by replacing every occurrence of *x* in the function ***g*** with the function **f**.

Please note that the results of $\mathbf{f} \circ \boldsymbol{g}$ and $\boldsymbol{g} \circ \mathbf{f}$ are often NOT equal, but they can be.

The Composition of Functions (3 of 4)

Example 2:

Given $f(x) = x - 1$ and $g(x) = 4x - 3$, find $\mathbf{f} \circ \boldsymbol{g}$ and $\boldsymbol{g} \circ \mathbf{f}$.

We find $\mathbf{f} \circ \boldsymbol{g}$ by replacing every occurrence of x in the function **f** with the function ***g*** as follows:

$$
\begin{aligned}
(f \circ g)(x) &= f(\,g(x)\,) = f(4x - 3) \\
&= (4x - 3) - 1 \\
&= 4x - 4
\end{aligned}
$$

This is the function f where x was replaced by $4x - 3$.

The Composition of Functions (4 of 4)

Example 2 continued:

We find $\boldsymbol{g} \circ \mathbf{f}$ by replacing every occurrence of x in the function $\boldsymbol{g}$ with the function $\mathbf{f}$ as follows:

$$(g \circ f)(x) = g(\,\mathrm{f}(x)\,) = g(x - 1)$$

$$= 4(x - 1) - 3$$ This is the function g where x was replaced by $x - 1$.

$$= 4x - 4 - 3$$

$$= 4x - 7$$

Please note that the results of $\mathbf{f} \circ \boldsymbol{g}$ and $\boldsymbol{g} \circ \mathbf{f}$ are not the same, but they can be!