Concepts
Exponential Functions

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Memorize the definition of the common exponential function.
2. Memorize characteristics of the graphs of exponential functions.
3. Apply transformations to the common exponential function.
4. Graph the common exponential function and its transformations by hand.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

1. The Common Exponential Function (1 of 3)

The common exponential function is defined by $\boldsymbol{f(x) = b^x}$,

where b is called *base* and consists of any positive number except 1.

Domain: All real numbers or $(-\infty, \infty)$

Examples:

$p(x) = 2^x$

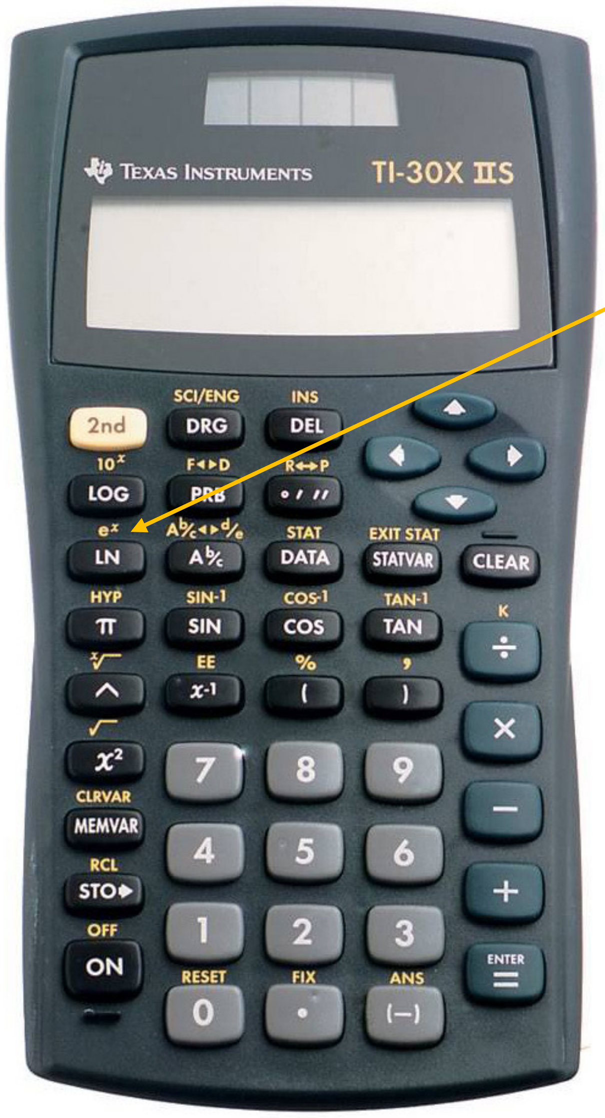
$k(x) = 5^x$

$g(x) = \boldsymbol{e}^x$

Special case of the common exponential function. The irrational number ***e***, approximately equal to ***2.72***, is called the natural base.

NOTE: In algebra, we never use 2.72 to represent the number ***e***. Instead, we use the calculator value exclusively.

The Basic Exponential Function (2 of 3)



Examining any calculator, we find that there is NO button containing the number ***e***. However, we can find the picture ***e***^{x} over the **LN** button. This means that we must use the 2nd button to access the number. To find the value of ***e***, we input the following:

2nd	LN	1	)	Enter

We find that ***e*** ≈ ***2.718281828***.
Note that we used the power of 1 to find the value of the number ***e***.

The calculator does not tell us that ***e*** is an irrational number. It simply fills up all available slots on its screen with decimal places. YOU must know that it is an irrational number with infinitely many decimal places.

The Basic Exponential Function (3 of 3)

Where does the number *e* come from?

The number ***e*** occurs quite frequently in the sciences and banking. It is an irrational number often rounded to *2.72*.

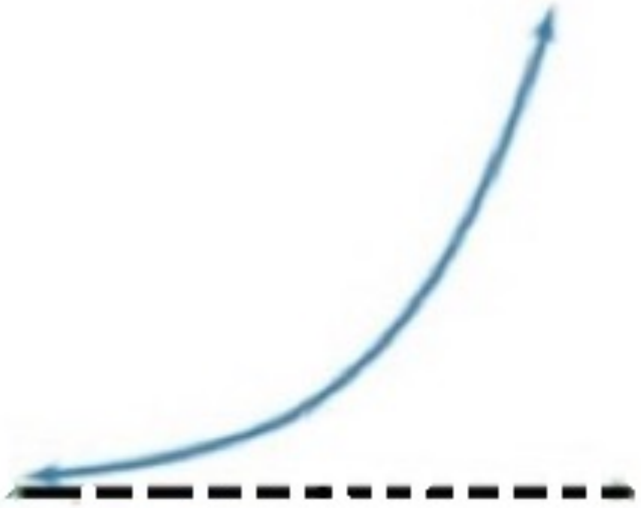
The number was discovered by the Swiss mathematician Jacob Bernoulli in 1683 while studying compound interest.

He wanted to know that happens to the expression $\left(1+\frac{1}{n}\right)^n$ when ***n*** gets infinitely large. He found that its value never becomes larger than 2.718281828... .

The the first appearance of ***e*** can be found in 1736 in the publication "Mechanica" written by the Swiss mathematician Leonard Euler. The number ***e*** is also known as **Euler's number**.

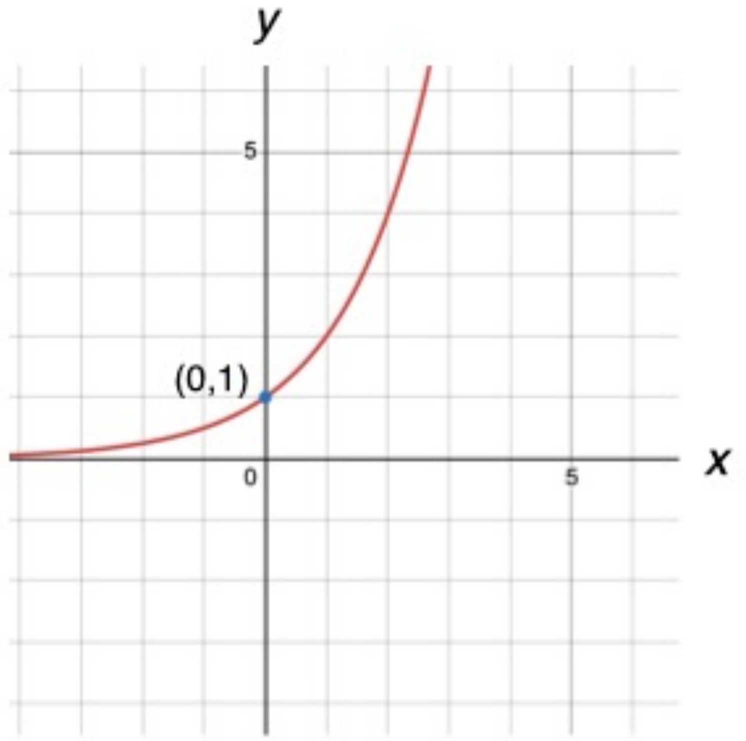
2. Graphs of Exponential Functions (1 of 2)

The graphs of exponential functions consist of SMOOTH curves. There is a *horizontal asymptote*. The graphs get closer and closer to it but never touch it. Below is the standard graph of any exponential function together with its asymptote.



Graphs of Exponential Functions (2 of 2)

The graph below is that of the common exponential function ***f(x)* = b**^{x}. The *x*-axis is always the *horizontal asymptote.* We know its equation to be *y = 0*. The *y*-intercept is always associated with the ordered pair (0, 1).



3. Transformations of the Common Exponential Function

Transformations* of the common exponential function $\boldsymbol{f(x)} = \mathbf{b}^{x}$ have the same type of graph, however, it can lie anywhere in the coordinate system (depending on the transformation).

*Remember that transformations may consist of vertical shifts, horizontal shifts, vertical stretching and compressing, and reflections across the *x*- and *y*-axes.

The domains of transformations of the common exponential function consist of *all real numbers*.

Note: Vertical shifts WILL affect the location of the horizontal asymptote!

4. Graph the Common Exponential Function and its Transformations by Hand

Graphing Strategy:

1. Find the equation of the *horizontal asymptote*. Use a dashed line to graph it unless it is the *x*-axis.
2. Find and plot the point associated with the *y*-intercept.
3. Find and plot additional points. Best is to find two to three points to either side of the *y*-intercept.
4. Use the information obtained in the steps above to graph the function keeping in mind the shape of the graph of exponential functions.