Concepts
Complex Numbers

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Define imaginary and complex numbers.
2. Find the conjugate of a complex number.
3. Perform arithmetic operations on complex numbers.
4. Write the square root of a negative number in terms of i.

1. Definition of Imaginary and Complex Numbers

Up to this point, we pretty much only discussed the set of real numbers. Actually, we have been mentioning “real” numbers a lot. They consist of integers, rational numbers, and irrational numbers. We will now discuss **imaginary numbers** and **complex numbers**.

The basic **imaginary number** is called *i* and is defined as

$i = \sqrt{-1}$ where $i^2 = -1$

Given imaginary numbers, we can now define another set of numbers, namely the **complex numbers**.

They consist of all numbers in the standard form $\boldsymbol{a + bi}$ where $\boldsymbol{a}$ and $\boldsymbol{b}$ are real numbers, and $\boldsymbol{i}$ is the basic imaginary number.

2. The Conjugate of a Complex Number

We discussed conjugates already. We stated that given an expression with exactly two terms, its conjugate is an expression with the same terms but the arithmetic operator in the middle of these terms is changed to the opposite one.

For example, the conjugate of $-2x - 3$ is $-2x + 3$.

Conjugates of complex numbers are similar. That is, for the complex number $\boldsymbol{a + bi}$, we define its **conjugate** to be $\boldsymbol{a - bi}$. Only the sign between the real and imaginary part changes!

For example, the conjugate of $-2 + 3i$ is $-2x - 3i$. Similarly, the conjugate of $5i$ is $-5i$.

3. Arithmetic Operations on Complex Number (1 of 2)

The form of a complex number $a + bi$ is just like $a + bx$. To add, subtract, and multiply two are more complex numbers, we use the same methods that we use for $a + bx$.

For example, when we add $(-3 - 2x) + (9 + 4x)$ we add the coefficients of variable terms, and we add the constants. That is,

$-3 - 2x + 9 + 4x = -3 + 9 + (-2 + 4)x = 6 + 2x$

Similarly, using the imaginary number i instead of x, we get

$(-3 - 2i) + (9 + 4i) = -3 - 2i + 9 + 4i = -3 + 9 + (-2 + 4)i = 6 + 2i$

Subtraction is done in a similar manner.

Arithmetic Operations on Complex Number (2 of 2)

For example, when we multiply $(-3 - 2x)(9 + 4x)$ we can use FOIL.

$-3(9) - 3(4x) - 2x(9) - 2x(4x) = -27 - 12x - 18x - 8x^2 = -27 - 30x - 8x^2$

Similarly, using the imaginary number i instead of x, we get

$-3(9) - 3(4i) - 2i(9) - 2i(4i) = -27 - 12i - 18i - 8i^2 = -27 - 30i - 8i^2$

Here we need to do some extra steps. Given $-27 - 30i - 8i^2$ and $i^2 = -1$, we can write $-27 - 30i - 8(-1) = -27 - 30i + 8 = -19 - 30i$.

4. The Square Root of a Negative Number

The square root of the number **– *b***, where ***b*** itself is positive is an imaginary number.

We usually rewrite this as $\sqrt{-b}$ = $i\sqrt{b}$.

Examples: $\sqrt{-3} = i\sqrt{3}$

$\sqrt{-4} = i\sqrt{4}$ = $2i$

Note that the imaginary number *i* is always written to the RIGHT of integers and fractions. It is usually written to the left of radicals!

Incidentally, if you were to evaluate $\sqrt{-3}$ or $\sqrt{-4}$ on the calculator, it would state “Domain Error.”