Concepts
Arithmetic Sequences and Series

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Learning Objectives

1. Memorize the definition of an arithmetic sequence and find the common difference.
2. Find the value of a term of an arithmetic sequence.
3. Memorize the definition of a finite arithmetic series and evaluate its sum.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

1. Definition of an Arithmetic Sequence (1 of 2)

An arithmetic sequence is a special sequence in which each term after the first is obtained by adding a fixed nonzero constant to the preceding term. An arithmetic sequence can be finite or infinite.

Example 1:

142, 146, 150, 154, 158, … is an arithmetic sequence because we add 4 to each preceding term to get the next term.

Example 2:

2, 8, 18, 32, 50, 72, … is NOT an arithmetic sequence because there is NO fixed number we add to the preceding term to get the next term.

Definition of an Arithmetic Sequence (2 of 2)

The fixed number we add to each preceding term to get the next term in an arithmetic sequence is called the **common difference** and is usually denoted by ***d***. It can be positive or negative.

We can find the common difference by subtracting the first term of an arithmetic sequence from the second term, that is, $d = a_2 - a_1$.

For example, in the arithmetic sequence 142, 146, 150, 154, 158, … , we would say $d = 146 - 142$.

2. The Value of a Term of an Arithmetic Sequence (1 of 2)

We can find the value of any term a_n of an *arithmetic sequence* as long as we know the first term a_1 and common difference d.

$a_n = a_1 + (n - 1)d$ where

a_n is the value of the term to be found
a_1 is the first term of the sequence
n is the position of the unknown term in the sequence (e.g., third term, ninth term)
d is the common difference

You can find a proof of this formula and those of some other formulas, which we will discuss shortly, in the appropriate weekly tasks folder.

The Value of a Term of an Arithmetic Sequence (2 of 2)

Example 3:

Find the value of a_{60} in an arithmetic sequence given the first term $a_1 = 100$ and the common difference $d = -30$.

Please note that a_{60} is the 60th term in the arithmetic sequence. We will use $a_n = a_1 + (n - 1)d$ with $n = 60$.

$$a_{60} = 100 + (60 - 1)(-30)$$
$$= 100 + (59)(-30)$$
$$= 100 - 1680$$
$$= -1580$$

The value of the 60th term is – 1580.

3. The Finite Arithmetic Series and its Sum (1 of 4)

In general, the **arithmetic series** is the sum of the terms of an *arithmetic sequence*. Given a finite arithmetic series, we can find this sum by writing out terms and then adding them.

Example 4:

Evaluate the sum of the arithmetic series $\sum_{k=1}^{5}(5k-9)$.

$k = 1$ $k = 2$ $k = 3$ $k = 4$ $k = 5$

$$\sum_{k=1}^{5}(5k-9) = [5(1) - 9] + [5(2) - 9] + [5(3) - 9] + [5(4) - 9] + [5(5) - 9]$$

$$= -4 + 1 + 6 + 11 + 16$$

$$= 30$$

The Finite Arithmetic Series and its Sum (2 of 4)

However, we can also use a **Summation Formula** instead of writing out the terms of a finite arithmetic series and then adding them. This formula is necessary when we are asked to add a large number of terms.

$S_n = \frac{n}{2}\left(a_1 + a_n\right)$ where

S_n is the name of the sum
n is the number of terms in the sum
a_1 is the first term of the sum
a_n is the last term of the sum

NOTE: The Summation Formula requires us to start with the first term.

The Finite Arithmetic Series and its Sum (3 of 4)

Example 5:

Evaluate the sum of the arithmetic series $\sum_{k=1}^{5}(5k-9)$ using the Summation Formula $S_n=\frac{n}{2}\left(a_1+a_n\right)$.

Let's find the values we need for this formula.

- The number of terms *n* in the sum is 5.
- We find the first term a_1 by evaluating (5*k* – 9) for *k* = 1.
  Specifically, a_1 = 5(1) – 9 = – 4
- We find the last term a_5 by evaluating (5*k*– 9) for *k* = 5.
  Specifically, a_5 = 5(5) – 9 = 16

The Finite Arithmetic Series and its Sum (4 of 4)

Example 5 continued:

Finally, given $a_1 = -4$ and $a_5 = 16$, we can find S_5. Specifically,

$$S_5 = \frac{5}{2}(-4+16)$$

$$= \frac{5}{2}(12)$$

$$= 30$$