Concepts and Examples
Volume and Surface Area

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Memorize the units of volumes and surface areas.
2. Memorize and use the volume and surface area formula of rectangular solids.
3. Memorize and use the volume and surface area formula of cylinders.
4. Use the volume and surface area formula of spheres. Does not need to be memorized.
5. Use the volume formula of right circular cones. Does not need to be memorized.
6. Use the volume formula of right pyramids. Does not need to be memorized.

1. Introduction to Volumes and Surface Areas

Volume is the amount of space occupied by a three-dimensional (3D) object. **Volumes are measured in cubic units, such as in^3, ft^3, yd^3, cm^3, m^3, km^3, etc.**

The surface area of a 3D object is the measure of the total area that the surface of the object occupies. Often, we define two different *surface areas*. The *total surface area* (TSA) and the *lateral surface area* (LSA).

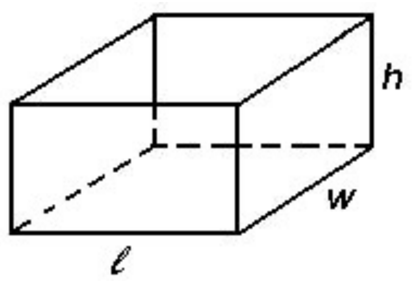
The total surface area is the sum of the areas of the top, the bottom, the front, the back, the right side, and the left side.

The lateral surface area is only the sum of the front, the back, the right side, and the left side. That is, we do not add in the areas of the top and the bottom!

Surface Areas are measured in square units, such as in^2, ft^2, yd^2, cm^2, m^2, km^2, etc.

2. Volume and Surface Area of Rectangular 3D Objects (1 of 4)

A rectangular 3D object has six faces that are rectangles. All angles right angles. Usually, we call its length ℓ, its width ***w***, and its height ***h***. When all three measurements are equal it is often called a cube.



Volume Formula:

$V = \ell \cdot w \cdot h$ (the area of the rectangular bottom times its height!)

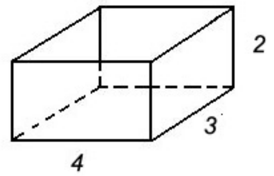
or $V = \ell wh$

Volume and Surface Area of Rectangular 3D Objects (2 of 4)

Example 1:

Determine the volume, the lateral surface area, and the total surface area of a rectangular 3D object whose length ℓ is 4 ft, its width w is 3 ft, and its height h is 2 feet.

Volume:



Required formula: $V = \ell wh$

Given $\ell = 4$, $w = 3$, and $h = 2$, we get

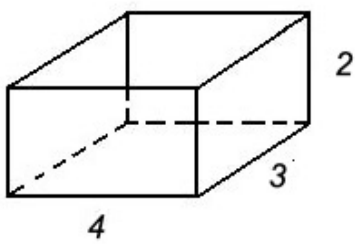
$V = 4(3)(2) = 24$

The volume of the rectangular prism is 24 ft^3. **Please note that the volume units are cubed.**

Volume and Surface Area of Rectangular 3D Objects (3 of 4)

Example 1 continued:

Lateral Surface Area:



Required formulas: $A_{front\ \&\ back} = 2\ell h$

$A_{both\ sides} = 2wh$

Given $\ell = 4$, $w = 3$, and $h = 2$, we get

$A_{front\ \&\ back} = 2(4)(2) = 16$ $A_{both\ sides} = 2(3)(2) = 12$

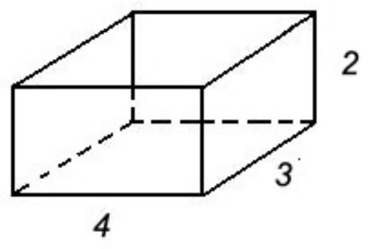
$LSA = 16 + 12 = 28$

The lateral surface area of the rectangular prism is 28 ft^2. **Please note that the surface area units are squared.**

Volume and Surface Area of Rectangular 3D Objects (4 of 4)

Example 1 continued:

Total Surface Area:



Required formulas: $A_{botton\ \&\ top} = 2\ell w$

$A_{front\ \&\ back} = 2\ell h$

$A_{both\ sides} = 2wh$

Given $\ell = 4$, $w = 3$, and $h = 2$, we get

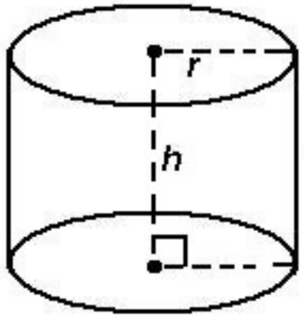
$A_{botton\ \&\ top} = 2(4)(3) = 24$ $A_{front\ \&\ back} = 2(4)(2) = 16$ $A_{both\ sides} = 2(3)(2) = 12$

$TSA = 24 + 16 + 12 = 52$

The total surface area of the rectangular prism is 52 ft^2. **Please note that the surface area units are squared.**

3. Volume and Surface Area of Right Circular Cylinders (1 of 5)

The cylinder is one of the basic 3-dimensional shapes in geometry. It has two parallel circular bases with radius ***r*** at a distance. The distance between the two circular bases is called the height ***h*** of the cylinder which makes a 90^o angle with the circular bases.



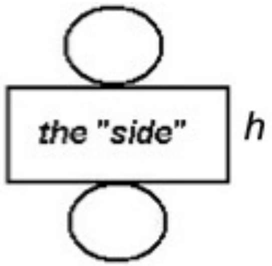
Volume Formula: $V = \pi \cdot r^2 \cdot h$

This is the area of the cylinder bottom times its height!

Volume and Surface Area of Right Circular Cylinders (2 of 5)

Surface Area Formula:

Below is a picture of a cylinder cut open lengthwise and then around the top and bottom.



The **total surface area (TSA)** consists of the area of two circles and a rectangle. The length of the rectangle is actually the circumference of the circles, and the width is the height of the cylinder. The **lateral surface area (LSA)** consists only of the "side."

$A_{botton\ \&\ top} = 2 \cdot \pi \cdot r^2$ $\qquad$ $A_{side} = 2 \cdot \pi \cdot r \cdot h$

$TSA = 2\pi r^2 + 2\pi rh$ (total surface area)

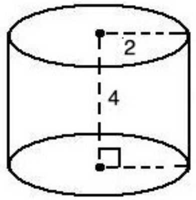
$LSA = 2\pi rh$ (lateral surface area, excludes areas of bottom and top)

Volume and Surface Area of Right Circular Cylinders (3 of 5)

Example 2:

Determine the volume, the lateral surface area, and the total surface area of a right circular cylinder, where $h = 4\ m$ and $r = 2\ m$. State the answer exactly. Then use the calculator to find the decimal approximation of the volume and the surface **area and round to one decimal place.**

Volume:



Required formula: $V = \pi r^2 h$

Given $h = 4$ and $r = 2$, we get

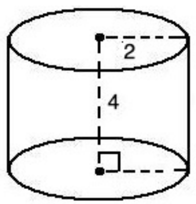
$V = \pi(2^2)(4) = \pi(4)(4) = 16\pi$ (exact answer)

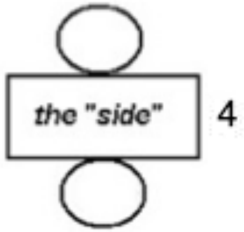
Using the calculator and the π button, we find the volume of the cylinder to be approximately 50.3 m^3. **Please note that the volume units are cubed.**

Volume and Surface Area of Right Circular Cylinders (4 of 5)

Example 2 continued:

Lateral Surface Area:





Required formula: $A_{side} = 2\pi rh$

Given $h = 4$ and $r = 2$, we get

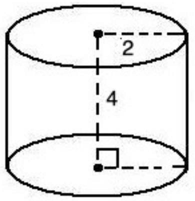
A_{side} = LSA = $2\pi(2)(4) = 16\pi$ (exact answer)

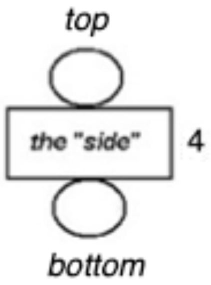
Using the calculator and the π button, we find the lateral surface area of the cylinder to be approximately 50.3 m^2. **Please note that the surface area units are squared.**

Volume and Surface Area of Right Circular Cylinders (5 of 5)

Example 2 continued:

Total Surface Area:





Required formulas: $A_{botton\ \&\ top} = 2\pi r^2$

$A_{side} = 2\pi rh$

Given $h = 4$ and $r = 2$, we get

$A_{botton\ \&\ top} = 2\pi(2^2) = 8\pi$

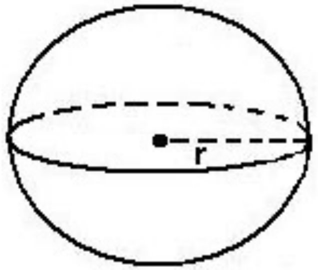
$A_{side} = 2\pi(2)(4) = 16\pi$

$TSA = 8\pi + 16\pi = 24\pi$ (exact answer)

Using the calculator and the π button, we find the total surface area of the cylinder to be approximately 75.4 m^2. **Please note that the surface area units are squared.**

4. Volume and Surface Area of Spheres (1 of 4)

The sphere is defined as the 3-dimensional round solid figure in which every point on its surface is equidistant from its center. The fixed distance is called the radius ***r*** of the sphere. We obtain a sphere from a rotation of the two-dimensional circle.



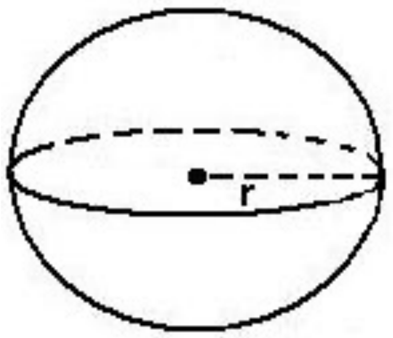
Volume Formula:

The formula for the volume of a sphere is obtained via calculus where we assume that a sphere is made up of numerous thin circular disks which are arranged one over the other.

$$V = \frac{4}{3} \cdot r^3 \cdot \pi$$

Volume and Surface Area of Spheres

Total Surface Area (TSA) Formula:



The formula for the total surface area of a sphere is also obtained via calculus. It is as follows:

$TSA = 4 \cdot r^2 \cdot \pi$

Volume and Surface Area of Spheres (3 of 4)

Example 3:

Find the the volume and the total surface area of a sphere with a radius of 12 in. Give the exact answers showing `pi`. Give the approximations using the `pi` button on your calculator and round to one decimal place.

Volume:

Required formula: $V = \frac{4}{3} \cdot r^3 \cdot \pi$

Given $r = 12$, we get

$V = \frac{4}{3} \cdot 12^3 \cdot \pi = \frac{4}{3} \cdot 1728 \cdot \pi = 2304\pi$ (exact answer)

Using the calculator and the π button, we find the volume of the sphere to be approximately 7238.2 in^3. **Please note that the volume units are cubed.**

Volume and Surface Area of Spheres (4 of 4)

Example 3 continued:

Total Surface Area:

Required formula: $TSA = 4 \cdot r^2 \cdot \pi$

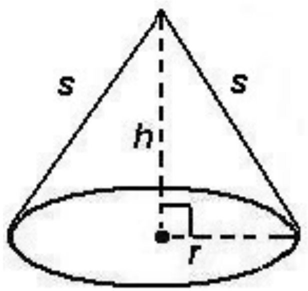
Given $r = 12$, we get

$TSA = 4 \cdot 12^2 \cdot \pi = 4 \cdot 144 \cdot \pi = 576\pi$ (exact answer)

Using the calculator and the π button, we find the total surface area of the sphere to be approximately 1809.6 in^2. **Please note that the volume units are squared.**

5. Volume of Right Circular Cones (1 of 2)

A cone is a 3-dimensional shape having a circular base with radius ***r*** and the sides ***s*** narrow smoothly to a point above the base. This point is known as apex. A right circular cone is a cone where the axis ***h*** of the cone meets the circular base at a right angle.



Volume Formula:

The formula for the volume of a cone is obtained via calculus.

$V = \frac{1}{3} \cdot r^2 \cdot h \cdot \pi$

Volume of Right Circular Cones (2 of 2)

Example 4:

Find the the volume of a right circular cone with a radius of 5 ft and height 4 ft. Give an approximation using the `pi` button on your calculator and round to two decimal places.

Volume:

Required formula: $V = \frac{1}{3} \cdot r^2 \cdot h \cdot \pi$

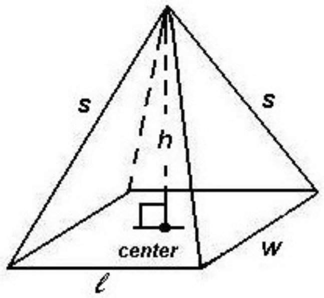
Given $r = 5$, and $h = 4$, we get

$V = \frac{1}{3} \cdot 5^2 \cdot 4 \cdot \pi = \frac{1}{3} \cdot 100 \cdot \pi \cong 104.7197551$ Calculator value that now needs to be rounded to two decimal places!

We determine the volume of the right circular cone to be approximately 104.72 ft^3. **Please note that the volume units are cubed.**

6. Volume of Right Pyramids (1 of 2)

A pyramid is a 3-dimensional shape having a rectangular base with length ℓ and width ***w***. The sides ***s*** narrow smoothly to a point above the base. A right pyramid is a pyramid where the axis ***h*** of the pyramid meets the rectangular base at a right angle.



Volume Formula:

The formula for the volume of a pyramid is obtained via calculus.

$$V = \frac{1}{3} \cdot \ell \cdot w \cdot h$$

Volume of Right Pyramids (2 of 2)

Example 5:

A stone pyramid in Egypt has a square base that measures 150 m on each side. The height is 93 m. What is the volume of the pyramid? Round your answer to the nearest hundredth.

Volume:

Required formula: $V = \frac{1}{3} \cdot l \cdot w \cdot h$

Given $l = 150$, $w = 150$, and $h = 93$, we get

$V = \frac{1}{3} \cdot 150 \cdot 150 \cdot 93 = \frac{1}{3} \cdot 2092500 = 697500$ Exact calculator value!

We determine the volume of the right pyramid to be *697500.00* ft^3. **Please note that the volume units are cubed.**