Concepts and Examples Measuring Length

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Use dimensional analysis to change units of measure.
2. Memorize units of length in the U.S. Measuring System and use dimensional analysis to change units.
3. Memorize units of length in the Metric Measuring System and use dimensional analysis to change units.
4. Change to and from the Metric Measuring System.

1. Dimensional Analysis

In this lesson we will be working with length measured in the U.S. Measuring System and the Metric Measuring System. We will use *Dimensional Analysis* to change from one unit of measure to another, such as changing from feet to inches or from meters to millimeters, etc.

Strategy:

- Set up the original amount as a fraction with its unit of measure in the numerator.

- Find a unit ratio containing the new unit of measure in the numerator and the original unit of measure in the denominator.

- Multiply the original number in fraction form with the unit ratio cancelling units as appropriate.

You must memorize the following measures.

Symbol	Unit	Meaning
1 in	inch	
1 ft	foot	12 in
1 yd	yard	3 ft
1 mi	mile	1760 yd (5280 ft)

Example 1:

Change 76.53 feet to inches using dimensional analysis. Round to two decimal places if necessary.

First, we'll write the original measure as the fraction $\frac{76.53 \, ft}{1}$.

We memorized that 1 foot contains 12 inches. We express this as a unit ratio in which the new unit of measure (in) is in the numerator and the original unit of measure (ft) is in the denominator.

$$\frac{12 \, in}{1 \, ft}$$

Now, we'll multiply the original measure in fraction form with the unit ratio.

Example 1 continued:

$$\frac{76.53\,ft}{1}\left(\frac{12\,in}{1\,ft}\right) = \frac{76.53(12)\,in}{1}$$

$$= 918.36\,in$$

Notice that we cross-canceled the original units of measure so that only the new unit of measure remains. We find that 76.53 ft equals exactly 918.36 in.

Example 2:

Change 1.75 miles to inches using dimensional analysis. Round to two decimal places if necessary.

First, we'll write the original measure as the fraction $\dfrac{1.75 \ mi}{1}$.

Here we must use several unit fractions. We know that 1 mile contains 1,760 yards, 1 yard contains 3 feet, and 1 foot contains 12 inches. We will use the following unit ratios:

$\dfrac{1760 \ yd}{1 \ mi}$ and $\dfrac{3 \ ft}{1 \ yd}$ and $\dfrac{12 \ in}{1 \ ft}$

Now, we'll multiply the original measure in fraction form with the unit ratios.

Example 2 continued:

We use the calculator to get $\dfrac{1.75\,mi}{1}\left(\dfrac{1760\,yd}{1\,mi}\right)\left(\dfrac{3\,ft}{1\,yd}\right)\left(\dfrac{12\,in}{1\,ft}\right) = \dfrac{1.75\,(1760)(3)(12)\,in}{1}$

$$= 110880\,in$$

Notice that we cross-canceled the original units of measure so that only the new unit of measure remains. We find that 1.75 mi equals exactly 110,880 in.

The millimeter, the centimeter, the meter, and the kilometer are the most commonly used metric measures. You must memorize them.

Symbol	Unit	Meaning
1 mm	millimeter	
1 cm	centimeter	10 mm
1 m	meter	100 cm
1 km	kilometer	1000 meter

Less commonly used metric linear measures:

1 decimeter (dm) = 10 centimeters (cm)

1 dekameter (dkm) = 10 meters (m)

1 hectometer (hm) = 10 dekameters (dkm)

Length in the Metric Measuring System

Example 3:

Change 0.378 meters to millimeters using dimensional analysis. Round to two decimal places if necessary.

First, we'll write the original measure as the fraction $\frac{0.378\,m}{1}$.

We memorized that 1 meter contains 1000 millimeter. We express this as a unit ratio in which the new unit of measure (mm) is in the numerator and the original unit of measure (m) is in the denominator.

$$\frac{1000\,mm}{1m}$$

Now, we'll multiply the original measure in fraction form with the unit ratio.

Example 3 continued:

$$\frac{0.378\,\cancel{m}}{1}\left(\frac{1000\,mm}{1\,\cancel{m}}\right) = \frac{0.378(1000)\,mm}{1(1)}$$

$$= 378\,mm$$

Notice that we cross-canceled the original units of measure so that only the new unit of measure remains. We find that 0.378 m equals exactly 378 mm.

Length in the Metric Measuring System

Example 4:

Change 789 meters to kilometers using dimensional analysis. Round to two decimal places if necessary.

First, we'll write the original measure as the fraction $\frac{789\,m}{1}$.

We memorized that 1 kilometer contains 1000 meter. We express this as a unit ratio in which the new unit of measure (mm) is in the numerator and the original unit of measure (m) is in the denominator.

$$\frac{1\,km}{1000\,m}$$

Now, we'll multiply the original measure in fraction form with the unit ratio.

Example 4 continued:

We use the calculator to get
$$\frac{789 \, m}{1}\left(\frac{1 \, km}{1000 \, m}\right) = \frac{789 \, km}{1(1000)}$$
$$= 0.789 \, km$$

We find that 789 m equals exactly 0.789 km.

You must memorize the following conversions.

1 inch (in) = 2.54 centimeter (cm)
This is an exact measure!
1 yard (yd) ≈ 0.91 meter (m)
This is an approximation!
1 mile (mi) ≈ 1.61 kilometer (km)
This is an approximation!

Example 5:

Change 55 miles to kilometers using dimensional analysis. Round to a whole number if necessary.

First, we'll write the original measure as the fraction $\frac{55\ mi}{1}$.

We memorized that 1 mile is <u>approximately</u> equal to 1.61 kilometer. We express this as a unit ratio in which the new unit of measure (km) is in the numerator and the original unit of measure (mi) is in the denominator.

$$\frac{1.61\ km}{1\ mi}$$

Now, we'll multiply the original measure in fraction form with the unit ratio.

Convert to and from the Metric Measuring System

Example 5 continued:

$$\frac{55 \; mi}{1}\left(\cdot \frac{1.61 \, km}{1 \, mi}\right) \cong 89 \, km$$

Notice that we cross-canceled the original units of measure so that only the new unit of measure remains. We find that 55 mi equals approximately 89 km.

Example 6:

Change 33 centimeters to inches using dimensional analysis. Round to two decimal places if necessary.

First, we'll write the original measure as the fraction $\dfrac{33\ cm}{1}$.

We memorized that 1 inch is <u>exactly</u> equal to 2.54 centimeters. We express this as a unit ratio in which the new unit of measure (in) is in the numerator and the original unit of measure (cm) is in the denominator.

$$\dfrac{1\ in}{2.54\ cm}$$

Now, we'll multiply the original measure in fraction form with the unit ratio.

Example 6 continued:

We use the calculator to get $\dfrac{33\ cm}{1}\left(\dfrac{1\ in}{2.54\ cm}\right)\cong 12.99$

Notice that we cross-canceled the original units of measure so that only the new unit of measure remains. We find that 33 cm is approximately equal to 12.99 in.