Concepts
The Slope of a Line

Based on power point presentations by Pearson Education, Inc.

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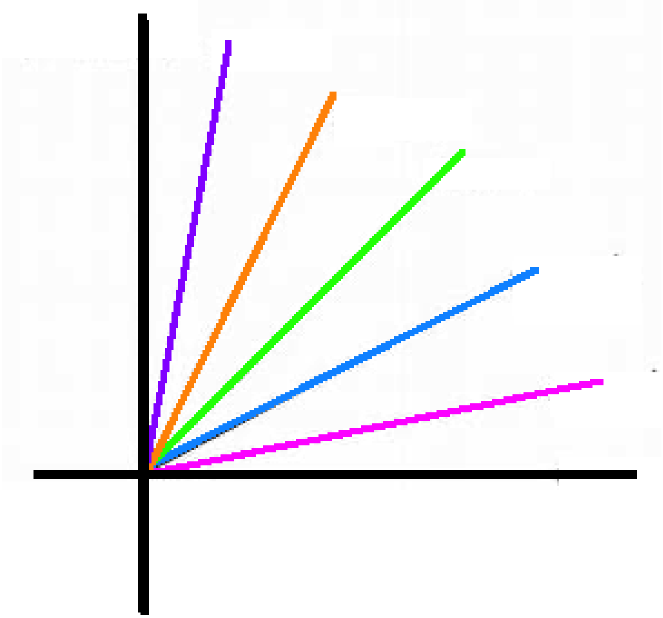
Learning Objectives

1. Define and find the slope of a line.
2. Define the slope-intercept equation of a line.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

1. Definition of the Slope of a Line (1 of 6)

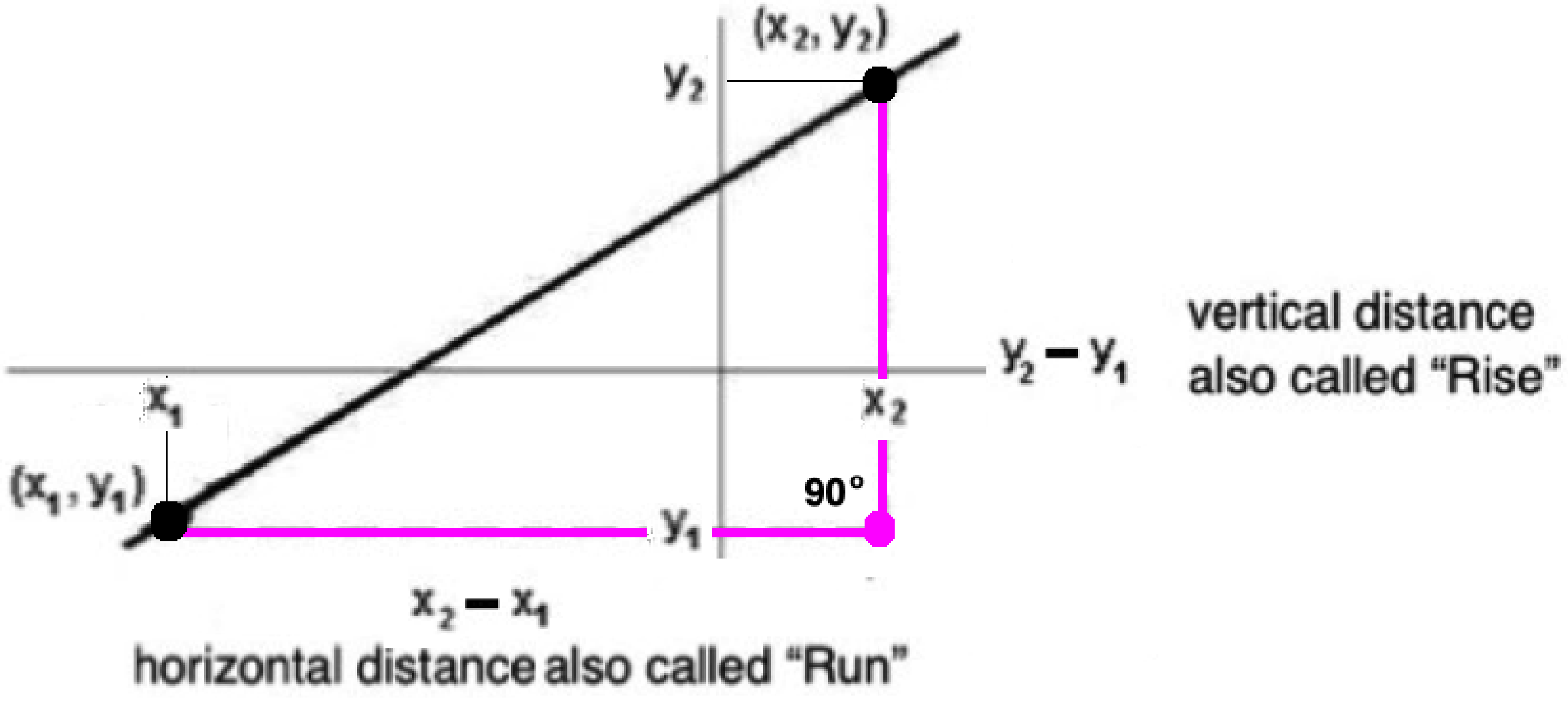
In the last lesson, we discussed the graphs of linear equations in two variables. We found out that their graphs are either increasing or decreasing lines. Let's look at a picture of several increasing lines.



We notice that each line has a different steepness. For example, the purple line is much steeper than the pink line.

Definition of the Slope of a Line (2 of 6)

In mathematics, the steepness of a line is measured by an entirely man-made **Slope Formula**. Specifically, at one point it was defined to be the change in vertical distance divided by the change in horizontal distance as we "travel" from one point on a line in a rectangular coordinate system to another point. Usually, these two points are defined by the ordered pairs (x_1, y_1) and (x_2, y_2).



Definition of the Slope of a Line (3 of 6)

The slope of a line is indicated by the lower-case letter *m*. Why *m*? No one knows for sure. Some mathematicians claim the *m* comes from the French word "monter" which means "to climb".

The slope of the line passing through two distinct ordered pairs (x_1, y_1) and (x_2, y_2) is defined by the following formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

NOTE: Regardless of the sign of the *x*-coordinates or the *y*-coordinates, the minus sign between the *y*-values and the *x*-values in the slope calculation must always be there.

We can also say $m = \frac{\text{Rise}}{\text{Run}}$ or $m = \frac{\textit{change in y}}{\textit{change in x}}$

Definition of the Slope of a Line (4 of 6)

Example 1:

Find the slope of the line passing through the points determined by the ordered pairs (4, −2) and (−1, 5).

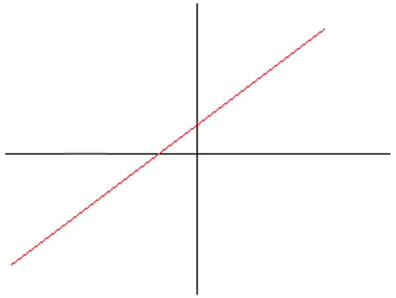
We will let (4, −2) equal (x_1, y_1) and (−1, 5) equal (x_2, y_2). However, you can also let (−1, 5) equal (x_1, y_1) and (4, −2) equal (x_2, y_2). In either case, you will get the same answer.

Let's say that (4, −2) equals (x_1, y_1) and (−1, 5) equals (x_2, y_2). Be sure not to get confused!

Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-2)}{-1-4} = \frac{5+2}{-5} = \frac{7}{-5} = -\frac{7}{5}$

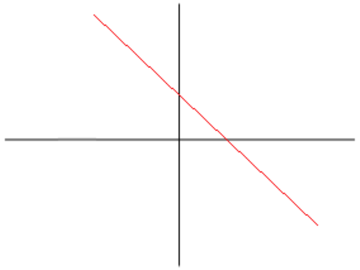
2. Definition of the Slope of a Line (5 of 6)

We will not state how certain slope properties affect the characteristics of a line.



POSITIVE SLOPE

All increasing lines have a positive slope.



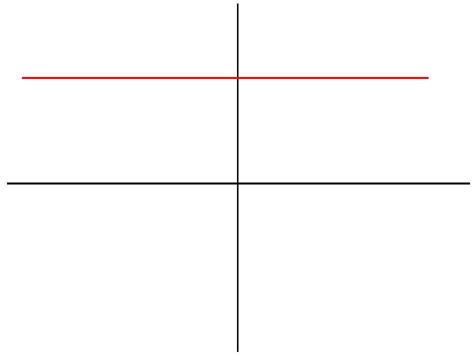
NEGATIVE SLOPE

ALL decreasing lines have a negative slope.

Definition of the Slope of a Line (6 of 6)

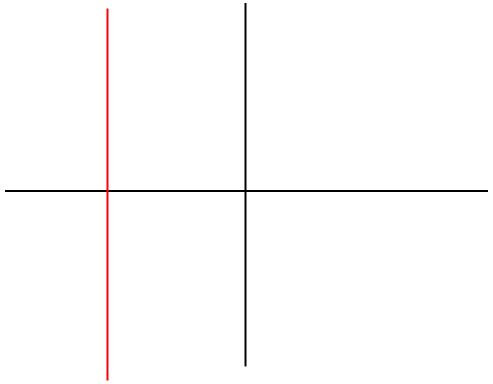
SLOPE OF 0

All horizontal lines have a slope of 0. Slopes of 0 occur when the numerator in the slope formula is 0.



UNDEFINED SLOPE

All vertical lines have an undefined slope. Undefined slopes occur when the denominator in the slope formula is 0.



2. The Slope-Intercept Equations of Lines and their Graphs

We have already discussed the *general form* of the equation of a line. It is $\boldsymbol{Ax + By + C = 0}$, where $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ are real numbers, but $\boldsymbol{A}$ and $\boldsymbol{B}$ cannot be 0.

Now we are going to discuss a different form of the equation of a line by manipulating $\boldsymbol{Ax + By + C = 0}$ using the *Principle of Equality* and its axioms.

What we will do is isolate the $\boldsymbol{y}$ on one side of the equal sign. First, we will subtract $\boldsymbol{Ax}$ and $\boldsymbol{C}$ from both side of the equation to get

$$\boldsymbol{By = -Ax - C}$$

The Slope-Intercept Equations of Lines and their Graphs

Next, we will divide both sides of the equation by ***B*** to get

$$y = \frac{-Ax - C}{B}$$

As you can see, ***y*** is now isolated on one side. But we are not quite done!

We will now distribute ***B*** to every term in the numerator on the right side of the equal sign as follows:

$$y = \frac{-Ax}{B} - \frac{C}{B} = -\frac{A}{B}x - \frac{C}{B}$$

NOTE: In algebra, we usually take the variable out of numerator and place it on the level of the fraction bar! We do the same with negative signs!

The Slope-Intercept Equations of Lines and their Graphs

Example 1:

Rewrite the equation of the line $-18x - 2y + 11 = 0$ so that the y-variable is isolated on the left side of the equation

Let's do the following manipulations:

$-2y + 11 = 18x$ (added $18x$ to both sides)

$-2y = 18x - 11$ (subtracted 11 from both sides)

$y = \frac{18x - 11}{-2}$ (divided both sides by -2)

As you can see, y is now isolated on one side. But we are not quite done!

The Slope-Intercept Equations of Lines and their Graphs

Example 1 continued:

We will now distribute – 2 to every term in the numerator on the right side of the equal sign as follows:

$y = \frac{18x}{-2} - \frac{11}{-2}$

and $y = -9x + \frac{11}{2}$

The Slope-Intercept Equations of Lines and their Graphs

In mathematics, the equation $y = -\frac{A}{B}x - \frac{C}{B}$ is called the **slope-intercept form** of a linear equation.

However, we make some replacements. That is, we assign the letter ***m*** to $-\frac{A}{B}$ and the letter ***b*** and to $-\frac{C}{B}$.

Therefore, from now on we will use ***y = mx + b*** when discussing the slope-intercept equation of a line. Please note that ***m*** is actually the slope of the line and ***b*** is the *y*-intercept.

The Slope-Intercept Equations of Lines and their Graphs

Examples of equations of lines in *slope-intercept form*:

$y = -18x + 11$ (where $m = -18$ is the slope of the line and $b = 11$ is the y-intercept)

$y = 2x$ (where $m = 2$ is the slope of the line and $b = 0$ is the y-intercept)

$y = -x - 1$ (this is still considered slope-intercept form) because the equation can be written as $y = -x + (-1)$. Therefore, $m = -1$ is the slope of the line and $b = -1$ is the y-intercept.

The Slope-Intercept Equations of Lines and their Graphs

Example 2:

Graph the linear equation $y = -3x - 6$ by hand. This linear equation is in *slope-intercept form*!

NOTE:

In a previous algebra course you may have learned a method of graphing called the "Slope-Intercept" Method. We will not use this method in our course. All linear equations in two variables can be graphed using the Point-by-Point Plotting Method or the Intercept Method or a combination of both.

The Slope-Intercept Equations of Lines and their Graphs

Example 2 continued:

Since we are not told which graphing method to use, let's use the *Intercept Method*.

Find the ordered pair associated with the y-intercept.

Let $x = 0$ and solve for y.

$y = 3(0) - 6$ (this is a linear equation in one variable)

$y = -6$

The y-intercept is -6, so the ordered pair associated with it is $(0, -6)$.

The Slope-Intercept Equations of Lines and their Graphs

Example 2 continued:

Find the ordered pair associated with the x-intercept.

Let $y = 0$ and solve for x

$0 = -3x - 6$ (this is a linear equation in one variable)

$3x = -6$

$x = -2$

The x-intercept is -2, so the ordered pair associated with it is $(-2, 0)$.

The Slope-Intercept Equations of Lines and their Graphs

Example 2 continued:

Graph the equation by drawing a line through the points created by the ordered pairs associated with the *y*- and *x*-intercepts.