Concepts
The Slope of a Line

Based on power point presentations by Pearson Education, Inc.

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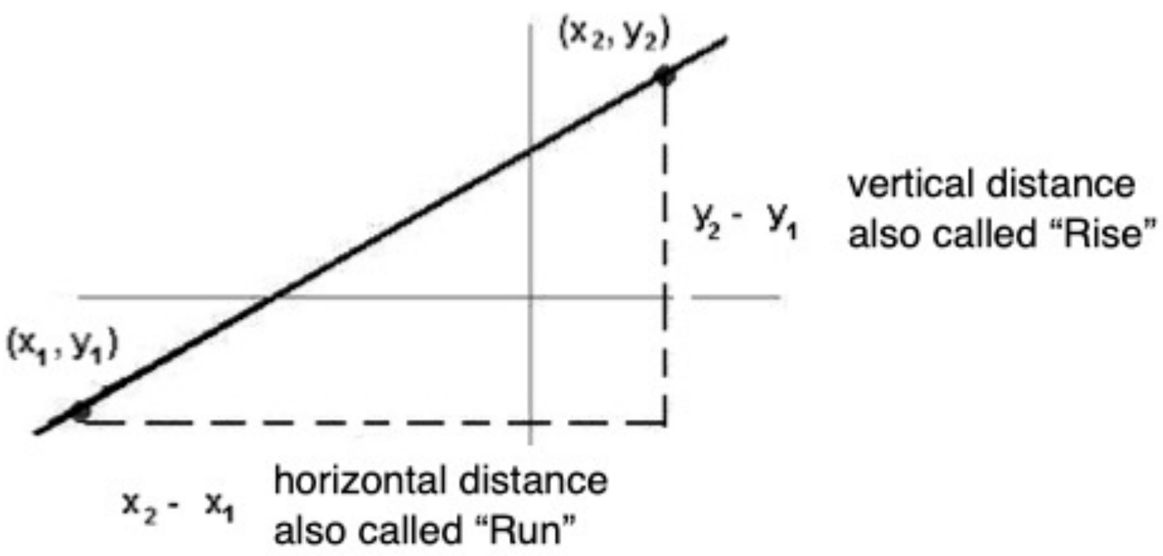
Learning Objectives

1. Define and calculate the slope of a line.
2. Identify the slopes of increasing, decreasing, vertical, and horizontal lines.
3. Identify the slope and the y-intercept in the equation of a line.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

1. Definition of the Slope of a Line (1 of 2)

Before we continue with our discussion on functions, lets first discuss the slope of a line. In layman's terms, the slope is a measure of the **steepness of a line**. It is said to be the change in vertical distance divided by the change in horizontal distance as we "travel" from one point, say (x_1, y_1), to another, say (x_2, y_2) lying on the same line in a rectangular coordinate system.



Definition of the Slope of a Line (2 of 2)

In mathematics, the slope of a line is indicated by using the lower-case letter *m*. Why *m*? No one knows for sure. Some mathematicians claim the *m* comes from the French word “monter” which means “to climb”.

The slope of the line through two distinct points (x_1, y_1) and (x_2, y_2) lying in a coordinate system is formally defined as

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

NOTE: Regardless of the sign of the *x*-coordinates or the *y*-coordinates, the minus sign between the *y*-values and the *x*-values in the slope calculation must always be there.

We can also say $m = \frac{\text{Rise}}{\text{Run}}$ or $m = \frac{\textit{change in } y}{\textit{change in } x}$

Definition of the Slope of a Line (3 of 3)

Example 1:

Find the slope of the line passing through the points determined by the ordered pairs (4, −2) and (−1, 5).

We will let (4, −2) equal (x_1, y_1) and (−1, 5) equal (x_2, y_2). However, you can also let (−1, 5) equal (x_1, y_1) and (4, −2) equal (x_2, y_2). In either case, you will get the same answer.

Let's say that (4, −2) equals (x_1, y_1) and (−1, 5) equals (x_2, y_2). Be sure not to get confused!

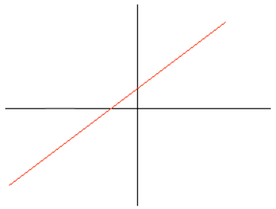
Then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-2)}{-1-4} = \frac{5+2}{-5} = \frac{7}{-5} = -\frac{7}{5}$

2. Identify the Slopes of Lines (1 of 2)

We learned that the slope of a line is its steepness. Specifically, the slope of the line through two distinct points (x_1, y_1) and (x_2, y_2) lying in a coordinate system is formally defined as

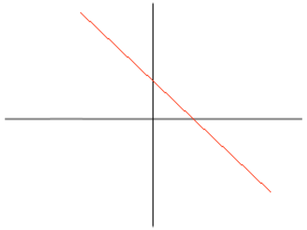
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Now, we will investigate how the slope affects the characteristics of a line.



SLOPE OF INCREASING (RISING) LINES

An increasing line has a positive slope.



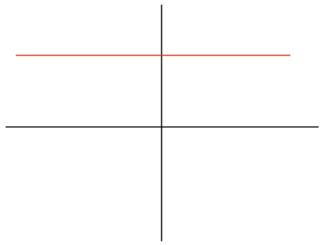
SLOPE OF DECREASING (FALLING) LINES

A decreasing line has a negative slope.

Identify the Slopes of Lines (2 of 2)

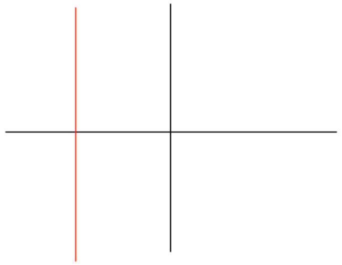
SLOPE OF HORIZONTAL LINES

All horizontal lines have a slope of 0.



SLOPE AND VERTICAL LINES

All vertical lines have an undefined slope.



3. The Slope-Intercept Form of the Equation of a Line (1 of 2)

We have already discussed the general form of the linear equation. It is $Ax + By + C = 0$, where A, B, and C are real numbers, but A and B cannot both be 0.

Now we are going to discuss a different form of the linear equation. It is called the **slope-intercept form** and is defined as $\boldsymbol{y = mx + b}$, where m is the slope and b is the y-intercept.

Examples of linear equations in slope-intercept form:

$y = -18x + 11$ (here $b = 11$)

$y = 5x - 7$ (here $b = -7$)

$y = 2x$ (here $b = 0$)

The Slope-Intercept Form of the Equations of a Line (2 of 2)

Example 2:

Given the *general form* of the line $-18x - y + 11 = 0$, change it to *slope-intercept form*. Let's do the following manipulations:

$-y + 11 = 18x$ (added $18x$ to both sides)

$-y = 18x - 11$ (subtracted 11 from both sides)

$y = -18x + 11$ (multiplied both sides by -1)

As you can see, y is now isolated on one side. The given equation is now in slope-intercept form with $m = -18$ and $b = 11$,