Concepts and Examples
Writing Mathematical Sentences; Solving Formulas

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Change problems given in words to mathematical sentences.
2. Solve a formula for one of its variables.

1. Change Problems given in Words to Mathematical Sentences (1 of 2)

A **mathematical sentence** is another word for equation. It contains certain mathematical expressions and an equal sign. Often, we want to change a problem given to us in words, into a mathematical sentence. We usually try to do this following the steps below. Please note that a healthy dose of critical thinking is definitely necessary!

Step 1 - Read the problem given in words carefully several times until you can state in your own words what is given and what the problem is looking for.

Step 2 – There are always one or more unknown quantity involved. Let x (or any other variable) represent one of the unknown quantities in the problem.

Step 3 - Write a mathematical sentence in terms of the variable from Step 2 that models the verbal conditions of the problem.

Change Word Problems to Mathematical Sentences (2 of 2)

Key Words to look for in word problems:

- Addition: the sum of, plus, increased by, more than, added to, exceeds, longer, total, heavier, older, wider, taller, gain, greater than, more, gain
- Subtraction: less than, decreased by, subtracted from, the difference between, diminished by, take away, reduced by, less, minus, shrinks, younger, lower, shorter, narrower, slower, loss
- Multiplication: times (two times, three times, etc.), multiply, of, the product of, multiplied by, twice, double, triple
- Division: divide, divided by, divided into, how big is each part, how many parts can be made from, the quotient of
- Equals: is

Example 1: Write a Mathematical Statement

Write the following as a mathematical sentence and solve:

5 more than a number is 12. What is this number?

"this number" is what we are finding, and we will call it x

"more than" means addition

"is" means equals

In summary, we write $5 + x = 12$.

We can solve this as follows:

$x = 12 - 5$

$x = 7$

The number we are asked to find is 7.

Example 2: Write a Mathematical Statement (1 of 2)

Write the following as a mathematical sentence and solve:

4 times the difference of a number and 3 is 12. What is this number?

"this number" is what we are finding, and we will call it *n.*

"4 times" means multiplication

"difference" means subtraction

"is" means equals

In summary, we write $4(n - 3) = 12$.

NOTE: We had to use parentheses because we were asked to find "4 times the difference".

Example 2: Write a Mathematical Statement (2 of 2)

We can solve $4(n - 3) = 12$ as follows:

$4n - 12 = 12$

$4n = 24$

$n = 6$

The number we are asked to find is 6.

Example 3: Write a Mathematical Statement

Write the following as a mathematical sentence and solve:

The sum of 12, 7, and a third number is 17. What is the third number?

"the third number" is what we are finding, and we will call it t.

"sum" means addition

"is" means equals

We write $12 + 7 + t = 17$ and the we will combine like terms to get $19 + t = 17$.

We can solve this as follows:

$t = 17 - 19$

$t = -2$

The number we are asked to find is -2 .

2. Solve a Formula for one of its Variables (1 of 2)

In mathematics, a **formula** is an equation that uses several variables to express a certain relationship.

For example, ***P = 2(b + s)*** is some formula consisting of the variables ***P***, ***b***, and ***s***.

Formulas are used in the natural sciences, engineering, and in the social sciences (economics, psychology, sociology, political science). Sometimes we want to solve a formula for one of its variables to make repeated calculations easier.

Solve a Formula for one of its Variables (2 of 2)

Strategy for Solving a Formula for one of its Variables:

Step 1 - Treat the variable you want to solve for as if it is the only variable in the equation while treating the other variables just like you would any constant.

Step 2 - "Isolate" the variable you want to solve for by moving away all other terms associated with it. We will treat these terms as constants. Note: Usually we "isolated" the variable on the left side of the equal sign.

Step 3 - If necessary, combine like terms.

Example 4: Solve a Formula for One of its Variables (1 of 2)

Solve the formula ***P = 2(b + s)*** for ***b***.

We will isolate the variable ***b*** by moving away all terms associated with it. We will treat these terms as constants.

First, we must use the *Distributive Property* to "open up" the parentheses.

We get ***P = 2b + 2s***.

We notice that the term ***2s*** is associated with ***2b*** through addition. Therefore, we must use the *Subtraction Axiom* and subtract ***2s*** from both sides of the equation.

P – 2s = 2b + 2s – 2s

and ***P – 2s = 2b***

Example 4: Solve a Formula for One of its Variables (2 of 2)

Now, to further isolate the variable ***b***, we will move away its coefficient ***2***. That is, we will use the *Division Axiom* and divide both sides of the equation by 2 to get

$\frac{P-2s}{2} = b$ and the formula ***P = 2(b + s)*** is now solved for ***b***.

Incidentally, we could rewrite this equation as $b = \frac{P-2s}{2}$. Remember, in mathematics we like to "isolate" variables on the left side of the equal sign.

Example 5: Solve a Formula for One of its Variables

Solve the formula $M = S - C$ for S.

We will isolate the variable S by moving away all other terms associated with it. We will treat these terms as constants.

C is associated with S through subtraction. Therefore, we must use the *Addition Axiom* and add C to both sides of the equation.

$$M + C = S - C + C$$

then $M + C = S$ and the formula $M = S - C$ is now solved for S.

Incidentally, we could rewrite this equation as $S = M + C$. Remember, in mathematics we like to "isolate" variables on the left side of the equal sign.