



Volume and Surface Area of 3-D Figures

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Memorize the units of volumes and surface areas.
2. Memorize and use the volume and surface area formula of rectangular solids.
3. Memorize and use the volume and surface area formula of cylinders.
4. Use the volume and surface area formula of spheres. Does not need to be memorized.
5. Use the volume formula of right circular cones. Does not need to be memorized.
6. Use the volume formula of right pyramids. Does not need to be memorized.

1. Introduction to Volumes and Surface Areas

Volume is the amount of space occupied by a three-dimensional figure. **Volumes are measured in cubic units, such as in^3 , ft^3 , yd^3 , cm^3 , m^3 , km^3 , etc.**

Often, we define two different *surface areas*. The *total surface area* (TSA) and the *lateral surface area* (LSA).

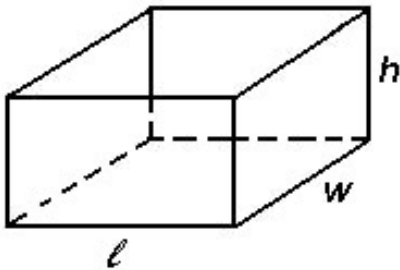
The total surface area is the sum of the areas of the top, the bottom, the front, the back, the right side, and the left side.

The lateral surface area is only the sum of the front, the back, the right side, and the left side. That is, we do not add in the areas of the top and the bottom!

Surface Areas are measured in square units, such as in^2 , ft^2 , yd^2 , cm^2 , m^2 , km^2 , etc.

2. Volume and Surface Area Formulas of Rectangular Prisms (1 of 2)

A rectangular prism is a 3-dimensional object which has six faces that are rectangles. All of its angles are right angles. Usually, we call its length l , its width w , and its height h . When all three measurements are equal it is often called a cube.



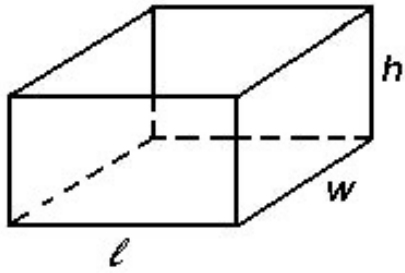
Volume Formula:

$$V = l \cdot w \cdot h \text{ (the area of the rectangular bottom times its height!)}$$

$$\text{or } V = lwh$$

Volume and Surface Area Formulas of Rectangular Prisms (2 of 2)

Surface Area Formula:



$$A_{\text{bottom \& top}} = 2 \cdot l \cdot w$$

$$A_{\text{front \& back}} = 2 \cdot l \cdot h$$

$$A_{\text{both sides}} = 2 \cdot w \cdot h$$

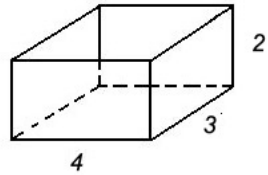
$$TSA = 2lw + 2wh + 2lh \text{ (total surface area)}$$

$$LSA = 2wh + 2lh \text{ (lateral surface area, excludes areas of bottom and top)}$$

Example 1: Find Volume and Surface Area (1 of 2)

Determine the volume and the total surface area of a rectangular prism whose length l is 4 ft, its width w is 3 ft, and its height h is 2 feet.

Volume:



Required formula: $V = lwh$

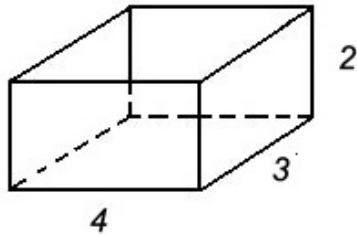
Given $l = 4$, $w = 3$, and $h = 2$, we get

$$V = 4(3)(2) = 24$$

The volume of the rectangular prism is 24 ft^3 . **Please note that the volume units are cubed.**

Example 1: Find Volume and Surface Area (2 of 2)

Total Surface Area:



Required formulas: $A_{\text{bottom \& top}} = 2lw$ and
 $A_{\text{front \& back}} = 2lh$ and $A_{\text{both sides}} = 2wh$

Given $l = 4$, $w = 3$, and $h = 2$, we get

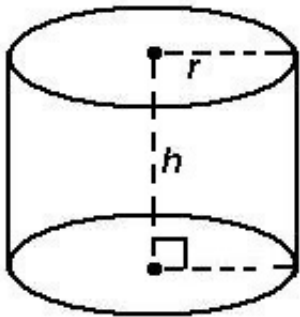
$$A_{\text{bottom \& top}} = 2(4)(3) = 24 \quad A_{\text{front \& back}} = 2(4)(2) = 16 \quad A_{\text{both sides}} = 2(3)(2) = 12$$

$$TSA = 24 + 16 + 12 = 52$$

The total surface area of the rectangular prism is 52 ft². **Please note that the surface area units are squared.**

3. Volume and Surface Area Formulas of Right Circular Cylinders (1 of 2)

The cylinder is one of the basic 3-dimensional shapes in geometry. It has two parallel circular bases with radius r at a distance. The distance between the two circular bases is called the height h of the cylinder which makes a 90° angle with the circular bases.



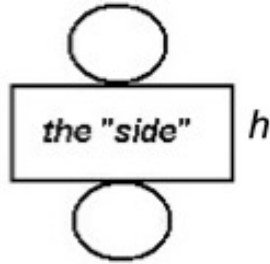
Volume Formula: $V = \pi \cdot r^2 \cdot h$

This is the area of the cylinder bottom times its height!

Volume and Surface Area of Right Circular Cylinders (2 of 2)

Surface Area Formula:

Below is a picture of a cylinder cut open lengthwise and then around the top and bottom.



The **total surface area (TSA)** consists of the area of two circles and a rectangle. The length of the rectangle is actually the circumference of the circles, and the width is the height of the cylinder. The **lateral surface area (LSA)** consists only of the "side."

$$A_{\text{bottom \& top}} = 2 \cdot \pi \cdot r^2 \qquad A_{\text{side}} = 2 \cdot \pi \cdot r \cdot h$$

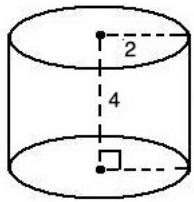
$$TSA = 2\pi r^2 + 2\pi r h \text{ (total surface area)}$$

$$LSA = 2\pi r h \text{ (lateral surface area, excludes areas of bottom and top)}$$

Example 2: Find Volume and Surface Area (1 of 2)

Determine the volume and the total surface area of a right circular cylinder, where $h = 4 \text{ m}$ and $r = 2 \text{ m}$. State the answer exactly. Then use the calculator to find the decimal approximation of the volume and the surface area and round to one decimal place.

Volume:



Required formula: $V = \pi r^2 h$

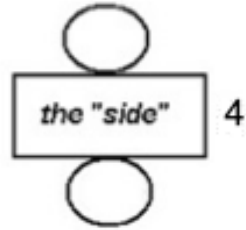
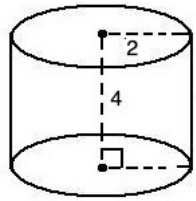
Given $h = 4$ and $r = 2$, we get

$$V = \pi(2^2)(4) = \pi(4)(4) = 16\pi \text{ (exact answer)}$$

Using the calculator and the π button, we find the volume of the cylinder to be approximately 50.3 m^3 . **Please note that the volume units are cubed.**

Example 2: Find Volume and Surface Area (2 of 2)

Total Surface Area:



Required formulas: $A_{\text{bottom \& top}} = 2\pi r^2$ and $A_{\text{side}} = 2\pi rh$

Given $h = 4$ and $r = 2$, we get

$$A_{\text{bottom \& top}} = 2\pi(2^2) = 8\pi$$

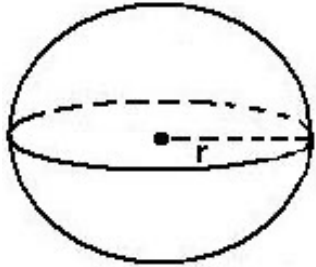
$$A_{\text{side}} = 2\pi(2)(4) = 16\pi$$

$$TSA = 8\pi + 16\pi = 24\pi \text{ (exact answer)}$$

Using the calculator and the π button, we find the total surface area of the cylinder to be approximately 75.4 m^2 . **Please note that the surface area units are squared.**

4. Volume and Surface Area Formulas of Spheres (1 of 2)

The sphere is defined as the 3-dimensional round solid figure in which every point on its surface is equidistant from its center. The fixed distance is called the radius r of the sphere. We obtain a sphere from a rotation of the two-dimensional circle.



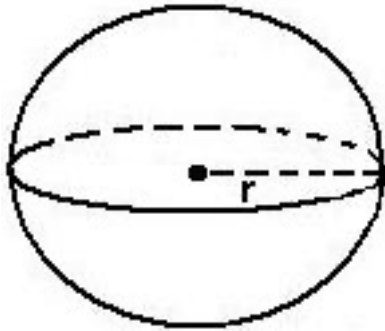
Volume Formula:

The formula for the volume of a sphere is obtained via calculus where we assume that a sphere is made up of numerous thin circular disks which are arranged one over the other.

$$V = \frac{4}{3} \cdot r^3 \cdot \pi$$

Volume and Surface Area of Spheres (2 of 2)

Total Surface Area (TSA) Formula:

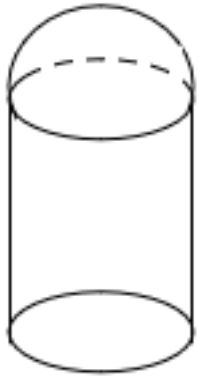


The formula for the total surface area of a sphere is also obtained via calculus. It is as follows:

$$TSA = 4 \cdot r^2 \cdot \pi$$

Example 3: Find the Volume (1 of 3)

Assume that you have a cylinder-shaped building with a domed roof as follows:



The dome is in the shape of a hemisphere (half of a sphere)! Find the volume of the entire structure. Find an exact answer first and then use a calculator to give a decimal approximation rounded to two decimal places.

Example 3: Find the Volume (2 of 3)

Here we must find volumes for two different shapes and then add the results. The diameter of the structure is 8 ft, therefore, the radius is 4 ft. This is what we are going to use in our formulas!

Required formulas: $V = \pi r^2 h$ and $V = \frac{4}{3} r^3 \pi$

Volume of the hemisphere

We are given $d = 8$ so that $r = 4$. Therefore, $V_{sphere} = \frac{4}{3}(4)^3\pi = \frac{256\pi}{3}$, which is the exact answer.

Given a hemisphere, which is equivalent to half of a sphere, we find its volume to be exactly $\frac{128\pi}{3} \text{ ft}^3$. **Please note that the volume units are cubed.**

Example 3: Find the Volume (3 of 3)

Volume of the cylinder

We are given $h = 12$ and $d = 8$ so that $r = 4$. Therefore,

$$V_{cylinder} = \pi(4)^2(12) = 192\pi, \text{ which is the exact answer.}$$

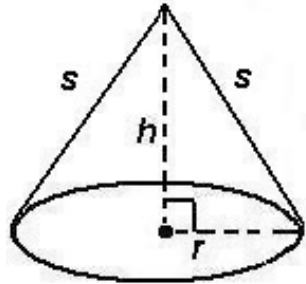
We find its volume of the cylinder to be to be exactly $192\pi \text{ ft}^3$. **Please note that the volume units are cubed.**

$$\text{Then } V_{building} = V_{sphere} + V_{cylinder} = \frac{128\pi}{3} + 192\pi = \frac{704\pi}{3}, \text{ which is the exact answer.}$$

Using the calculator and the π button, we find the volume of the building to be approximately 737.23 ft^3 . **Please note that the volume units are cubed.**

5. Volume Formula of Right Circular Cones

A cone is a 3-dimensional shape having a circular base with radius r and the sides s narrow smoothly to a point above the base. This point is known as apex. A right circular cone is a cone where the axis h of the cone meets the circular base at a right angle.



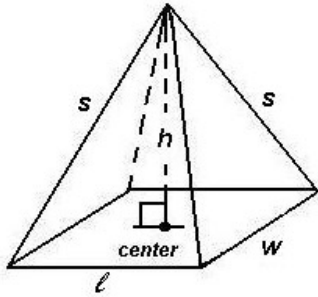
Volume Formula:

The formula for the volume of a cone is obtained via calculus.

$$V = \frac{1}{3} \cdot r^2 \cdot h \cdot \pi$$

6. Volume Formula of Right Pyramids

A pyramid is a 3-dimensional shape having a rectangular base with length l and width w . The sides s narrow smoothly to a point above the base. A right pyramid is a pyramid where the axis h of the pyramid meets the rectangular base at a right angle.



Volume Formula:

The formula for the volume of a pyramid is obtained via calculus.

$$V = \frac{1}{3} \cdot l \cdot w \cdot h$$