Circumference and Area of Circles

Based on power point presentations by Pearson Education, Inc.
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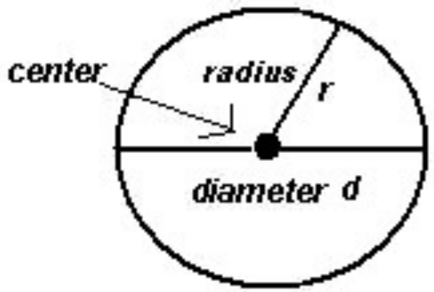
Learning Objectives

1. Memorize the definition of a circle.
2. Memorize and use the circumference formula of circles.
3. Memorize and use the area formula of circles.

1. Definition of a Circle

A circle is a 2-dimensional shape made by drawing a curve that is always the same distance from a center.



Radius

The radius ***r*** of a circle is the length of the line from the center of the circle to any point on its edge. The plural form is radii (pronounced "ray-dee-eye").

Diameter

The diameter ***d*** of a circle is a line segment between to points on the circle which passes through the **center** of the circle. The diameter is twice as long as the radius, ***d = 2r***, which is a line segment between one point on the circle and the center of the circle.

2. Circumference of Circles

The circumference *C* of circles is their perimeter. There exists a special relationship between the circumference of a circle and its diameter. That is, if we divide the circumference of ANY circle by its diameter, the quotient is always the same number, namely the number π (pi).

We can express this as, $\frac{C}{d} = \pi$ and given that ***d* = 2*r***, we can also state $\frac{C}{2r} = \pi$.

Remember that π is is a non-repeating, non-terminating decimal approximately equal to 3.141592654. **In this course, always use the π - button on your calculator and NOT the decimal approximation 3.14.**

When we solve both equations above for *C*, which is the circumference, we get two formulas for the circumference of circles, namely

$C = d\pi$ or $C = 2\pi r$

Example 1: Circumference of Circles

Find the circumference *C* of a circle whose diameter is 16 cm. First give an exact answer (express in terms of π) and then find the decimal equivalent rounded to a whole number.

Required formula: $C = d\pi$

Given is a diameter of $d = 16$, then $C = 16\pi$, which is the exact answer.

To find the decimal equivalent of this answer, we use the following calculator input:

16	×	π	**ENTER**

Note: Always use the π - button!

We find that the circumference *C* is approximately 50 cm.

Example 2: Circumference of Circles

Find the diameter *d* of a circle whose circumference is 36 meters. Round the answer to the nearest hundredth.

Required formula: $C = d\pi$

We are given $C = 36$. Therefore, $36 = d\pi$ and $d = \frac{36}{\pi}$ which is the exact answer.

To find the decimal equivalent of this answer, we use the following calculator input:

36	÷	π	ENTER

Note: Always use the π - button!

We find that the diameter *d* is approximately 11.46 m.

2. Area of Circles

There also exists a special relationship between the area *A* of a circle and the square of its radius. That is, if we divide the area by the square of the radius, the quotient is always the number π.

We can express this as $\frac{A}{r^2} = \pi$.

We can now solve for the area *A* to get the formula for the area of a circle, namely

$$A = \pi r^2$$

Example 3: Area of Circles

Find the area *A* of a circle whose diameter is 12 centimeters. Round the answer to the nearest hundredth.

Required formulas: $A = \pi r^2$ and $d = 2r$

We are given $d = 12$ so that $r = 6$. Therefore, $A = 6^2\pi$ and $A = 36\pi$, which is the exact answer.

To find the decimal equivalent of this answer, we use the following calculator input:

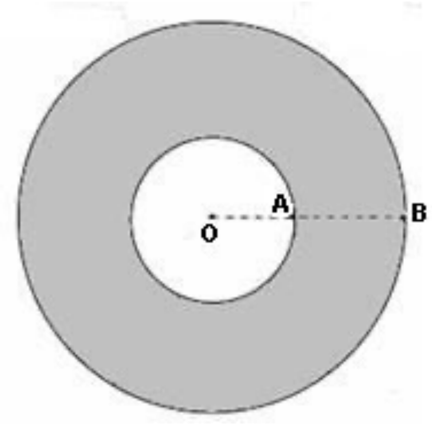
36	×	π	ENTER

Note: Always use the π - button!

We find that the area *A* is approximately 113 cm^2. **Please note that the area units are squared.**

Example 4: Area of Circles (1 of 2)

Given the picture below, find the area of the shaded ring rounded to the nearest hundredth. Assume that the point O is the center of both the small and the large circle. The distance OA, which is the radius of the small circle, is 5 meters and the distance OB, which is the radius of the large circle, is 11 meters. First give an exact answer (express in terms of π) and then find the decimal equivalent rounded to a whole number.



We can find the area of the shaded ring by subtracting the area of the small circle from the area of the large circle!

Example 4: Area of Circles (2 of 2)

Required formula: $A = \pi r^2$

Area of large circle:

We are given $r = 11$. Therefore, $A = 11^2\pi$ and $A = 121\pi$, which is the exact answer.

Area of small circle:

We are given $r = 5$. Therefore, $A = 5^2\pi$ and $A = 25\pi$, which is the exact answer.

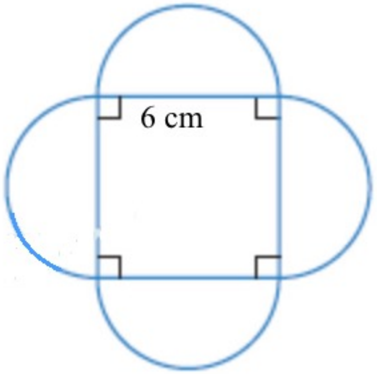
Then $A_{shaded} = 121\pi - 25\pi = 96\pi$, which is the exact answer.

Using the calculator, we find the area of the shaded ring to be approximately 302 m^2.

Please note that the area units are squared.

Example 5: Area of Circles (1 of 2)

Find the area of the following figure which consists of semi-circles attached to each side s of a square. First given an exact answer (express in terms of π) and then find the decimal equivalent rounded to two decimal places.



- A semi-circle is half of that of a circle! To find the area of a semi-circle we divide the area of a circle by 2.

Example 5: Area of Circles (2 of 2)

Required formulas: $A_{circle} = \pi r^2$ and $d = 2r$ *and* $A_{square} = s^2$

Area of the semi-circles

We are given $d = 6$ so that $r = 3$. Therefore, $A_{circle} = 3^2\pi = 9\pi$, which is the exact answer.

Given 4 semi-circles, which is equivalent to 2 circles each with area $A = 9\pi$, we find the total area of the semi-circles in the figure to be 18π.

Area of the square

Given side s of the square to be 6 cm, then $A_{square} = 6^2 = 36$.

Then $A_{figure} = 18\pi + 36$, which is the exact answer.

Using the calculator, we find the area of the given figure to be approximately 92.55 cm^2. **Please note that the area units are squared.**