Introduction to Polygons

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Name certain polygons according to the number of sides and find the sum of their angles.
2. Recognize the characteristics of triangles and name certain triangles.
3. Solve problems using similar triangles.
4. Recognize the characteristics of quadrilaterals and name certain quadrilaterals.

1. Definition of Polygons (1 of 2)

Polygon

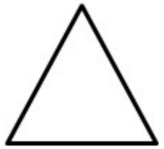
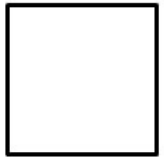
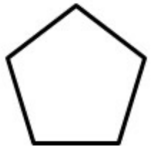
Any closed shape in the plane formed by three or more line segments that intersect only at their endpoints.

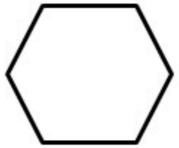
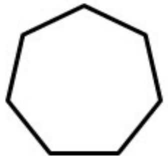
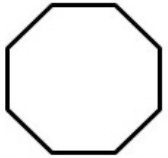
Regular Polygon

Has sides which are all the same length and angles of all the same measure.

Definition of Polygons (2 of 2)

Regular Polygons

Name	Picture
Triangle 3 sides	
Quadrilateral 4 sides	
Pentagon 5 sides	

Name	Picture
Hexagon 6 sides	
Heptagon 7 sides	
Octagon 8 sides	

The sum of the interior angles of a polygon with n sides is given by $180^{o}(n - 2)$. Note: “Interior angles” are the angles inside a shape!

Example 1: Find the Sum of the Angles of a a Polygon

Find the sum of the interior angles of an octagon.

The sum of the interior angles of a polygon with *n* sides is given by $180^o(n - 2)$.

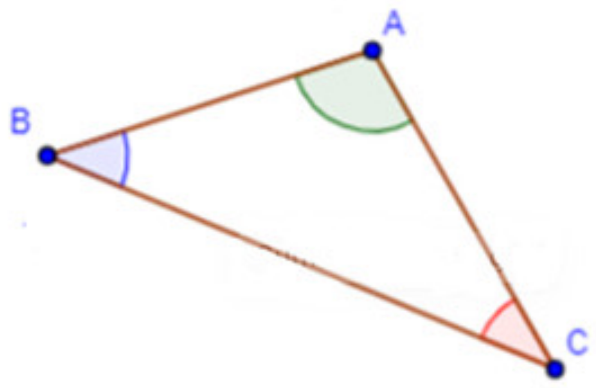
We know that an octagon has 8 sides. Using this formula with $n = 8$, we get the following:

$180^o(8 - 2) = 180^o(6) = 1080^o$

The sum of the interior angles of an octagon is 1080^o.

2. The Triangle (1 of 3)

One of the important polygons is the 3-sided polygon called a **triangle**. Due to the rigidity of its shapes, physicists proved that the triangle can withstand high amounts of force without being deformed. Therefore, architects and engineers use triangles when building bridges, roofs on houses, and other structures. They have found that using triangles when building structures adds strength by reducing lateral movement.



Interior Angles of a Triangle

The **interior angles** of a triangle are the angles inside the triangle. The sum of the measures of the three **interior angles** is always 180^o.

Sides of a Triangle

The longest side of a triangle is opposite the largest angle and vice versa. The shortest side of a triangle is opposite the smallest angle and vice versa

2. The Triangle (2 of 3)

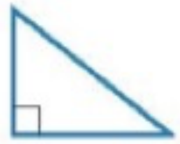
There are five types of triangles as follows:

Acute Triangle

All interior angle measures are greater than 0^o but less than 90^o.

Right Triangle

One interior angle measure is 90^o. This angle, also called right angle, is usually indicated by a rectangle as follows:



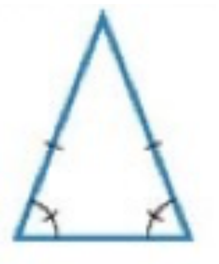
Obtuse Triangle

One interior angle measure is more than 90^o but less than 180^o.

2. The Triangle (3 of 3)

Isosceles Triangle

Two sides of this triangle have equal length and interior angles opposite these sides have equal measure.

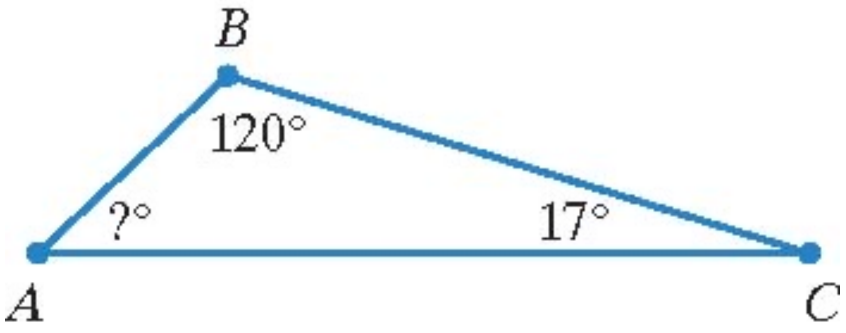


Equilateral Triangle

All sides of this triangle have equal length and all interior angle measures are 60^o.

Example 2: Use Angle Relationships in Triangles

Find the measure of $\angle A$ for the triangle *ABC*.



We know that $m\angle A + m\angle B + m\angle C = 180^o$

Given $m\angle B = 120^o$ and $m\angle C = 17^o$

then $m\angle A = 180^o - 120^o - 17^o = 43^o$

3. Similar Triangles (1 of 3)

Vocabulary

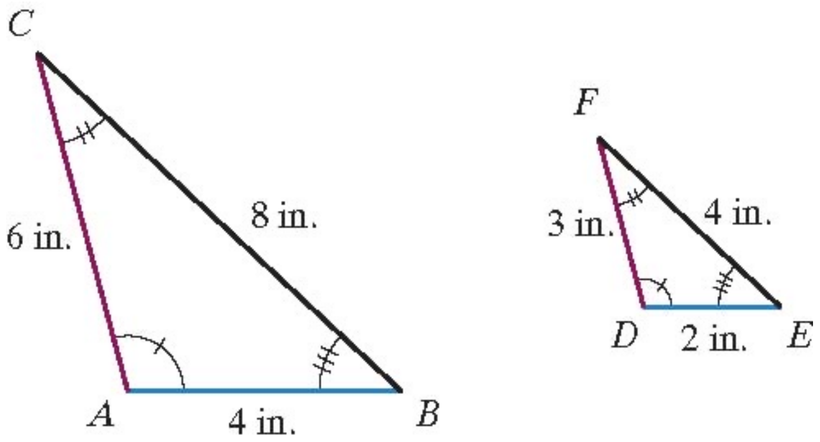
Corresponding angles - angles that have the same measure in two given triangles.

Corresponding sides - sides opposite the corresponding angles in two given triangles.

Similar triangles – two triangles with corresponding angles and the corresponding sides are proportional.

Similar Triangles

Below is an example of two similar triangles.



Let's examine them. The measures of angle ***C*** and angle ***F*** are equal because their arcs are intersected by 2 tick marks. We say that angles ***C*** and ***F*** are **corresponding angles**.

Likewise, the measures of angle ***A*** and angle ***D*** are equal because their arcs are intersected by 1 tick mark. ***A*** and ***D*** are also **corresponding angles**.

Finally, the measures of angle ***B*** and angle ***E*** are equal because their arcs are intersected by 3 tick marks. ***B*** and ***E*** are also **corresponding angles**.

Similar Triangles (3 of 3)

Often, we are asked to “solve” similar triangles. “Solving” similar triangles means that you must find the measure of a missing side or angle in one of the triangles. Following is the procedure:

Step 1

Read the problem and represent the unknown quantity by a variable (most common is ***x***, but it can be any letter).

Step 2

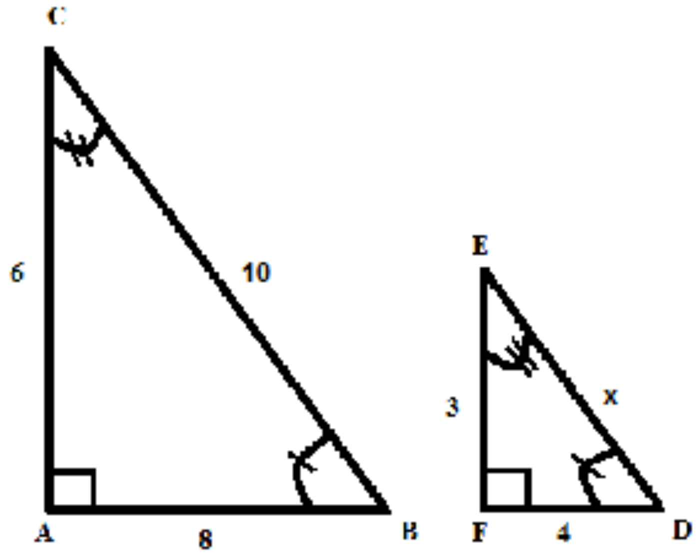
Set up a proportion by creating two ratios, each consisting of corresponding sides, one from the large triangle and one from the small triangle. Each quantity must occupy the same position in the ratios on either side of the equal sign.

Step 3

Apply *cross-multiplication* and find a solution for the variable.

Example 3: Solve Similar Triangles (1 of 2)

Given are two similar triangles with their sides measured in inches. Find *x* which is the measure of side *DE*.



NOTE: The picture shows that all three angles of the two triangles are equal (compare tick marks, right angle). Therefore, the triangles are similar, and their corresponding sides are proportional.

Example 3: Solve Similar Triangles (2 of 2)

We set up a proportion by creating two ratios, each consisting of corresponding sides, one from the large triangle and one from the small triangle. **Each quantity must occupy the same position in the ratios on either side of the equal sign.**

$$\frac{6}{3} = \frac{10}{x}$$

To solve this, we will use *cross-multiplication.*

$$\frac{6}{3} \times \frac{10}{x}$$

6*x* = 30

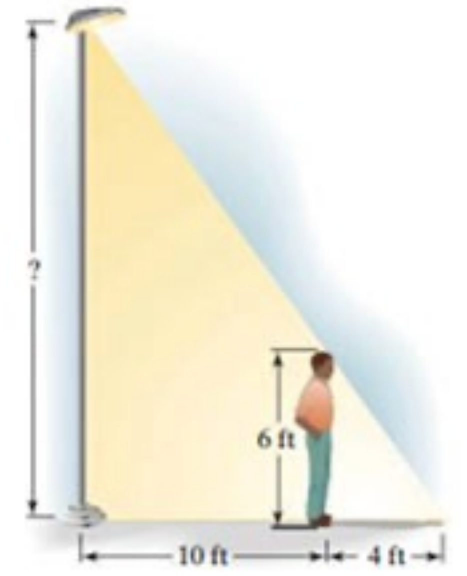
and *x* = 5

The missing length *x* is 5 inches.

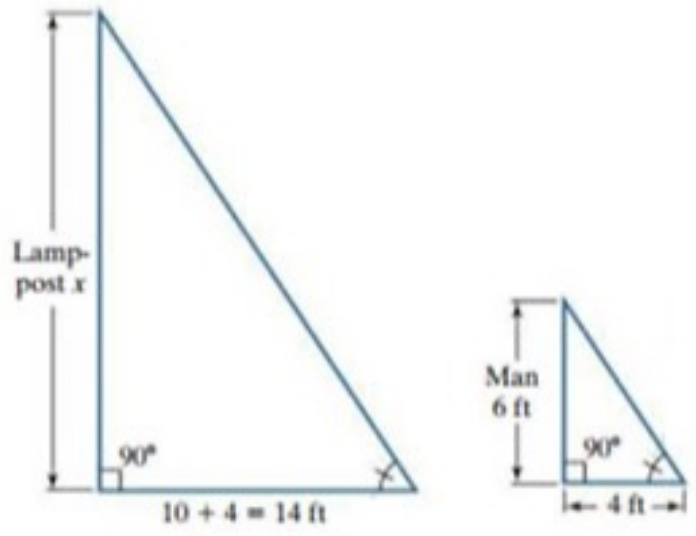
Example 4: Sole Similar Triangles (1 of 2)

A 6-foot tall man is standing 10 feet from a lamp post and his shadow is 4 feet long. How tall is the lamp post?

Let's draw some pictures.



Following are two similar triangles we can get from the picture on the left. Notice that the angles in both triangles are equal!



Example 4: Solve Similar Triangles (2 of 2)

We find the measure of the lamp post by setting up a proportion. We create two ratios, each consisting of corresponding sides, one from the large triangle and one from the small triangle. Each quantity must occupy the same position in the ratios on either side of the equal sign.

$$\frac{x}{6} = \frac{14}{4}$$

To solve this, we will use *cross-multiplication.*

$$\frac{x}{6} \times \frac{14}{4}$$

4x = 84 and x = 21

The lamp post is 21 feet tall.

4. The Quadrilaterals (1 of 2)

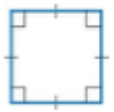
Another important polygon is the 4-sided polygon called a **quadrilateral**. There are five types of quadrilaterals as follows:

Rectangle

Both pairs of opposite sides are parallel, and each pair has the same measure. All interior angles are right angles.

Square

A rectangle with all sides having equal length.



The Quadrilaterals (2 of 2)

Parallelogram

Both pairs of opposite sides are parallel and have the same measure. Opposite interior angles have the same measure. No interior angles are right angles.

Rhombus

Parallelogram with all sides having equal lengths.

Trapezoid

Exactly one pair of parallel sides.