Linear and Constant Functions

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Define linear functions.
2. Graph linear functions using the Intercept Method and the Point-by-Point Plotting Method.
3. Define and graph constant functions.

1. The Linear Function (1 of 2)

The graph of the linear function is a straight line.

The linear function is defined as ***f(x) = mx + b***,

where $m \neq 0$ is the slope of the graph of the function and *b* is the *y*-intercept.

Domain: *All Real Numbers*

Range: *All Real Numbers*

Please note that the definition of a linear function is nothing but the *slope-intercept form* of a linear equation in two variables, which is $y = mx + b$.

The Linear Function (2 of 2)

Special Case of the Linear Function - *The Identity Function*

The *Identity Function* is also called the *common* linear function. It is defined as

f*(x) = x*, where $m = 1$ and $b = 0$

2. Graphs of Linear Functions

Since linear functions are just linear equations in two variables, the graphs of linear functions are lines.

We graph linear functions just like linear equations in two variables. That is, we use either the *Intercept Method* *or* the *Point-by-Point Plotting Method* or a combination of both.

Example 1: Graph a Linear Function (1 of 3)

Graph the *Identity Function* $f(x) = x$.

Let's try the Intercept Method!

x-intercept:

Given $f(x) = x$, let $f(x) = y = 0$ and solve for x.

$$0 = x$$

The *x*-intercept is 0 and the point associated with the *x*-intercept is (0, 0), which is the origin (the point at which the two coordinate axes intersect).

y-intercept:

Given $f(x) = x$, let $x = 0$ and solve for $f(x) = y$.

$$f(0) = 0 = y$$

The *y*-intercept is 0 and the point associated with the y-intercept is also (0, 0).

Example 1: Graph a Linear Function

Most often the *Intercept Method* produces two different points. However, this does not always happen. In our case, the *Intercept Method* only produced one point, namely (0, 0).

Since we need at least two points to graph a line, we will use the *Point-by-Point Plotting Method* to find at least one more point. How about we find two points by letting x be equal to –2 and 2? This way we have a point above the origin and below the origin.

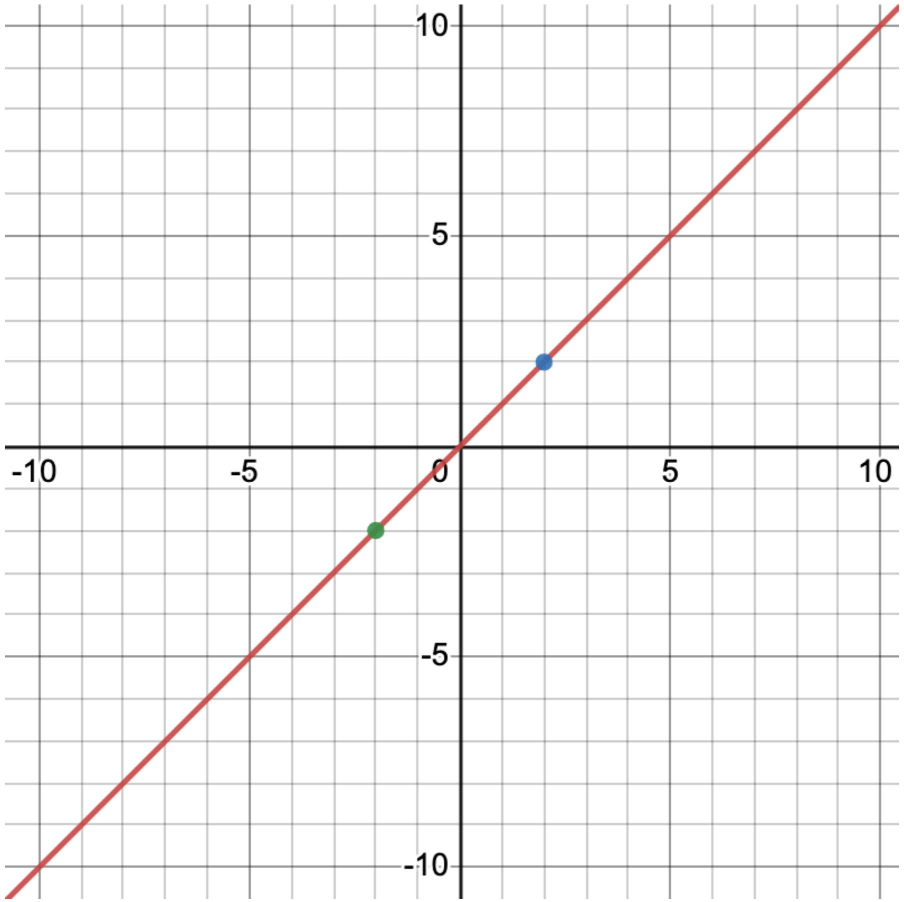
Given $f(x) = x$, we then find $f(-2) = y = -2$ and $f(2) = y = 2$.

The coordinates of the additional points are (–2, –2) and (2, 2).

NOTE: In the *Identity Function*, EVERY ordered pair has x- and y-coordinates that are equal!

Example 1: Graph a Linear Function (3 of 3)

Graph the identity function by drawing a line through the points found in the previous three slides.



Reminder:
We found the coordinates (–2, –2), (0, 0), and (2, 2).

Example 2: Graph a Linear Function (1 of 3)

Graph the linear function $g(x) = \frac{3}{2}x - 3$.

Let’s try the Intercept Method!

x-intercept:

Given $g(x) = \frac{3}{2}x - 3$, let $g(x) = y = 0$ and solve for x.

$$0 = \frac{3}{2}x - 3$$

$$3 = \frac{3}{2}x$$

$$3\left(\frac{2}{3}\right) = x$$

$$2 = x$$

The *x*-intercept is 2 and the point associated with it is (2, 0).

Example 2: Graph a Linear Function (2 of 3)

y-intercept:

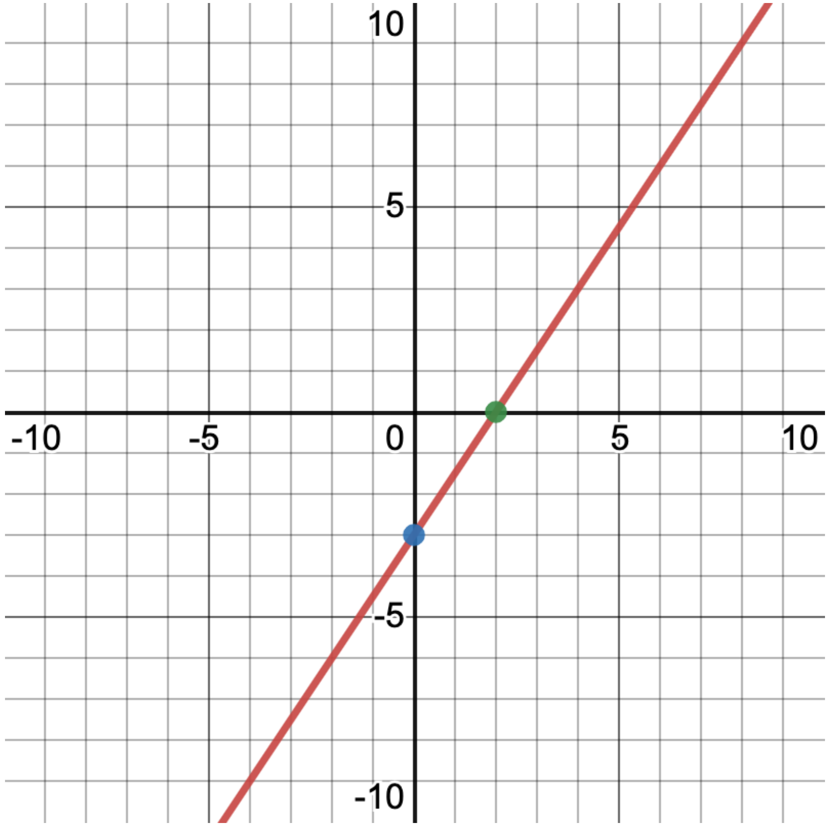
Given $g(x)=\frac{3}{2}x-3$, let x = 0 and solve for $g(x) = y$.

$$g(0)=\frac{3}{2}(0)-3=y$$

The *y*-intercept is −3 and the point associated with it is (0, −3).

Example 2: Graph a Linear Function (3 of 3)

Graph the function by drawing a line through the two points associated with the intercepts.



Reminder:
We found the coordinates (2, 0) and (0, –3).

Example 3: Graph a Linear Function

Graph the linear function h(x) = 2x.

Let's try the Intercept Method!

x-intercept:

Given h(x) = 2x, let h(x) = y = 0 and solve for x

0 = 2x

0 = x (dividing both sides by 2!)

The x-intercept is 0 and the point associated with the x-intercept is (0, 0), which is the origin (the point at which the two coordinate axes intersect).

Example 3: Graph a Linear Function (2 of 4)

y-intercept:

Given h(x) = 2x, let x = 0 and solve for h(x) = y.

h(0) = 2(0) = y

The *y*-intercept is 0 and the point associated with the y-intercept is also (0, 0).

Example 3: Graph a Linear Function (3 of 4)

Most often the *Intercept Method* produces two different points. However, this does not always happen. In our case, the *Intercept Method* only produced one point, namely (0, 0).

Since we need at least two points to graph a line, we will use the *Point-by-Point Plotting Method* to find at least one more point. How about we find two points by letting x be equal to –2 and 2? This way we have a point above the origin and below the origin.

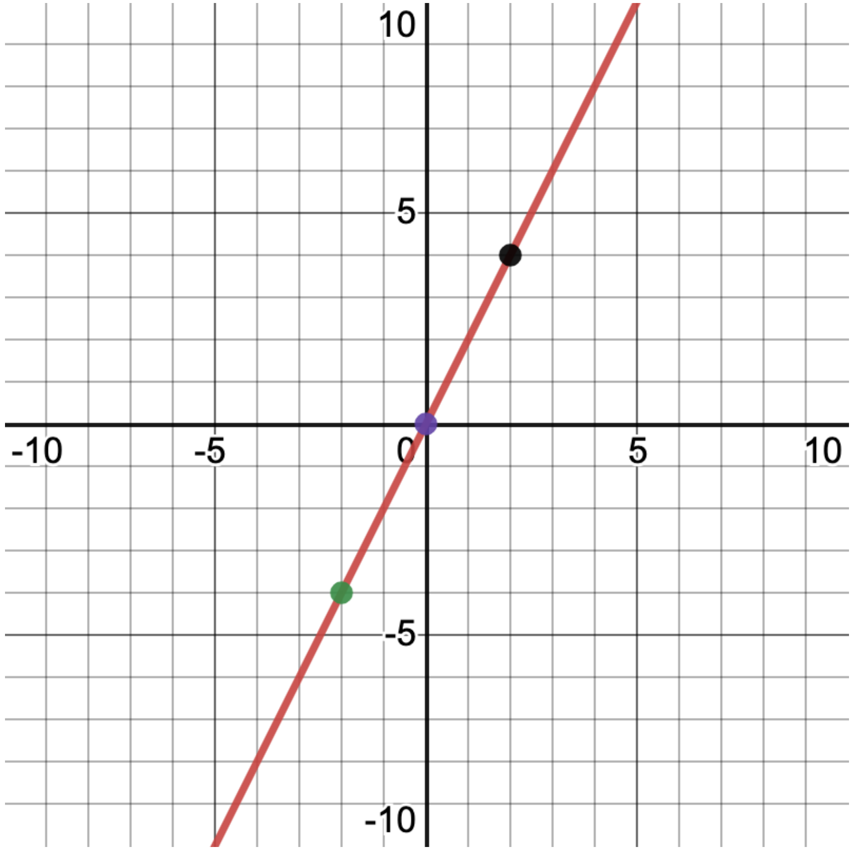
Given $h(x) = 2x$, we calculate $h(-2) = y = 2(-2) = -4$

and $h(2) = y = 2(2) = 4$

The coordinates of the additional points are (–2, – 4) and (2, 4).

Example 3: Graph a Linear Function (4 of 4)

Graph the function by drawing a line through the points found in the previous three slides.



Reminder:

We found the coordinates (–2, –4), (0, 0), and (2, 4).

3. The Constant Function and its Graph

The *Constant Function* is defined as **f(*x*) = *b***,

where *m* = 0 is the slope of the graph of the function and *b* is the *y*-intercept.

Domain: *All Real Numbers* or $(-\infty, \infty)$ in *Interval Notation*.

Range: Consists of ONE number, namely *b* or **{b}** in *Interval Notation.*

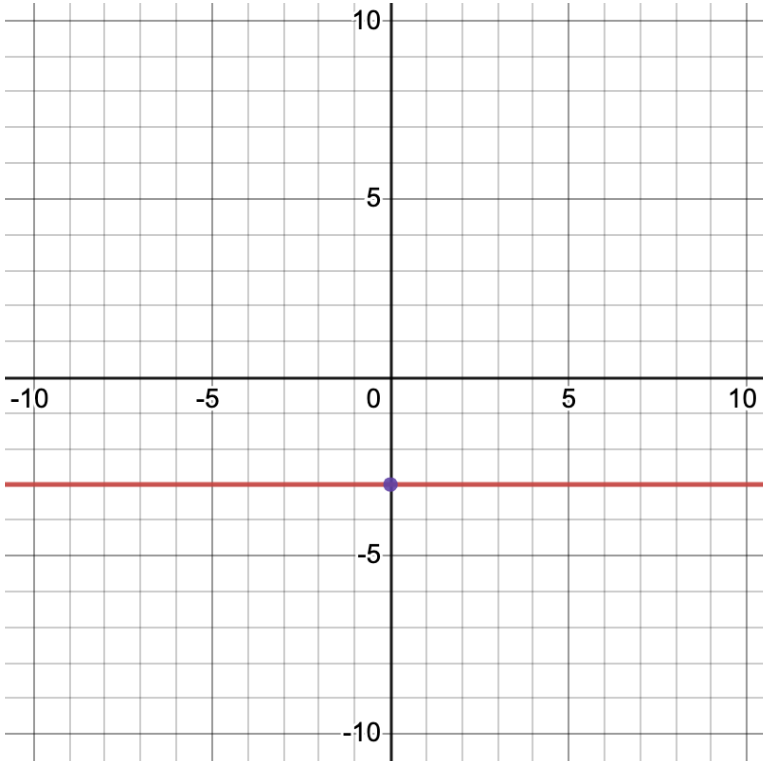
The graph of a constant function is a horizontal line. It is nothing but *y* = *b*, an equation of the horizontal line we learned previously.

Example 4: Graph a Constant Function

Graph the *Constant Function* k(x) = – 3.

As stated already, the graph of the *constant function* is a horizontal line.

We know that the y-intercept is – 3, therefore the point associated with the y-intercept must be (0, – 3).



In the picture, we drew the point (0, – 3), which is the point associated with the y-intercept.

Then, we used our knowledge that the graph of the *constant function* is a horizontal line to draw a line through (0, – 3) parallel to the x-axis.