The Slope of a Line

Based on power point presentations by Pearson Education, Inc.

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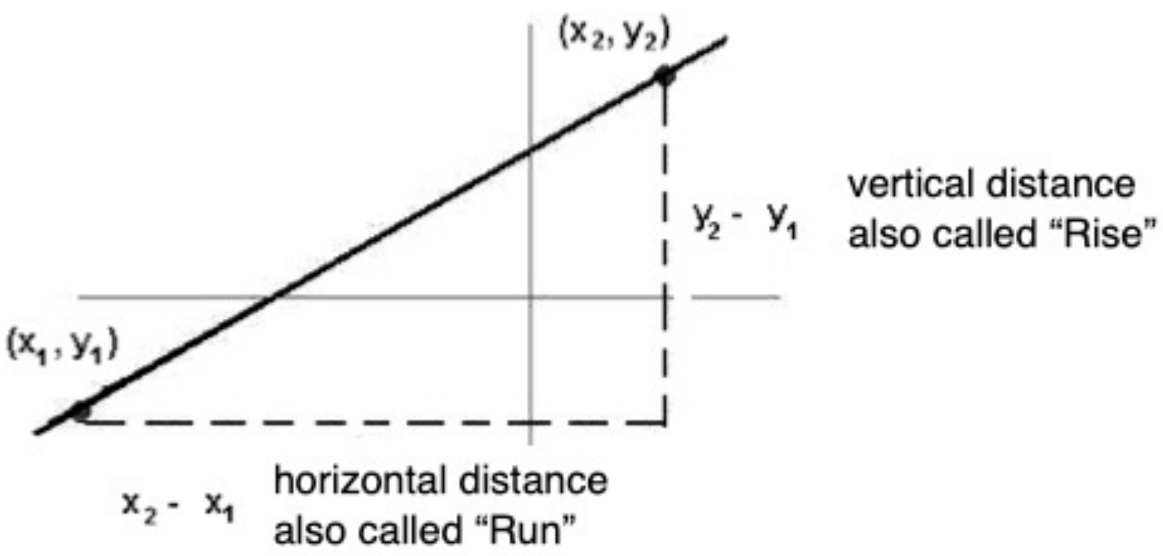
Learning Objectives

1. Define and calculate the slope of a line.
2. Identify the slopes of increasing, decreasing, vertical, and horizontal lines.
3. Identify the slope and the y-intercept in the equation of a line.
4. Write the slope-intercept form of the equation of a line, if possible.

1. Definition of the Slope of a Line (1 of 2)

Before we continue with our discussion on functions, lets first discuss the slope of a line. In layman's terms, the slope is a measure of the **steepness of a line**. It is said to be the change in vertical distance divided by the change in horizontal distance as we "travel" from one point, say (x_1, y_1), to another, say (x_2, y_2) lying on the same line in a rectangular coordinate system.



Definition of the Slope of a Line (2 of 2)

In mathematics, the slope of a line is indicated by using the lower-case letter *m*. Why *m*? No one knows for sure. Some mathematicians claim the *m* comes from the French word “monter” which means “to climb”.

The slope of the line through two distinct points (x_1, y_1) and (x_2, y_2) lying in a coordinate system is formally defined as

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

NOTE: Regardless of the sign of the *x*-coordinates or the *y*-coordinates, the minus sign between the *y*-values and the *x*-values in the slope calculation must always be there.

We can also say $m = \frac{\text{Rise}}{\text{Run}}$ or $m = \frac{\textit{change in y}}{\textit{change in x}}$

Example 1: Calculate the Slope of a Line

Find the slope of the line passing through the points (4, −2) and (−1, 5).

Here you can say that (4, −2) is (x_1, y_1) and (−1, 5) is (x_2, y_2). However, you can also state that (4, −2) is (x_2, y_2) and (−1, 5) is (x_1, y_1). In either case, you will get the same answer.

Let's say that (4, −2) is (x_1, y_1) and (−1, 5) is (x_2, y_2). Be sure not to get confused! Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-2)}{-1-4} = \frac{7}{-5} = -\frac{7}{5}$$

NOTE: Regardless of the sign of the *x*-coordinates or the *y*-coordinates, the minus sign between the *y*-values and the *x*-values in the slope calculation must always be there.

Example 2: Calculate the Slope of a Line

Find the slope of the line passing through the points (− 1, 3) and (− 4, − 6).

Here you can say that (− 1, 3) is (x_1, y_1) and (− 4, − 6) is (x_2, y_2). However, you can also state that (− 4, − 6) is (x_1, y_1) and (− 1, 3) is (x_2, y_2). In either case, you will get the same answer.

Let's say that (− 4, − 6) is (x_1, y_1) and (− 1, 3) is (x_2, y_2). Be sure not to get confused! Then

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{3-(-6)}{-1-(-4)}=\frac{3+6}{-1+4}=\frac{9}{3}=3$$

NOTE: Regardless of the sign of the *x*-coordinates or the *y*-coordinates, the minus sign between the *y*-values and the *x*-values in the slope calculation must always be there.

Example 3: Calculate the Slope of a Line

Find the slope of the line passing through the points (6, 3) and (6, 4).

Let's say that (6, 3) is (x_1, y_1) and (6, 4) is (x_2, y_2). Be sure not to get confused! Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-3}{6-6} = \frac{1}{0}$$

Since there is a 0 in the denominator, this particular slope is undefined.

Example 4: Calculate the Slope of a Line

Find the slope of the line passing through the points (1, 5) and (− 9, 5).

Let's say that (1, 5) is (x_1, y_1) and (− 9, 5) is (x_2, y_2). Be sure not to get confused! Then

$$m = \frac{y_2 - y_1}{x_2 - y_{x1}} = \frac{5-5}{-9-1} = \frac{0}{-10} = 0$$

Since there is a 0 in the numerator, this particular slope equals 0. Please note the difference between Example 3 and Example 4!

2. Identify the Slopes of Lines (1 of 2)

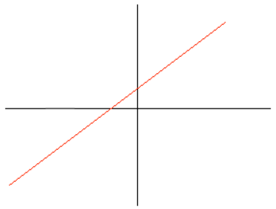
We learned that the slope of a line is its steepness. Specifically, the slope of the line through two distinct points (x_1, y_1) and (x_2, y_2) lying in a coordinate system is formally defined as

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Now, we will investigate how the slope affects the characteristics of a line.

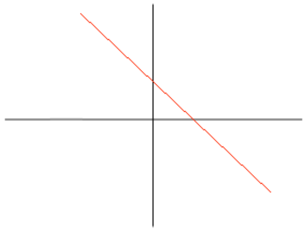
SLOPE OF INCREASING (RISING) LINES

An increasing line has a positive slope.



SLOPE OF DECREASING (FALLING) LINES

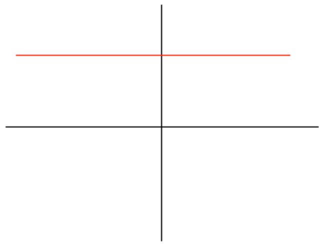
A decreasing line has a negative slope.



Identify the Slopes of Lines (2 of 2)

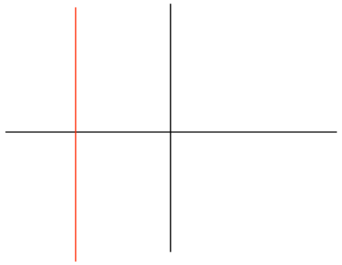
SLOPE OF HORIZONTAL LINES

All horizontal lines have a slope of 0.



SLOPE AND VERTICAL LINES

All vertical lines have an undefined slope.



Example 5: Identify the Slopes of Lines

Identify the slopes of the graphs of the following lines. State whether the lines are increasing, decreasing, horizontal, or vertical.

1. $y = 3x + 9$

   $m = 3$, the slope is positive, therefore, the line is an increasing

2. $y = -5x - 2$

   $m = -2$, the slope is negative, therefore, the line is an decreasing

3. $y = 6$

   horizontal line, $m = 0$

4. $x = -1$

   vertical line, m is undefined

3. The Slope-Intercept Form of the Equation of a Line (1 of 2)

We have already discussed the general form of the linear equation. It is $Ax + By + C = 0$, where A, B, and C are real numbers, but A and B cannot both be 0.

Now we are going to discuss a different form of the linear equation. It is called the **slope-intercept form** and is defined as $\boldsymbol{y = mx + b}$, where m is the slope and b is the y-intercept.

Examples of linear equations in slope-intercept form:

$y = -18x + 11$ (here $b = 11$)

y= 5x – 7 (here $b = -7$)

$y = 2x$ (here $b = 0$)

The Slope-Intercept Form of the Equations of a Line (2 of 2)

While the two forms look totally different, it is actually quite easy to change from one form to the other.

Let's use $-18x - y + 11 = 0$, for example. It is a linear equation in general form. Let's do the following manipulations:

$-y + 11 = 18x$ (added $18x$ to both sides)

$-y = 18x - 11$ (subtracted 11 from both sides)

$y = -18x + 11$ (multiplied both sides by -1)

As you can see, y is now isolated on one side. The equation is now in slope-intercept form with $m = -18$ and $b = 11$,

Example 6: Identify the Slope and the *y*-Intercept

Identify the slope, the *y*-intercept, and the ordered pair associated with the *y*-intercept given the linear equation $5x + 4y - 9 = 0$.

The equation is in general form. We must change it to slope-intercept form $y = mx + b$.

We will move the *x*-term and the constant to the right side of the equation into its proper position next to the equal sign as follows

$4y = -5x + 9$

Next, we divide both sides of the equation by 4 to get the following:

$$y = -\frac{5}{4}x + \frac{9}{4}$$

We find that the slope is $-\frac{5}{4}$ and the *y*-intercept is $\frac{9}{4}$.

The ordered pair associated with the *y*-intercept is $\left(0, \frac{9}{4}\right)$.

Example 7: Write the the equation of a line (1 of 2)

Find the equation of a line with slope −5 that passes through the point (−1, −4). If possible, express in slope-intercept form.

We are going to use $m = -5$ and the point (−1, − 4) and place them into $y = mx + b$ as follows:

$-4 = -5(-1) + b$

This allows us to find *b.*

$-4 = 5 + b$

$-9 = b$

Example 7: Write the the equation of a line (2 of 2)

Given $m = -5$ and $b = -9$, we can now write the slope-intercept equation of a line with slope −5 that passes through the point (−1, −4) as follows:

$y = -5x - 9$

Example 8: Write the Equation of a Line (1 of 3)

Find the equation of the line whose graph passes through the points (4, −2) and (−1, 5). Express in slope-intercept form.

Since we are asked to express the equation in slope-intercept form $y = mx + b$, we must find m and b.

Let’s find m first. Here you can say that (4, −2) is (x_1, y_1) and (−1, 5) is (x_2, y_2). However, you can also state that (4, −2) is (x_2, y_2) and (−1, 5) is (x_1, y_1). In either case, you will get the same answer.

Let’s say that (4, −2) is (x_1, y_1) and (−1, 5) is (x_2, y_2). Be sure not to get confused! Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-2)}{-1-4} = \frac{7}{-5} = -\frac{7}{5}$$

Example 8: Write the Equation of a Line (2 of 3)

Now we are going to use $m = -\frac{7}{5}$ and one of the given points, say (4, −2), and replace the *m, x, y* in *y = mx + b* as follows:

$$-2 = -\frac{7}{5}(4) + b$$

This allows us to find *b*.

$$-2 = -\frac{7}{5}(4) + b$$

$$-2 + \frac{28}{5} = b$$

To combine fractions, we need a common denominator. We will write – 2 as $-\frac{10}{5}$.

and find $b = \frac{18}{5}$.

Example 8: Write the Equation of a Line (3 of 3)

Given $m = -\frac{7}{5}$ and $b = \frac{18}{5}$, we can now write the equation of the line whose graph passes through the points (4, −2) and (−1, 5) in slope-intercept form as follows:

$$y = -\frac{7}{5}x + \frac{18}{5}$$

NOTE: In algebra we usually leave the equation in fraction form. We usually DO NOT change improper fractions to mixed numbers. Also, we usually do not change fractions to decimals, however, there are some exceptions to this convention

Example 9: Write the the equation of a line (1 of 2)

Find the equation of the line that passes through the points (4, 2) and (−1, 2). If possible, express in slope-intercept form.

Since we are asked to express the equation in slope-intercept form $y = mx + b$, we must find m and b.

Let's find m first. Here you can say that (4, 2) is (x_1, y_1) and (−1, 2) is (x_2, y_2). However, you can also state that (4, 2) is (x_2, y_2) and (−1, 2) is (x_1, y_1). In either case, you will get the same answer.

Let's say that (4, 2) is (x_1, y_1) and (−1, 2) is (x_2, y_2). Be sure not to get confused! Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-2}{-1-4} = \frac{0}{-5} = 0$$

Example 9: Write the the equation of a line (2 of 2)

Now we are going to use $m = 0$ and one of the given points, say (4, 2) and place them into $y = mx + b$ as follows:

$2 = 0(4) + b$

This allows us to find b.

$2 = 0 + b$

$2 = b$

Given $m = 0$ and $b = 2$, we can now write the slope-intercept equation of the line that passes through the points (4, 2) and (−1, 2) as follows:

$y = 0x + 2$

and lastly, we will write $y = 2$ which is the equation of a horizontal line.

Example 10: Find the Equation of a Line

Find the equation of the line whose graph passes through the points (−2, 6) and (−2, −8). If possible, express in slope-intercept form.

Since we are asked to express the equation in slope-intercept form $y = mx + b$, we must find m and b.

Let's find m first. Here you can say that (−2, 6) is (x_1, y_1) and (−2, −8) is (x_2, y_2). However, you can also state that (−2, 6) is (x_2, y_2) and (−2, −8) is (x_1, y_1). In either case, you will get the same answer.

Let's say that (−2, 6) is (x_1, y_1) and (−2, −8) is (x_2, y_2). Be sure not to get confused! Then

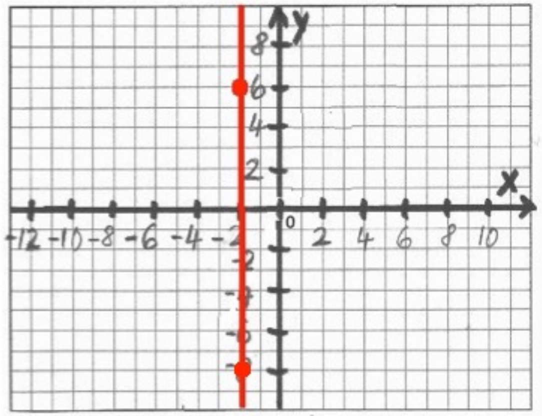
$$m = \frac{-8-6}{-2-(-2)} = \frac{-14}{-2+2} = \frac{-14}{0}$$

We note that the slope is undefined.

Example 10: Write the the equation of a line (2 of 2)

We learned that only vertical lines have undefined slopes. Their equation is in the form $\boldsymbol{x = a}$ where $\boldsymbol{a}$ is the x-intercept.

To find the equation of this line, we will plot the two given points $(-2, 6)$ and $(-2, -8)$ and connect them.



We find that this line intersects the x-axis at the point $(-2, 0)$. Therefore, the x-intercept must be -2.

The equation of the line is $x = -2$.