Mathematical Statements; Solving and Using Formulas

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Change word problems to mathematical statements.
2. Solve and use mathematical formulas.

1. Strategy for Writing Mathematical Statements (1 of 2)

- Read the problem carefully several times until you can state in your own words what is given and what the problem is looking for. Let x (or any variable) represent on of the unknown quantities in the problem.
- Write a mathematical statement in x that models the verbal conditions of the problem.

Strategy for Writing Mathematical Statements (2 of 2)

Key Words to look for in word problems:

- Addition: the sum of, plus, increased by, more than, added to, exceeds, longer, total, heavier, older, wider, taller, gain, greater than, more, gain
- Subtraction: less than, decreased by, subtracted from, the difference between, diminished by, take away, reduced by, less, minus, shrinks, younger, lower, shorter, narrower, slower, loss
- Multiplication: times (two times, three times, etc.), multiply, of, the product of, multiplied by, twice, double, triple
- Division: divide, divided by, divided into, how big is each part, how many parts can be made from, the quotient of
- Equals: is

Example 1: Write a Mathematical Statement

Write the following as a mathematical statement: 5 more than a number is 12. Then solve the equation

"more than" means addition

"a number" is what we are finding, and we will call it x

"is" means equals

$5 + x = 12$

and $x = 7$ which is the number we were asked to find.

Example 2: Write a Mathematical Statement

Write the following as a mathematical statement: 4 times the difference of a number and 3 is 12. Then solve the equation.

"4 times" means multiplication

"difference" means subtraction

"a number" is what we are finding, and we will call it *x.*

"is" means equals

$4(x - 3) = 12$

NOTE: We had to use parentheses because we were asked to find "4 times the difference".

Then $4x - 12 = 12$

$4x = 24$

and $x = 6$ which is the number we were asked to find.

Example 3: Write a Mathematical Statement

Write the following as a mathematical statement: the sum of 12, 7, and a third number is 17. Then solve the equation

"sum" means addition

"a number" is what we are finding, and we will call it x.

"is" means equals

$12 + 7 + x = 17$

$19 + x = 17$

$x = -2$ which is the number we were asked to find.

Example 4: Write a Mathematical Statement

After Ben pumped 5 gal of gas, the pump showed a price of $18.10. When he was finished pumping, the price displayed was $52.49. How many gallons of gas did he pump? Use your calculator!

To solve this problem, we will set up a proportion with prices in the numerators and gallons in the denominators.

Let *x* be the number of gallons pumped.

$$\frac{18.10}{5} = \frac{52.49}{x}$$

Using the *cross-multiplication principle*, we have ***18.10x = 5(52.49)***.

Then, ***18.10x = 262.45***

and ***x = 14.5***

Ben pumped a total of 14.5 gallons of gas.

2. Solve and Use Mathematical Formulas

A mathematical **formula** is an equation that uses variables to express a relationship between two or more quantities. They are used in the natural sciences, engineering, and in the social sciences (economics, psychology, sociology, political science).

To solve a formula for one of its variables, we treat a specified variable as if it is the only variable in the equation. The other variables are treated just like numbers.

We isolate the specified variable on one side of the equation, usually the left side, and leave everything else on the other side of the equation.

Solving formulas in this manner often shortens our work when doing repeated formula evaluations.

Example 5: Solve a Formula for One of its Variables

Solve the formula $M = S - C$ for S.

We will isolate the variable S by "moving" away all terms associated with it.

The variable C is associated with S through subtraction. Therefore, we must do the opposite operation and add C to both sides of the equation.

$M + C = S - C + C$

and $M + C = S$

or $S = M + C$

Example 6: Solve a Formula for One of its Variables (1 of 2)

Solve the formula ***P = 2(b + s)*** for ***b***.

We will isolate the variable ***b*** by "moving" away all terms associated with it.

First, we must use the *Distributive Property* to "open up" the parentheses.

We get ***P = 2b + 2s***

We notice that the term ***2s*** is associated with ***2b*** through addition. Therefore, we must do the opposite operation and subtract ***2s*** from both sides of the equation.

P – 2s = 2b + 2s – 2s

and

P – 2s = 2b

Example 7: Solve a Formula for One of its Variables (2 of 2)

To further isolate the variable ***b***, we will now move away its coefficient ***2***. That is, we will multiply both sides of the equation by its reciprocal, which is $\frac{1}{2}$.

$$\left(\frac{1}{2}\right)(P-2s)=\left(\frac{1}{2}\right)(2b)$$

and $b=\dfrac{P-2s}{2}$

Example 8: Use a Mathematical Formula (1 of 2)

The formula $T = 4x^2 + 330x + 3310$ models the average cost of tuition and fees, T, for public U.S. Colleges for school years ending x years after 2000.

Use the formula to project the average cost of tuition and fees at public U.S. colleges for the school year ending in 2014.

HINT: x represents the years after 2000

What does x equal in the for the school year ending in 2014?

$x = 2014 - 2000 = 14$, therefore, we will substitute 14 for x in the formula.

Example 8: Use a Mathematical Formula (2 of 2)

$$T = 4x^2 + 330x + 3310$$

$$T = 4(14)^2 + 330(14) + 3310$$

$$T = 4(196) + 330(14) + 3310$$

$$T = 784 + 4620 + 3310$$

$$T = 8714$$

Please note that when we substitute the variable with a number, we enclose the number in parentheses!

The formula indicates that the average cost of tuition and fees at public U.S. colleges will be $8714 for the school year ending in 2014.

Example 9: Use a Mathematical Formula

The formula $W = -66x^2 + 526x + 1030$ describes the number of calories needed per day by women in age group x.

According to the model, how many calories per day are needed by women in age group 4?

HINT: x represents age groups!

Using $x = 4$, we get

$W = -66(4)^2 + 526(4) + 1030$

$W = -66(16) + 526(4) + 1030$

$W = -1056 + 2104 + 1030$

$W = 2078$

The model indicates that 2078 calories are needed per day by women in age group 4.