Linear Equations in One Variable

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Memorize the basic principles of equations.
2. Memorize the definition of linear equations.
3. Solve linear equations involving integers and decimals.
4. Solve linear equations containing rational numbers a/b.
5. Use the cross-multiplication principle.

1. Basic Principles of Equations

Any equation can be transformed into an equivalent equation by using one or more of the following axioms (statements that are self-evident and require no proof):

Addition Axiom - Add the same quantity to **BOTH** sides of the equal sign.

Subtraction Axiom - Subtract the same quantity from **BOTH** sides of the equal sign.

Multiplication Axiom – Multiply **BOTH** sides of the equal sign by the same **nonzero** quantity.

Division Axiom – Divide **BOTH** sides of the equal sign by the same **nonzero** quantity.

2. Definition of a Linear Equation in One Variable

Given the real numbers $\boldsymbol{A}$ and $\boldsymbol{B}$ with $\boldsymbol{A \neq 0}$, the *general form* of a linear equation in one variable, say x, is

$$\boldsymbol{Ax + B = 0}$$

Examples of Linear Equations in One Variable in General Form:

$5x + 9 = 0$ where $A = 5$ and $B = 9$

$3x = 0$ where $A = 3$ and $B = 0$

$-x - 7 = 0$ where $A = -1$ and $B = -7$

Please note that $-x$ means $-1x$ or $-1 \cdot x$, but in mathematics a coefficient of 1 is usually not stated!

3. Solve Linear Equations involving Integers or Decimals

When we are asked to "solve" a linear equation, say in x, we are actually asked to determine all values of x that result in a true statement when substituted into the equation. Such values are called **solutions**.

Solution Strategy:

1. Simplify the algebraic expression on each side of the equal sign by removing grouping symbols and combining like terms.
2. Using the Addition, Subtraction, Multiplication, and/or Division Axioms (often several times) to completely isolate the variable on one side of the equal sign. Its coefficient MUST be 1. It does not matter on which side you isolate it, right or left. Although, in mathematics the left side is preferred. This is the proposed solution of the linear equation.
3. Check the proposed solution in the original equation. If it results in a true statement, then the proposed solution is an actual solution.

Example 1: Solve a Linear Equation

Solve $x + 5 = 11$.

We must isolate the variable on one side of the equal sign. Its coefficient must be 1.

We use the **Subtraction Axiom** and subtract 5 from both sides of the equal sign.

$x + 5 – 5 = 11 – 5$ (if you are comfortable, you do not have to show this step)

and ***x* = 6** which is the proposed solution.

Check:

Given $x = 6$, is $6 + 5 = 11$ a true statement. Indeed, both sides equal 11. Therefore, ***x = 6* is also the actual solution**. We can also express the answer as {6}. This is called *Roster Form*!

Example 2: Solve a Linear Equation

Solve $x - 4 = 20$.

We must isolate the variable on one side of the equal sign. Its coefficient must be 1.

We use the **Addition Axiom** and add 4 to both sides of the equal sign.

$x - 4 + 4 = 20 + 4$ (if you are comfortable, you do not have to show this step)

and $\boldsymbol{x = 24}$ which is the proposed solution.

Check:

Given $x = 24$, is $24 - 4 = 20$ a true statement. Indeed, both sides equal 20. Therefore, ***x = 24* is also the actual solution**. We can also express the answer in roster form as {24}.

Example 3: Solve a Linear Equation (1 of 2)

Solve $2x = 10$.

We must isolate the variable on one side of the equal sign. Its coefficient must be 1.

We can either use the **Division Axiom** or the **Multiplication Axiom**.

Let's use the **Division Axiom** first. That is, let's divide both sides of the equal sign by 2.

$$\frac{2x}{2} = \frac{10}{2}$$

and $x = 5$ which is the proposed solution.

Example 3: Solve a Linear Equation (2 of 2)

Now let's use the **Multiplication Axiom**. That is, let's multiply both sides of the equal sign by the reciprocal of 2 which is $\frac{1}{2}$.

$$\frac{1}{2}(2x) = \frac{1}{2}(10)$$

and $x = 5$ just like when we used the *Division Axiom*.

As you can see, the coefficient of x became 1 when we multiplied $2x$ by $\frac{1}{2}$.

Check:

Given $x = 5$, is $2(5) = 10$ a true statement. Indeed, both sides equal 10. Therefore, ***x = 5*** **is also the actual solution**.

Example 4: Solve a Linear Equation

Solve $2(x - 4) - 5x = -5$.

Simplify the algebraic expression on each side:

$2(x - 4) - 5x = -5$ This is the given equation.

$2x - 8 - 5x = -5$ Used the distributive property.

$-3x - 8 = -5$ Combined like terms on the left.

Collect variable terms on one side and constants on the other side.

$-3x - 8 + 8 = -5 + 8$ Add 8 to both sides (Addition Axiom).

$-3x = 3$ Combined like terms (simplified).

Example 4: Solve a Linear Equation (2 of 2)

Isolate the variable and solve.

$$-\frac{1}{3}(-3x) = -\frac{1}{3}(3)$$ Multiply both sides by the reciprocal of – 3 (Multiplication Axiom).

and $x = -1$ Simplified the fractions.

While $x = -1$ is considered the “proposed” solution, if you decide to check it in the original equation, you will find that the proposed solution is also the actual solution.

Example 5: Solve a Linear Equation (1 of 2)

Solve $4(2x + 1) = 29 + 3(2x - 5)$.

We must isolate the variable on one side of the equal sign.

Simplify the algebraic expression on each side. That is, we use the *Distributive Property of Multiplication* to remove parentheses.

$4(2x + 1) = 29 + 3(2x - 5)$	This is the given equation.
$8x + 4 = 29 + 6x - 15$	Used the distributive property.
$8x + 4 = 14 + 6x$	Combined like terms on the right.
$2x + 4 = 14$	Subtracted 6*x* from both sides (Subtraction Axiom).
$2x = 10$	Subtracted 4 from both sides (Subtraction Axiom).

Example 5: Solve a Linear Equation (2 of 2)

Isolate the variable and solve.

$$\frac{2x}{2} = \frac{10}{2}$$

and $x = 5$ Divided both sides by 2 (Division Axiom).

While $x = 5$ is considered the "proposed" solution, if you decide to check it in the original equation, you will find that the proposed solution is also the actual solution.

Example 6: Solve a Linear Equations

Solve $2x + 6 = 2(x + 4)$.

We must isolate the variable on one side of the equal sign.

$2x + 6 = 2(x + 4)$ This is the given equation.

$2x + 6 = 2x + 8$ Used the distributive property.

$6 = 8$ Subtracted $2x$ from both sides (Subtraction Axiom).

The original equation is equivalent to **6 = 8**, which is false for every value of x.

Therefore, the given equation has no solution. The solution set is the empty set, and Ø is the symbol we often use.

Example 7: Solve a Linear Equation

Solve $4x + 6 = 6(x + 1) - 2x$.

We must isolate the variable on one side of the equal sign.

$4x + 6 = 6(x + 1) - 2x$	This is the given equation.
$4x + 6 = 6x + 6 - 2x$	Used the distributive property.
$4x + 6 = 4x + 6$	Combined like terms (simplified).
$6 = 6$	Subtracted 4x from both sides (Subtraction Axiom).

We get a true statement, but no *x* is left. When we get such a statement, we say that there are infinitely many solutions.

4. Solve Linear Equations Involving Rational Numbers a/b

Easiest Strategy:

- Eliminate all denominators by multiplying **BOTH** sides of the equation (preferably) by the smallest number that all denominators divide into evenly.
- Then proceed with the strategy for solving linear equations.

Example 8: Solve a Linear Equation (1 of 2)

Solve $\frac{x-3}{4}=\frac{5}{14}-\frac{x+5}{7}$.

The smallest number that the denominators 4, 14, and 7 divide into evenly is 28. We will now multiply both sides of the equation by 28.

$$28\left(\frac{x-3}{4}\right)=28\left(\frac{5}{14}\right)-28\left(\frac{x+5}{7}\right)$$
$$7(x-3)=2(5)-4(x+5)$$

Note that we are multiplying BOTH sides of the equation by 28. This means that ALL terms must be multiplied by 28!

Example 8: Solve a Linear Equation (2 of 2)

We must isolate the variable on one side of the equal sign.

$7(x-3)=2(5)-4(x+5)$	Equation from previous slide.
$7x-21=10-4x-20$	Used the distributive property.
$7x-21=-4x-10$	Combined like terms (simplified).
$11x-21=-10$	Added $4x$ to both sides (Addition Axiom).
$11x=11$	Added 10 to both sides (Addition Axiom).
$x=1$	Divided both sides by 11 (Division Axiom).

Example 9: Solve a Linear Equation

Solve $\frac{3}{4} - x = \frac{7}{8}$

The smallest number that the denominators 4, 1, and 8 divide into evenly is 8. We will multiply both sides of the equation by 8.

$$8\left(\frac{3}{4} - x\right) = 8\left(\frac{7}{8}\right)$$

$$8\left(\frac{3}{4}\right) - 8x = 8\left(\frac{7}{8}\right)$$ Used the distributive property on the left.

6 – 8x = 7 Multiplied the fractions.

– 8x = 1 Subtracted 6 from both sides (Subtraction Axiom).

$$x = -\frac{1}{8}$$ Divided both sides by – 8 (Division Axiom).

5. Cross-Multiplication Principle

If two numbers $\frac{a}{b}$ and $\frac{c}{d}$ are equal, that is $\frac{a}{b} = \frac{c}{d}$, then $\boldsymbol{a \cdot d = b \cdot c}$ where $b \neq 0$ and $d \neq 0$. We call $\boldsymbol{a \cdot d}$ and $\boldsymbol{b \cdot c}$ the cross products.

Please note that this cross-multiplication principle does not have to be used, but it often makes the calculations easier.

We could simply eliminate all denominators by multiplying both sides of the equation by the smallest number that all denominators divide into evenly.

Example 10: Solve a Linear Equation

Solve $\frac{F}{3} = \frac{9}{5}$ using the cross-multiplication principle.

Instead of starting out multiplying both sides of the equation by the smallest number both denominators divide into evenly, we'll use cross-multiplication. That is, we make a cross across the equal sign as follows:

Using the cross-multiplication principle, we can now state ***5F = 9(3)*** and ***5F = 27.***

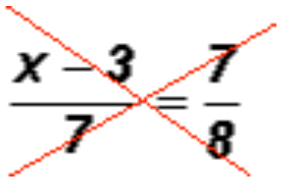
We will solve for ***F*** by multiplying both sides by the reciprocal of ***5****.*

We get $F = \frac{27}{5}$.

Example 11: Solve a Linear Equation (1 of 2)

Solve $\frac{x-3}{7} = \frac{7}{8}$ using the cross-multiplication principle.

Instead of starting out multiplying both sides of the equation by the smallest number both denominators divide into evenly, we'll use cross-multiplication. That is, we make a cross across the equal sign as follows:



Using the cross-multiplication principle, we can now state ***8(x – 3) = 7(7)***.

Next, we must use the Distributive Property to remove the parentheses to get

8x – 24 = 49

Example 11: Solve a Linear Equation (2 of 2)

Now, we will collect like terms on the same side of the equal sign to get

8x = 73

We will solve for ***x*** by multiplying both sides by the reciprocal of ***8***.

We get $x = \frac{73}{8}$.