Irrational and Real Numbers

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Define irrational numbers.
2. Evaluate irrational numbers derived from radicals.
3. Evaluate irrational numbers derived from nature.
4. Define real numbers and recognize their subsets.

1. Definition of Irrational Numbers

So far, we discussed natural numbers, integers, and rational numbers.

An **irrational number** is any type of number that CANNOT be written as a rational number in the form $\frac{a}{b}$.

Irrational numbers are non-terminating, non-repeating decimal numbers.

Irrational numbers come from many sources, such as radicals and logarithms. They also occur in nature.

2. Irrational Numbers derived from Radicals

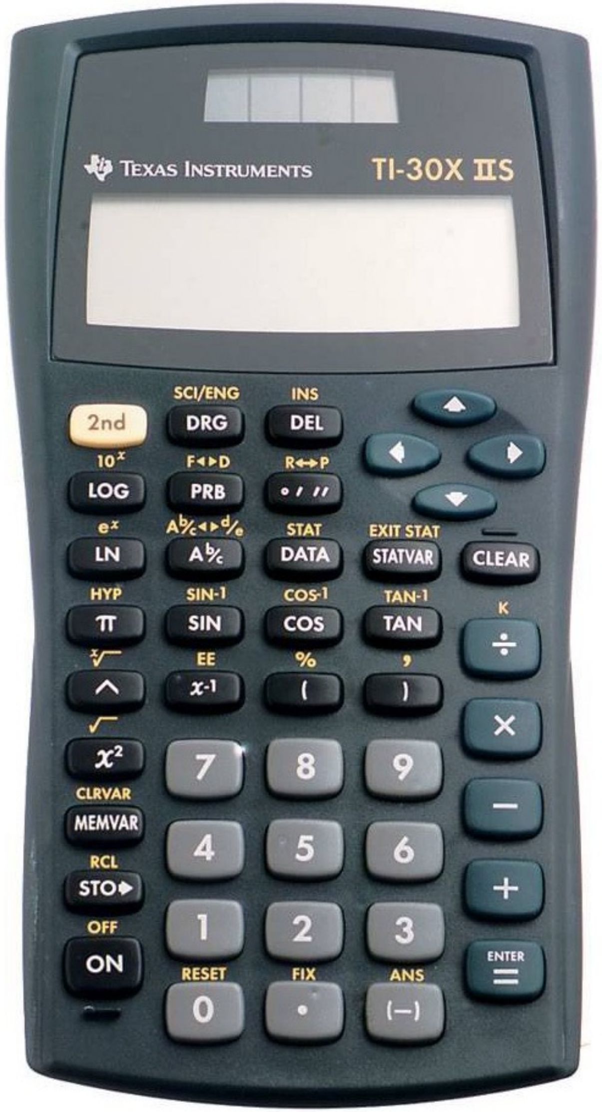
We already learned that certain radicals are rational numbers.

For example, $\sqrt{81}$, because there is a rational number that produces 81 when multiplied by itself, namely 9.

However, there are infinitely many radicals which are NOT rational numbers. For example, $\sqrt{10}$, because there is NO rational number that produces 10 when multiplied by itself

Therefore, we must state that $\sqrt{10}$ is an irrational number (not rational!!!).

Example 1: Evaluate Radicals



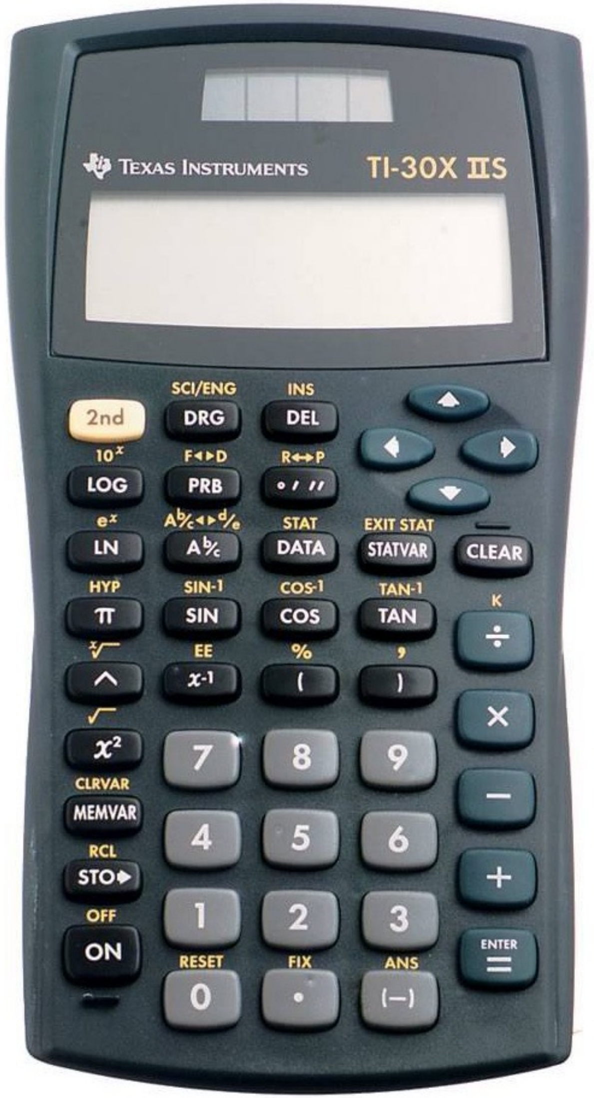
Evaluate $\sqrt{10}$ on the calculator.

Press the 2nd button. Then the x^2 button. This invokes the square root function which resides above the x^2 button. You get $\sqrt{(}$. Finally type **10** and press the right parenthesis) button to close the parentheses. Press the ENTER button.

We find that $\sqrt{10}$ equals **3.16227766…** .

The decimal portion is never-ending making $\sqrt{10}$ an irrational number, a number that cannot be written as a rational number in the form $\frac{a}{b}$.

Example 2: Evaluate Radicals



Evaluate $\sqrt[3]{40}$ on the calculator.

Here we must use a calculator such as the TI-30X IIS.

Type **3** (the index). Then press the 2nd button. Then the caret ^ button. This invokes the root function which resides above the caret ^ button. Finally type **40**. Press the ENTER button.

We find that $\sqrt[3]{40}$ equals **3.419951893…** .
The decimal portion is never-ending making $\sqrt[3]{40}$ an irrational number, a number that cannot be written as a rational number in the form $\frac{a}{b}$.

3. Irrational Numbers Derived from Nature (1 of 4)

The Number π

In high school, we learned that the circle is a geometric figure with a special relationship between its distance around the circle (circumference) and its diameter. That is, if we divide the circumference of any circle by its diameter, the quotient is always the same number. The name of this number is the Greek letter Pi (pronounced like pie) and its symbol is π.

The number π is the irrational number 3.141592654… approximately equal to 3.14.

Irrational Numbers Derived from Nature (2 of 4)

Let's evaluate the number π using the calculator. To find out the value of we input the following:

π	ENTER

We find that π = 3.141592654... .

NOTE: The calculator does not tell us that π is an irrational number. It simply fills up all available slots on its screen with decimal places. YOU must know that it is an irrational number with infinitely many decimal place.

Irrational Numbers Derived from Nature (3 of 4)

The Number *e*

The number ***e*** was discovered by the Swiss mathematician Jacob Bernoulli in 1683 while studying compound interest. He wanted to know that happens to the expression $\left(1+\frac{1}{n}\right)^n$ when *n* gets infinitely large.

The the first appearance of ***e*** was in the publication "Mechanica" in 1736 by the Swiss mathematician Leonard Euler. The number ***e*** is also known as **Euler's number**.

The number ***e*** is the irrational number 2.718281828 … approximately equal to 2.72.

Irrational Numbers Derived from Nature (4 of 4)

Examining any calculator, we find that there is NO button containing the number ***e***. However, we can find its picture over the LN button. This means that we must use the 2nd button to access the number. To find the value of ***e***, we input the following:

2nd	*LN*	*1*	*)*	*Enter*

We find that ***e*** = 2.718281828 … .

Note that we used the power of 1 to find the value of the number ***e***.

NOTE: The calculator does not tell us that ***e*** is an irrational number. It simply fills up all available slots on its screen with decimal places. YOU must know that it is an irrational number with infinitely many decimal places.

Example 3: Evaluate an Expression Containing *e* and π

a. Evaluate $\frac{3}{\pi}+4$ using the calculator. Round to 2 decimal places. Use the π button on the calculator and not 3.14.

We find that $\frac{3}{\pi}+4\approx 4.95$.

b. Evaluate $e^2 - 1$ using the calculator. Round to 3 decimal places. Use the ***e*** function on the calculator and not 2.72.

We find that $e^2 - 1 \approx 6.389$

c. Evaluate $\ln\left(\frac{1}{e^3}\right)$ using the calculator. Round to 2 decimal places.

Use the ***e*** function on the calculator and not 2.72.

We find that the natural logarithmic expression is exactly equal to − 3.

4. Definition of the Real Numbers (1 of 2)

The **Real Numbers** consist of the following sets of numbers:

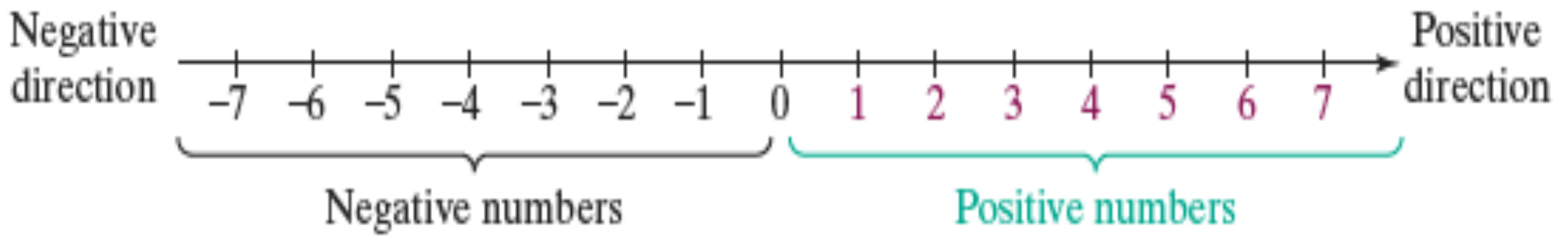
- The set of **Natural Numbers:** {1, 2, 3, 4, 5,…}
- The set of **Whole Numbers:** {0, 1, 2, 3, 4, 5,…}
- The set of **Integers:** {…., −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5,…}
- The set of **Rational Numbers:** numbers that can be written as the quotient of two integers with the denominator not equal to 0.
- The set of **Irrational Numbers:** numbers that cannot be expressed as a quotient of integers.

Definition of the Real Numbers (2 of 2)

The **real number line** is a graph used to represent the set of real numbers.

An arbitrary point, called the **origin**, is labeled 0. The distance from 0 to 1 is called the **unit distance**.

Numbers to the right of the origin are **positive** and numbers to the left of the origin are **negative**. On the real number line, the real numbers increase from left to right.



Example 4: Recognize Subsets of the Real Numbers

Consider the following set of numbers:

$$\left\{-9, -1.3, 0, 0.\overline{3}, \pi, 9, 10\right\}$$

List the numbers in the set that are natural numbers:

The natural numbers in the set are 9 and 10.

List the numbers in the set that are whole numbers:

The whole numbers in the set are 0, 9, and 10.

Example 5: Recognize Subsets of the Real Numbers

Consider the following set of numbers.

$$\left\{-9, -1.3, 0, 0.\overline{3}, \pi, 9, 10\right\}$$

List the numbers in the set that are integers:

The numbers in the set that are integers are – 9, 0, 9, and 10.

List the numbers in the set that are rational numbers:

The numbers in the set that are rational numbers are

$-9,\ -1.3,\ 0,\ 0.\overline{3} = \frac{1}{3},\ 9,$ and 10.

Example 6: Recognize Subsets of the Real Numbers

Consider the following set of numbers.

$$\left\{-9, -1.3, 0, 0.\overline{3}, \pi, 9, 10\right\}$$

List the numbers in the set that are irrational numbers.

The numbers in the set that are irrational numbers is π (Pi) .