Introduction to Radicals and Logarithms

Based on power point presentations by Pearson Education, Inc.
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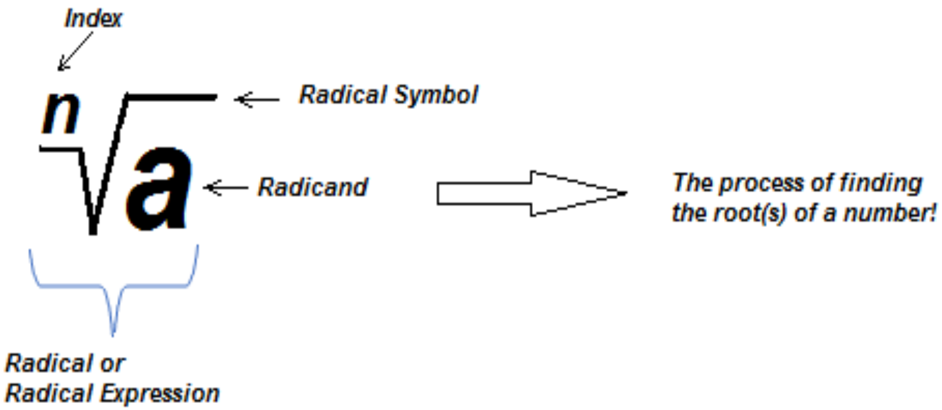
Learning Objectives

1. Define and evaluate radicals.
2. Define and evaluate logarithms.

1. Definition of Radical Expressions

Radical expressions or simply “radicals” are related to exponential expressions in that they reverse the operation of raising a number to a power. The radical symbol $\sqrt{}$ indicates this process. Evaluating a radical expression is also called “finding a root."



Example 1: Evaluate Radicals

Evaluate $\sqrt{81}$ by hand.

We are asked to evaluate a square root. It has index 2. Since square roots occurs frequently in mathematics, we do not write the index. Seeing a radical without an index, always means that it is 2.

We are asked to reverse the operation of raising a number to the 2nd power.

We know that $9 \cdot 9 = 81$, therefore, $\sqrt{81} = 9$.

The solution is a rational number, more specifically an integer.

Please note that $(-9) \cdot (-9)$ also equals 81. However, BY DEFINITION a radical strictly asks us to find the **principal root**, which is a positive number.

Example 2: Evaluate Radicals

Evaluate $-\sqrt{36}$ by hand.

First of all, $-\sqrt{36}$ = –1 · $\sqrt{36}$.

Given $\sqrt{36}$, we are asked to reverse the operation of raising a number to the 2nd power. We know that $6^2 = 36$, therefore, $\sqrt{36}$ = 6 then $-\sqrt{36}$ = – 6 .

Example 3: Evaluate Radicals

Evaluate $\sqrt[3]{27}$ by hand.

We are asked to evaluate a cube root. We know that
3(3)(3) = 27.
1 2 3 (Three 3's matching 3 in the index)

Therefore, $\sqrt[3]{27}$ = 3.

The solution is a rational number, more specifically an integer.

Example 4: Evaluate Radicals

Evaluate $\sqrt[5]{-32}$ by hand.

We are asked to evaluate a fifth root. We know that
(– 2)(– 2)(– 2)(–2)(– 2) = – 32.
1 2 3 4 5 (Five (– 2)'s matching 5 in index)

Therefore, $\sqrt[5]{-32}$ = – 2.

The solution is a rational number, more specifically an integer.

Example 5: Evaluating Radicals $\sqrt[3]{\frac{125}{27}} = \frac{\sqrt[3]{125}}{\sqrt[3]{27}} = \frac{5}{3}$

Evaluate $\sqrt[3]{\frac{125}{27}}$ by hand.

We are allowed to distribute the radical to the numerator and denominator as follows:

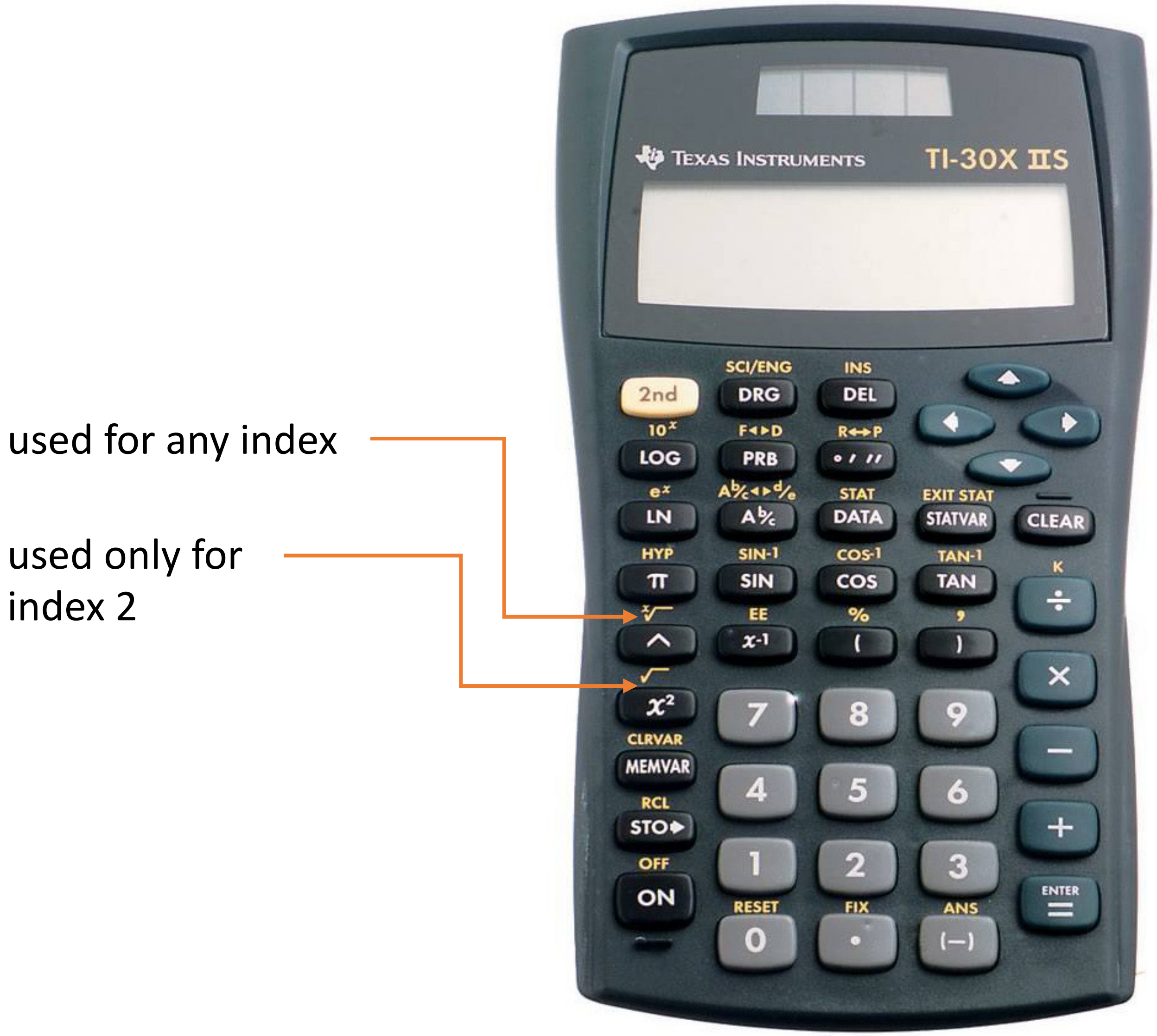
$$\frac{\sqrt[3]{125}}{\sqrt[3]{27}}$$

❗ We know that 125 = 5(5)(5) and 27 = 3(3)(3) and the three 5's and three 3's match the the 3 in the index.

Therefore, $\sqrt[3]{\frac{125}{27}} = \frac{\sqrt[3]{125}}{\sqrt[3]{27}} = \frac{5}{3}$.

The solution is a rational number.

Example 6: Evaluating Radicals



Evaluate $\sqrt{105625}$ on the calculator.

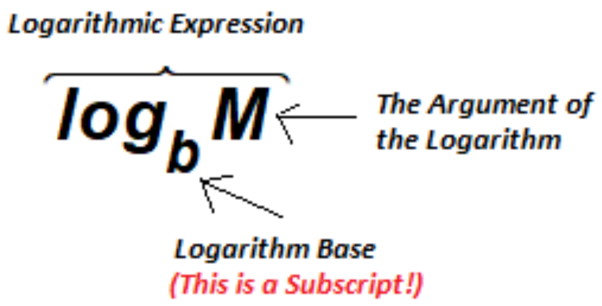
Press the 2nd button. Then the x^2 button. This invokes the square root function which resides above the x^2 button. You get $\sqrt{(}$. Finally type **105625** and press the right parenthesis) button to close the parentheses. Press the ENTER button.

We find that $\sqrt{105625}$ equals **325**.

2. Definition of Logarithmic Expressions (1 of 3)

Logarithmic expressions or simply logarithms are also related to exponential expressions in that they allow us to write them in a different ways. Logarithms were developed by ancient mathematicians who were trying to find unknown exponents.

Please examine the general logarithmic expression carefully!



This is expressed as “log base b of M”.

Example:

Evaluate $\mathbf{\log_{10} 1000}$.

The logarithmic expression asks us to find the power to which 10 must be raised to get 1000! The answer is 3, right?

Definition of Logarithmic Expressions (2 of 3)

The logarithm bases that occur most frequently in applications are ***10*** and ***e***. Please note that ***e*** is the famous number 2.718281828 …. containing infinitely many decimal places and often rounded to 2.72.

$\log_{10} x = \log x$

a logarithm of base 10 is referred to as the common logarithm. The 10 is usually left off.

$\log_{e} x = \ln x$

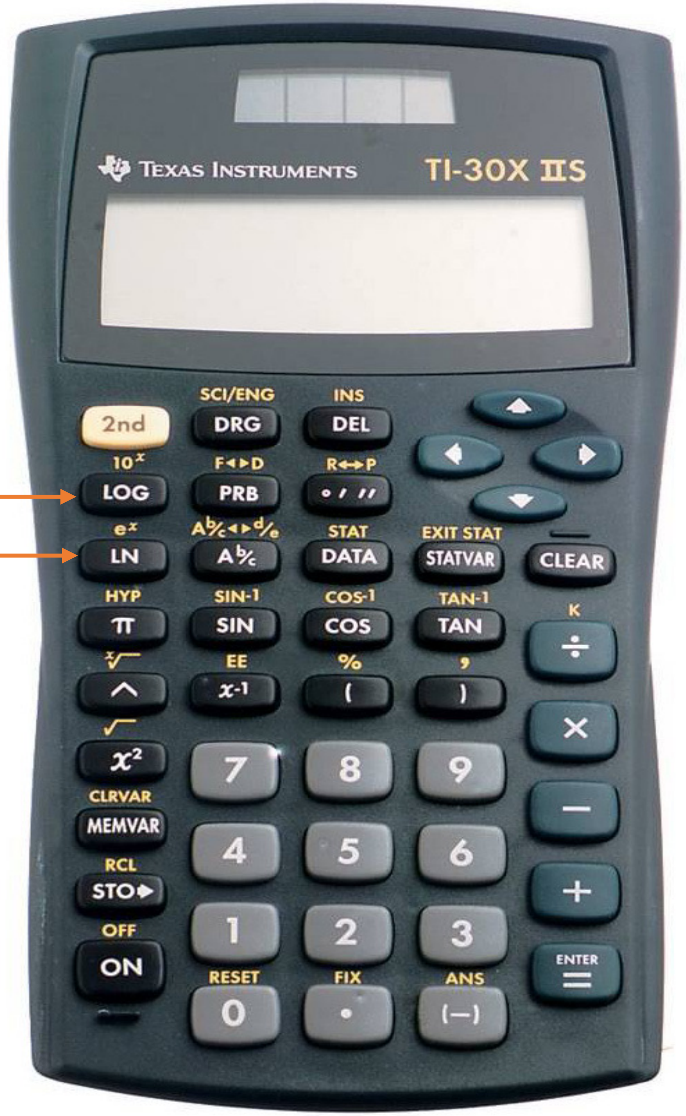
a logarithm of base ***e*** is referred to as the natural logarithm.

NOTE: The ***e*** is usually left off and the word "log" changes to "ln". Please note that the letter "l" in "ln" is a lower case L. It is neither the upper case letter "i" nor the number 1.

Definition of Logarithmic Expressions (3 of 3)

Log_{10} button (LOG)

log_e button (LN)



Example 7: Evaluate a Logarithm

Evaluate $\mathbf{log_{10}\ 100}$ on the calculator.

The logarithmic expression asks us to find the power to which 10 must be raised to get 100! The answer is 2, right?

Calculator Input:

LOG	*100*	*)*	*ENTER*

Left parenthesis will open when you press the *LOG* button. Specifically, you will see **log (**. After you enter the argument 100, you MUST type the right parenthesis, namely ***)***, before you press ENTER.

We indeed find that $\mathbf{log_{10}\ 100 = 2}$ which is a rational number.

Example 8: Evaluate a Logarithm

Evaluate **ln 1** on the calculator.

Remember, ln = $\log_e$. The logarithmic expression asks us to find the power to which *e* must be raised to get 1!

Calculator input:

LN	1	)	ENTER

Left parenthesis will open when you press the *LN* button. Specifically, you will see **ln (**. After you enter the argument ***e***, you MUST type the right parenthesis, namely ***)***, before you press ENTER.

We find that **ln 1 = 0** which is a rational number. Think about it, there IS a power to which ***e*** can be raised to get 1! Namely, 0, right?