Exponents; Advanced Order of Operations

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Define and evaluate exponential expressions.
2. Use the rules of exponents.
3. Use the advanced Order of Operations given grouping symbols, exponents, multiplication, division, addition, and subtraction.

1. Definition of Exponential Expressions

Exponential expressions are of the form b^x, where ***b*** is called a **base** and ***x*** **exponent** or **power**. The exponent states how many times to multiply base **b** by itself.

Given 5^2, the base is ***5*** and the exponent is ***2***, and it states to use 5 twice in a multiplication, so that $5^2 = 5 \cdot 5 = 25$.

5^2 is read as "five to the second power" or "five raised to the second power" or "five squared".

Example 1: Evaluate Exponential Expressions

Evaluate the exponential expression 3^2.

The exponent 2 tell us to use the base 3 twice in a multiplication.

$3^2 = 3(3) = 9$

Calculator Tip:

We can find second powers on the calculator in two different ways.

We simply press the caret **^** button on the calculator.

3	^	2	Enter

There is also a $\mathbf{x^2}$ button to strictly evaluate second powers.

3	x^2	Enter

Example 2: Evaluate Exponential Expressions

Evaluate the exponential expression 4^3.

The exponent 3 tell us to use the base 4 three times in a multiplication.

$4^3 = 4(4)(4) = 64$

Calculator Tip:

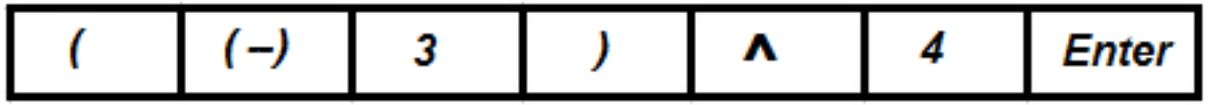
We simply press the caret **^** button on the calculator.

Example 3: Evaluate Exponential Expressions

Evaluate the exponential expressions $(-3)^2$.

$(-3)^2 = (-3)(-3) = 9$

Calculator Tip: The calculator requires parentheses around a negative number if it is to be raised to a power! Use the (–) key for the negative sign!



Example 4: Evaluate Exponential Expressions

Evaluate the exponential expression 100^1.

The exponent 1 tell us to use the base 100 one time in a multiplication.

$100^1 = 100$

NOTE: The value of any number with an exponent of *1* is the number itself. Customarily, the ***1*** is not written!

2. Rules of Exponents (1 of 3)

The Negative-Exponent Rule

If b is any real number other than 0 and n is a natural number, then

$$b^{-n} = \frac{1}{b^n}.$$

Example: $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

Rules of Exponents (2 of 3)

The Zero-Exponent Rule

If b is any real number other than 0, then $b^0 = 1$.

Examples: $(-4)^0 = 1$

$7^0 = 1$

Rules of Exponents (3 of 3)

Power Rule for Fractions

If a and b are real numbers, $b \neq 0$, or algebraic expressions, and n is an integer, then

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

In English this means, when a quotient is raised to a power, raise the numerator AND the denominator to that power.

Example: $\left(\frac{2}{5}\right)^4 = \frac{2^4}{5^4} = \frac{16}{625}$

3. Advanced Order of Operations

Earlier we learned, that the *Order of Operations* are rules that state the sequence in which multiple operations in an expression should be solved. We already discussed the order given multiplication, division, addition, and subtraction. We will now add to this the order given grouping symbols and exponents.

1. Grouping Symbols are evaluated first.
   Grouping symbols are parentheses ( ), brackets [ ], braces { }, and fraction bars.
2. Exponents are evaluated next.
3. Multiplications and Divisions are done next, in the order in which they occur, working from left to right.
4. Addition and Subtraction are done last, in the order in which they occur, working from left to right.

Example 5: Use the Advanced Order of Operations

Evaluate $6^2 – 24 \div 2^2 \cdot 3 + 1$.

There are no grouping symbols. Therefore, we begin by evaluating the exponent.

$6^2 – 24 \div 2^2 \cdot 3 + 1 =$

$= 36 – 24 \div 4 \cdot 3 + 1$ (evaluated the exponent)

$= 36 – 6 \cdot 3 + 1$ (evaluated the division)

$= 36 – 18 + 1$ (evaluated the multiplication)

$= 18 + 1$ (evaluated the subtraction)

$= 19$ (evaluated the addition)

Example 6: Use the Advanced Order of Operations

Evaluate $7^2 – 20 \div 2^2 \cdot (2 + 3)^3$.

There are grouping symbols. Therefore, we begin by evaluating the parentheses.

$7^2 – 20 \div 2^2 \cdot (2 + 3)^3 =$

$= 7^2 – 20 \div 2^2 \cdot 5^3$ (evaluated parentheses)

$= 49 – 20 \div 4 \cdot 125$ (evaluated the exponent)

$= 49 – 5 \cdot 125$ (evaluated the division)

$= 49 – 625$ (evaluated the multiplication)

$= – 576$ (evaluated the subtraction)

Example 7: Use the Advanced Order of Operations

Evaluate -5^0.

Note there is a difference between $(-5)^0$ and -5^0! In the first expression, there are parentheses around the -5 which means that it is raised to the 0 power.

$(-5)^0 = 1$

In -5^0 only 5 is raised to the 0 power. Therefore, you must use the *Order of Operations* rule that states that exponents are evaluated before multiplication.

$-5^0 =$
$= -1(5^0)$
$= -1(1)$
$= -1$

Example 8: Use the Advanced Order of Operations

Evaluate $(-4)^2$ and -4^2

$(-4)^2 = (-4) \cdot (-4) = 16$

In -4^2 only 4 is raised to the 2nd power. Therefore, you must use the *Order of Operations* rule that states that exponents are evaluated before multiplication.

$-4^2 =$

$= -1(4^2)$

$= -1(16)$

$= -16$

Example 9: Use the Advanced Order of Operations

Evaluate $(-4)^3$ and -4^3

$(-4)^3 =$
$= (-4) \cdot (-4) \cdot (-4)$
$= -64$

In -4^3 only 4 is raised to the 3rd power. Therefore, you must use the *Order of Operations* rule that states that exponents are evaluated before multiplication.

$-4^3 =$
$= -1(4^3)$
$= -1(64)$
$= -64$

Please compare these results with the ones from Example 8.

Example 10: Use the Advanced Order of Operations

Evaluate $6 - [24(-48 \div 3 + 1) + 8] + 3^4$.

There is addition and subtraction. Therefore, we begin by evaluating from left to right.

$6 - [24(-48 \div 3 + 1) + 8] - 3^4 =$

$= 6 - [24(-16 + 1) + 8] - 3^4$ (evaluated division in the innermost grouping symbol)

$= 6 - [24(-15) + 8] - 3^4$ (evaluated addition in the innermost grouping symbol)

$= 6 - [-360 + 8] - 3^4$ (evaluated the multiplication in the bracket grouping symbol)

$= 6 - [-352] - 3^4$ (evaluated the addition in the bracket grouping symbol)

$= 6 + 352 - 3^4$ (used the distributive property on the bracket grouping symbol)

$= 6 + 352 - 81$ (evaluated the exponent)

$= 277$ (evaluated addition and subtraction from left to right)