Examples
Exploring the Right Triangle

Based on power point presentations by Pearson Education, Inc.

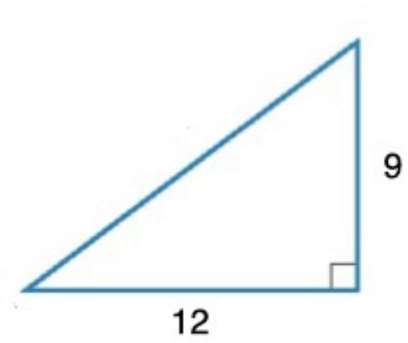
Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Memorize and use the *Pythagorean Theorem*.
2. Memorize trigonometric ratios and evaluate them given sides of right triangles.
3. Evaluate trigonometric ratios given angles in right triangles.
4. Find angle measures given trigonometric ratios.

Example 1: Use the Pythagorean Theorem (1 of 2)

Find the length of the hypotenuse *c* in the following right triangle. If necessary, use the calculator and round the answer to one decimal place.



Given legs $a = 9$ and $b = 12$.

Using the *Pythagorean Theorem*, we can write the following:

$9^2 + 12^2 = c^2$

$81 + 144 = c^2$

$c^2 = 225$

Example 1: Use the Pythagorean Theorem (2 of 3)

Now, we ended up with the squared variable c^2. But what we really need is the value of c. What we will do here is use something called the **Square Root Property** to solve for c. It says the following:

If a squared variable is equal to some number, then the square root of this variable is equal to the positive and negative square root of the number.

Mathematically, this is expressed as follows:

If $x^2 = d$ where d is any number, then x has exactly two solutions, namely $x = \sqrt{d}$ and $x = -\sqrt{d}$.

Example 1: Use the Pythagorean Theorem (3 of 3)

Therefore, by the *Square Root Property*, if $c^2 = 225$ then

$c = \sqrt{225} = 15$ and $c = -\sqrt{225} = -15$

NOTE: Since we are discussing triangles, and the measure of their sides is never negative, we can rule out any negative solutions.

In conclusion, the length of the hypotenuse *c* is exactly equal to 15.

Example 2: Use the Pythagorean Theorem (1 of 2)

Given a right triangle, assume that the lengths of the legs are x = 6 ft and y = 7.5 ft. Find the length of the hypotenuse z. If necessary, use the calculator and round the answer to one decimal place.

Using the *Pythagorean* Theorem to find the length of the hypotenuse z, we can write

$6^2 + 7.5^2 = z^2$

$36 + 56.25 = z^2$

$z^2 = 92.22$

Example 2: Use the Pythagorean Theorem (2 of 2)

Now, we will use the *Square Root Property* to get a value of z as follows:

If $z^2 = 92.22$ then $z = \sqrt{92.22}$ which, according to the calculator, is approximately equal to 9.6.

Remember, we do not need to consider the negative solution of the *Square Root Property* since we are dealing with lengths.

We find the length of the hypotenuse z is approximately equal to 9.6 feet.

Example 3: Use the Pythagorean Theorem

Given a right triangle, assume that the length of leg b = 4 m and the length of the hypotenuse c = 5 m. Find the length of the other leg, let's call it a. If necessary, use the calculator and round the answer to one decimal place.

Using the *Pythagorean Theorem*, we get

$a^2 + 4^2 = 5^2$

$a^2 + 16 = 25$

$a^2 = 25 - 16$

$a^2 = 9$

and $a = \sqrt{9} = 3$

We find the length of leg a to be exactly equal to 3 meters.

Example 4: Use the Pythagorean Theorem

Given a right triangle, assume that the length of leg $g = 7$ ft and the length of the hypotenuse $h = 10$ ft. Find the length of the other leg, let's call it *f*. If necessary, use the calculator and round the answer to one decimal place.

Using the *Pythagorean Theorem*, we get

$f^2 + 7^2 = 10^2$

$f^2 + 49 = 100$

$f^2 = 100 - 49 = 51$

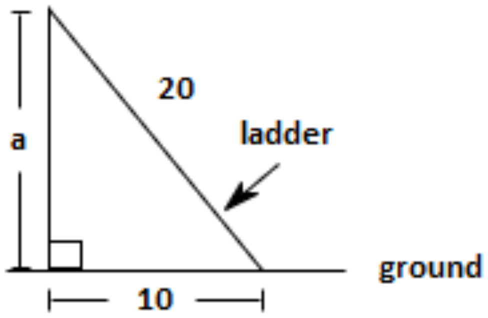
and $f = \sqrt{51}$ which, according to the calculator, is approximately equal to 7.1.

We find the length of leg *f* to be approximately equal to 7.1 feet.

Example 5: Use the Pythagorean Theorem (1 of 2)

A 20 ft ladder is placed against a wall of a building that makes a 90^o angle with the ground. The bottom of the ladder is 10 ft from the base of the building. How high up the wall does the ladder reach? Use the calculator and round to the nearest hundredth.

Let's first draw a picture. We let *a* be the number of feet the ladder reaches up the wall.



Example 5: Use the Pythagorean Theorem (2 of 2)

Since we are encountering a right triangle, we can use the *Pythagorean Theorem* to solve for *a*.

We know that leg *b* = 10 and hypothenuse *c* = 20 (ladder). Using the *Pythagorean Theorem*, we get

$a^2 + 10^2 = 20^2$

$a^2 + 100 = 400$

$a^2 = 400 - 100$

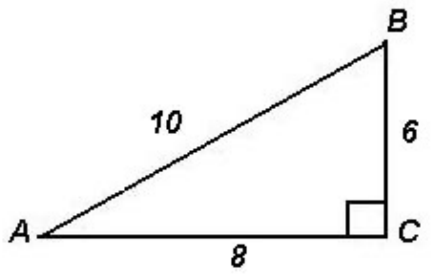
$a^2 = 300$

and $a = \sqrt{300}$ which, according to the calculator, is approximately equal to 17.32.

We find that the ladder reaches approximately 17.32 feet up the wall.

Example 6: Use the Trigonometric Ratios (1 of 2)

Given the following triangle, find the sine, cosine, and tangent ratios for angles A and B. Express your answers both as a fraction and a decimal.

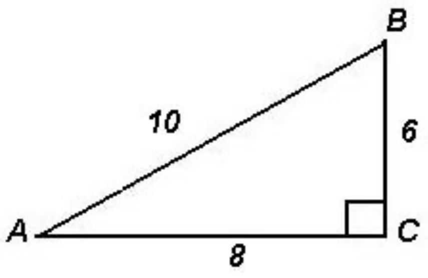


Ratios for Angle A:

$$\sin A = \frac{opp}{hyp} = \frac{6}{10} = \frac{3}{5} = 0.6 \qquad \cos A = \frac{adj}{hyp} = \frac{8}{10} = \frac{4}{5} = 0.8$$

$$\tan A = \frac{opp}{adj} = \frac{6}{8} = \frac{3}{4} = 0.75$$

Example 6: Use the Trigonometric Ratios (2 of 2)



Ratios for Angle B:

$$\sin B = \frac{opp}{hyp} = \frac{8}{10} = \frac{4}{5} = 0.8 \qquad \cos B = \frac{adj}{hyp} = \frac{6}{10} = \frac{3}{5} = 0.6$$

$$\tan B = \frac{opp}{adj} = \frac{8}{6} = \frac{4}{3} = 1.33$$

Please note that the location of the "side opposite" and the" side adjacent" changes with the location of the angle in the triangle!

Example 7: Evaluate Trigonometric Ratios Given Angles

a. Find the value of sin 34^o rounded to four decimal places, if necessary. Your calculator must be in degree mode!

   We find that sin 34^o is approximately equal to 0.5592. This value is a non-terminal decimal. That is, it is an irrational number.

b. Find the value of cos 55^o rounded to four decimal places, if necessary. Your calculator must be in degree mode!

   We find that cos 55^o is approximately equal to 0.5736. This value is a non-terminal decimal. That is, it is an irrational number.

c. Find the value of tan 39^o rounded to four decimal places, if necessary. Your calculator must be in degree mode!

   We find that tan 39^o is approximately equal to 0.8098. This value is a non-terminal decimal. That is, it is an irrational number.

Example 8: Evaluate Trigonometric Ratios Given Angles

a. Find the value of $\sin 30^o$ rounded to four decimal places if necessary. Your calculator must be in degree mode!

   We find that $\sin 30^o$ is exactly equal to 0.5. This value is a terminal decimal. That is, it is a rational number.

b. Find the value of $\cos 60^o$ rounded to four decimal places, if necessary. Your calculator must be in degree mode!

   We find that $\cos 60^o$ is exactly equal to 0.5. This value is a terminal decimal. That is, it is a rational number.

c. Find the value of $\tan 45^o$ rounded to four decimal places, if necessary. Your calculator must be in degree mode!

   We find that $\tan 45^o$ is exactly equal to 1. This value is a whole number.

Example 9: Find Angles (1 of 2)

a. Use a calculator to find the measure for angle A given that $\sin A = 0.86$. Round the angle to two decimal places. Your calculator must be in degree mode!

   We write this as $A = \sin^{-1}(0.86)$ and pronounce it as "A equals the arcsine of 0.86".

   We find that angle A is approximately equal to 59.32^{o}.

b. Use a calculator to find the measure for angle B given that $\cos B = 0.39$. Round the angle to two decimal places. Your calculator must be in degree mode!

   We write this as $B = \cos^{-1}(0.39)$ and pronounce it as ”B equals the arccosine of 0.39".

   We find that angle B is approximately equal to 67.05^{o}.

Example 9: Find Angles (2 of 2)

c. Use a calculator to find the measure for angle *C* given that tan *C* = 0.13. Round the angle to two decimal places. Your calculator must be in degree mode!

We write this as $C = \tan^{-1}$ (0.39) and pronounce it as ”B equals the arctangent of 0.13".

We find that angle *C* is approximately equal to 7.41^{o}.