Examples
Radical and Logarithm Expressions

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Evaluate radical expressions.
2. Evaluate logarithmic expressions.
3. Memorize and apply the *Change-of-Base Property* for logarithms.

Example 1: Evaluate a Radical Expression

Evaluate $\sqrt{81}$ without a calculator.

We are asked to evaluate the "square root of 81". It has index 2. Since square roots occurs frequently in mathematics, we do not write the index. Seeing a radical without an index, always means that it is 2.

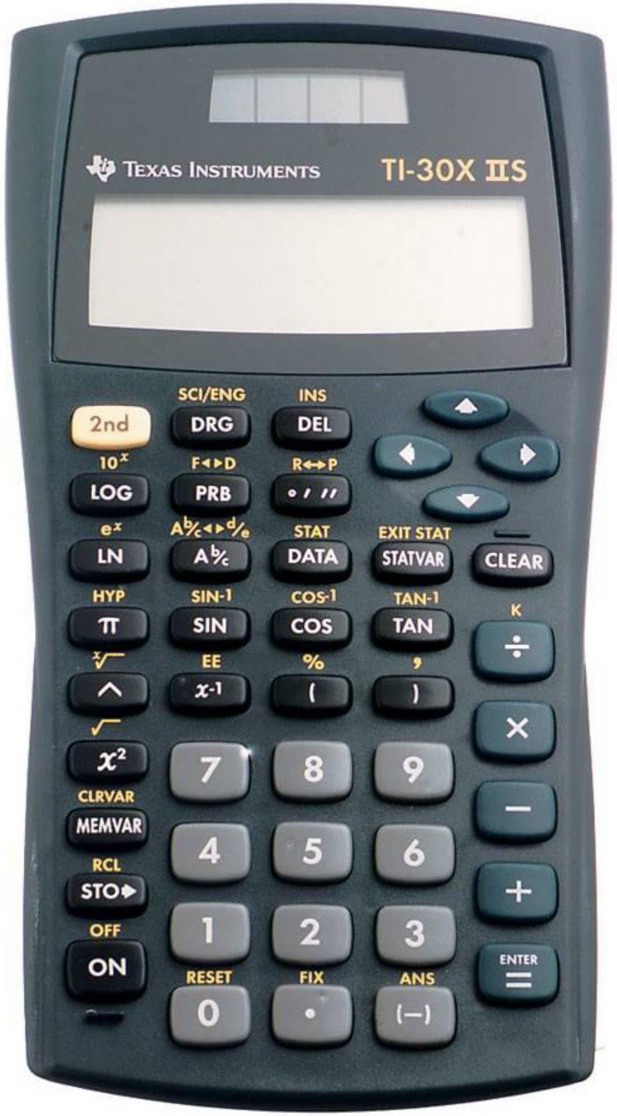
We are asked to reverse the operation of raising a number to the 2nd power.

We know that $81 = 9 \cdot 9 = 9^2$, therefore, $\sqrt{81} = \sqrt{9^2} = 9$.

The solution is a rational number, more specifically an **integer.**

Please note that $(-9) \cdot (-9)$ also equals 81. However, BY DEFINITION a radical expression with EVEN index always asks us to find a positive number. This number is called the *principal root*.

Example 2: Evaluate a Radical Expression



Evaluate $\sqrt{81}$ with a calculator.

We will use the TI-30X IIS.

- Press the 2nd button and then the x^2 button. We will see $\sqrt{(}$.
- Type **81** and press the right parentheses) button to close the set.
- Press the ENTER button.

The answer is **9** which is a rational number and more specifically an **integer**.

Example 3: Evaluate a Radical Expression

Evaluate $\sqrt{36}$ and $-\sqrt{36}$ without a calculator.

We are asked to evaluate the “square root of 36.” We know that 36 = 6 · 6 = 6^2.

Therefore, $\sqrt{36} = \sqrt{6^2} = 6$.

Since $-\sqrt{36}$ = –1 · $\sqrt{36}$, we find that $-\sqrt{36}$ = – 6 .

The solutions are rational numbers, more specifically **integers**.

Example 4: Evaluate a Radical Expression

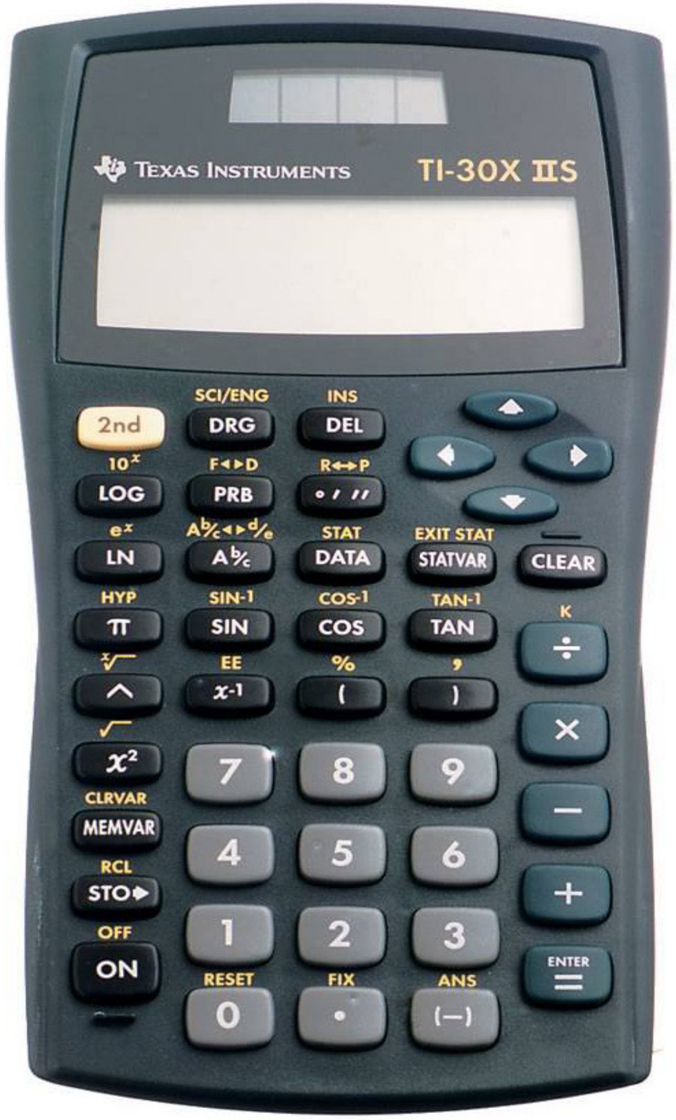
Evaluate $\sqrt[3]{64}$ without a calculator.

We are asked to evaluate the "cube root of 64". We know that $64 = 4 \cdot 4 \cdot 4 = 4^3$.

Therefore, $\sqrt[3]{64} = \sqrt[3]{4^3} = 4$.

The solution is a rational number, more specifically an **integer.**

Example 5: Evaluate a Radical Expression



Evaluate $\sqrt[3]{64}$ with a calculator.

We will use the TI-30X IIS.

- Type the index **3**.
- Press the 2nd button and then the ^ (caret) button. We will see 3 $\sqrt[x]{\ }$.
- Type **64**.
- Press the ENTER button.

The answer is **4** which is a rational number and more specifically an integer.

Example 6: Evaluate Radical Expressions

Evaluate $\sqrt[5]{-32}$ without a calculator.

We are asked to evaluate a fifth root. We know that

$(-2)(-2)(-2)(-2)(-2) = -32$

There are five (-2)'s matching 5 in index!

Therefore, $\sqrt[5]{-32} = -2$.

The solution is a rational number, more specifically an **integer**.

Example 7: Evaluate a Radical Expression

Evaluate $\sqrt[3]{\dfrac{125}{27}}$ without a calculator.

We are allowed to distribute the radical to the numerator and denominator as follows:

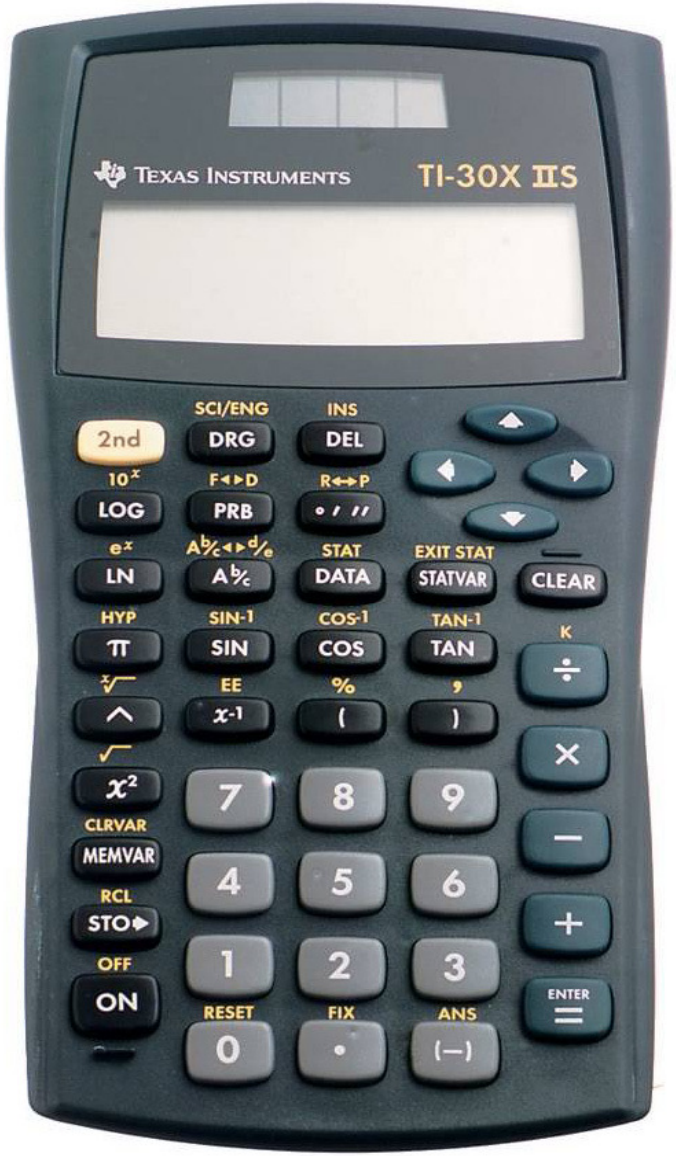
$$\frac{\sqrt[3]{125}}{\sqrt[3]{27}}$$

We know that 125 = 5(5)(5) and 27 = 3(3)(3) and the three 5's and three 3's match the the 3 in the index.

Therefore, $\sqrt[3]{\dfrac{125}{27}} = \dfrac{\sqrt[3]{125}}{\sqrt[3]{27}} = \dfrac{5}{3}$.

The solution is a **rational number.**

Example 8: Evaluate a Radical Expression



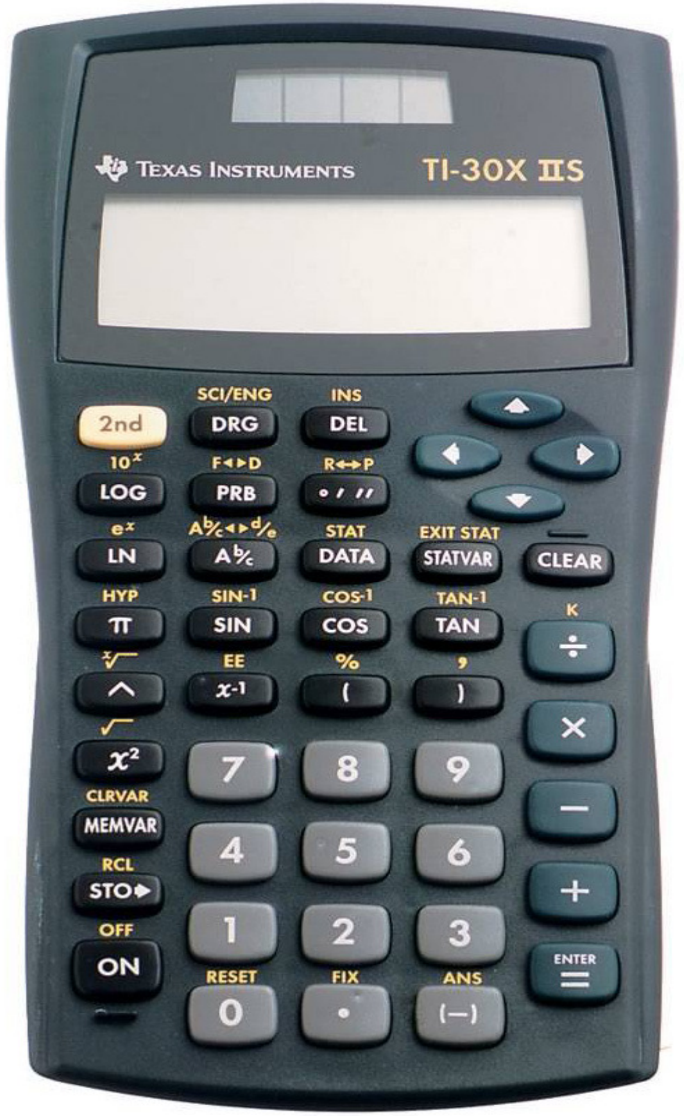
Evaluate $\sqrt{105625}$ with a calculator.

We will use the TI-30X IIS.

- Press the 2nd button and then the x^2 button. Wee will see $\sqrt{(}$.
- Type **105625** .
- Press the right parenthesis) button to close the set.
- Press the ENTER button.

The answer is **325** which is a rational number and more specifically an **integer**.

Example 9: Evaluate a Logarithmic Expression



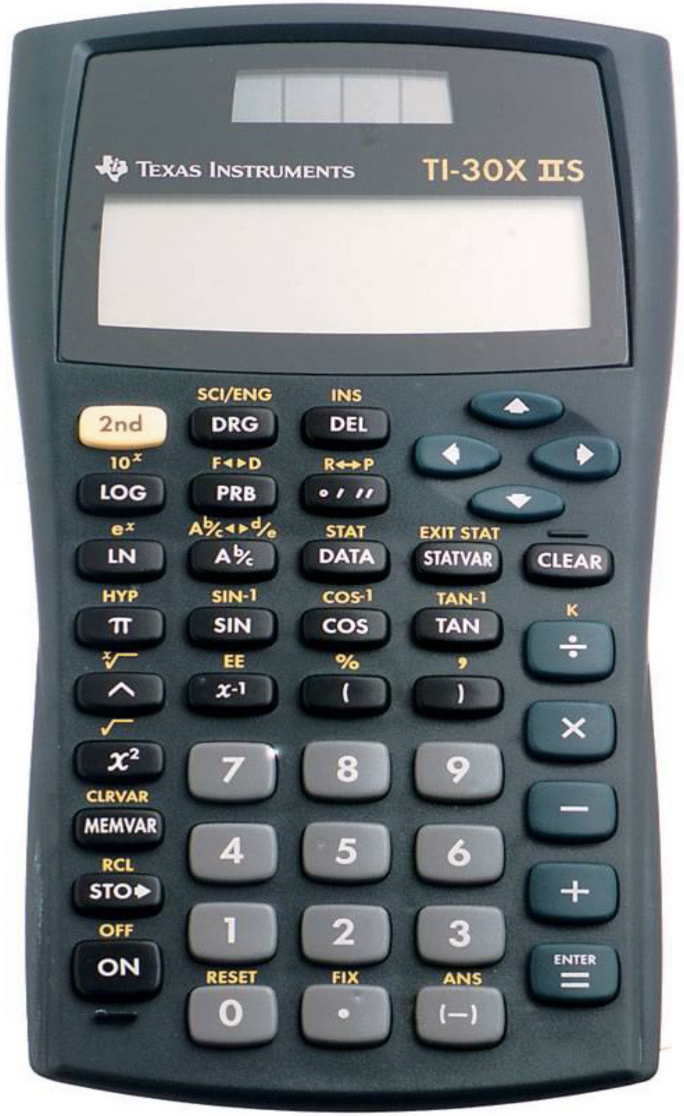
Evaluate **log 1000** with a calculator.

We will use the TI-30X IIS.

- Press the LOG button because we are dealing with a log base 10. You will see **log (**.
- Type **1000**.
- Press the right parenthesis button **)** to “close” the set.
- Press the ENTER button.

The answer is **3** which is a rational number and more specifically an **integer**.

Example 10: Evaluate a Logarithmic Expression



Evaluate **ln 1** with a calculator.

We will use the TI-30X IIS.

- Press the LN button because we are dealing with a log base ***e***. You will see **ln (**.
- Type **1**.
- Press the right parenthesis button **)** to “close” the set.
- Press the ENTER button.

The answer is **0** which is a rational number and more specifically an **integer**.

Example 11: Use the Change-of-Base Property

Evaluate $log_3\ 8$. Round the answer to two decimal places.

Let's use both versions of the *Change-of-Base Property* to illustrate that it does not matter which one we use. In either case, we must use a calculator.

Using log base 10: $log_3\ 8 = \dfrac{log\ 8}{log\ 3} \cong 1.89$

Using log base ***e***: $log_3\ 8 = \dfrac{ln\ 8}{ln\ 3} \cong 1.89$