Examples
Operations on Rational Numbers

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Multiply rational numbers.
2. Divide rational numbers.
3. Add and subtract rational numbers with like denominators.
4. Add and subtract rational numbers with unlike denominators.
5. Perform operations on mixed numbers.
6. Change complex fractions to simple form.

Example 1: Multiply Rational Numbers

Find the product of $-\frac{3}{4} \cdot \frac{8}{9}$ If possible, reduce to lowest terms.

We notice that the first fraction is negative. For multiplication purposes we will move the negative sign to the numerator. This is an acceptable operation. We could have also moved the negative sign to the denominator which is done less frequently.

$$\frac{-3}{4} \cdot \frac{8}{9}$$

Now we can multiply the respective numerators and denominators as follows:

$$\frac{-3 \cdot 8}{4 \cdot 9} = \frac{-24}{36} = \frac{-2}{3}$$

We always reduce the result to lowest terms.

Note: It is more acceptable to write $\frac{-2}{3}$ as $-\frac{2}{3}$

Example 2: Divide Rational Numbers (1 of 2)

Find the quotient of $21 \div \frac{5}{8}$. If possible, reduce to lowest terms.

The first number is an integer. Since the second number is a fraction, let's change 21 to fraction format, specifically $21 = \frac{21}{1}$. Then let's write the following:

$$\frac{21}{1} \div \frac{5}{8}$$

Now let's find the product of the first fraction and the reciprocal of the second fraction, which is $\frac{8}{5}$.

Example 2: Example 2: Divide Rational Numbers (2 of 2)

We get $\frac{21}{1} \cdot \frac{8}{5} = \frac{21 \cdot 8}{1 \cdot 5} = \frac{168}{5}$.

The result is reduced to lowest terms. I challenge you to find a factor common the the numerator and the denominator.

Now, should we change the improper fraction to a mixed number? Only if you are asked to do so. Otherwise, leave it alone.

Example 3: Divide Rational Numbers

a. Evaluate $\frac{-9}{0}$

The denominator of any fraction CANNOT contain the value 0. If the denominator of a fraction is 0, we say that the value of this fraction is undefined.

b. Evaluate $\frac{0}{-9}$

A numerator IS allowed to take on the value of 0. Any fraction with a numerator equal to 0 has an overall value of 0, as long as the denominator is not also 0.

c. Evaluate $\frac{0}{0}$

On one hand, its value is undefined because the denominator is equal to ***0***. On the other hand, its value is ***0*** because the numerator is ***0***. Since there is a disagreement, we will say that this value of is **indeterminate**.

NOTE: $\frac{0}{0} \neq 0$

Example 4: Add and Subtract Rational Numbers with Like Denominators

a. Find the sum of $\frac{3}{7}+\frac{2}{7}$. If necessary, reduce to lowest terms.

$$\frac{3}{7}+\frac{2}{7}=\frac{3+2}{7}=\frac{5}{7}$$ This is reduced to lowest terms!

b. Find the difference of $\frac{11}{12}-\frac{5}{12}$. If necessary, reduce to lowest terms.

$$\frac{11}{12}-\frac{5}{12}=\frac{11-5}{12}=\frac{6}{12}$$ This is NOT reduced to lowest terms!

We notice that 6 and 12 have the factor of 6 in common, and when we divide both by this common factor, we get $\frac{1}{2}$.

Example 5: Add Rational Numbers with Unlike Denominators

Find the sum of $\frac{2}{5}+\frac{3}{7}$. If necessary, reduce to lowest terms.

Let's find a number divisible by all denominators. Please understand that we can always find such a number by calculating the product of all denominators. In our case, that would be 5(7) = 35 and there is no smaller number! I challenge you to find one!

Let's change all denominators to **35** by multiplying the existing denominators by an appropriate number. Of course, we have to multiply the numerators by this number as well.

$$\frac{2}{5}+\frac{3}{7}=\frac{2}{5}\cdot\frac{7}{7}+\frac{3}{7}\cdot\frac{5}{5}$$

NOTE: We created equivalent fractions! The first fraction in the sum was multiplied by $\frac{7}{7}$ and the second fraction by $\frac{5}{5}$.

Example 5: Add Rational Numbers with Unlike Denominators
(2 of 2)

After multiplication, we end up with $\frac{14}{35} + \frac{15}{35}$.

All that's left to do now is add the numerators and put this sum over the denominator to get $\frac{29}{35}$ which is reduced to lowest terms.

Example 6: Add Rational Numbers with Unlike Denominators
(1 of 2)

Find the sum of $\frac{-3}{4} + \frac{-4}{7}$ and express as a rational number.

Both numbers are negative. Remember … "Adding a gambling loss to another gambling loss is still a gambling loss."

Note there is an operational sign next to a directional sign. Before we carry out the addition, we'll change to the following:

$\frac{-3}{4} - \frac{4}{7}$

Note that + (–) becomes (–).

Example 6: Add Rational Numbers with Unlike Denominators
(2 of 2)

The two rational numbers have unlike denominators. Therefore, let's find a number divisible by all denominators. Please understand that we can always find such a number by calculating the product of all denominators. In our case, that would be 4(7) = 28 and there is no smaller number! I challenge you to find one!

Let's change all denominators to **28** by multiplying the existing denominators by an appropriate number. Of course, we have to multiply the numerators by this number as well.

$$\frac{-3(7)}{4(7)} - \frac{4(\ \ )}{7(\ \ )} = \frac{-21}{28} - \frac{16}{28}$$
$$= \frac{-21-16}{28}$$
$$= \frac{-37}{28} = -\frac{37}{28}$$

NOTE: We created equivalent fractions! The first fraction in the sum was multiplied by $\frac{7}{7}$ and the second fraction by $\frac{4}{4}$.

Example 7: Add Mixed Numbers (1 of 2)

Find the sum of $-6\frac{1}{2}+\left(-2\frac{2}{3}\right)$ and express as a mixed number.

Both numbers are negative. Remember … "Adding a gambling loss to another gambling loss is still a gambling loss."

Note there is an operational sign next to a directional sign. Before we carry out the addition, we'll change to the following:

$-6\frac{1}{2}-2\frac{2}{3}$ Note that the double sign + (–) becomes (–).

Be aware that $-6\frac{1}{2}=-6-\frac{1}{2}$ and $-2\frac{2}{3}=-2-\frac{2}{3}$!!!

$-6\frac{1}{2}$ IS NOT EQUAL TO $-6+\frac{1}{2}$!!!

Example 7: Add Mixed Numbers (2 of 2)

Therefore, we get $-6\frac{1}{2}-2\frac{2}{3}=-\frac{13}{2}-\frac{8}{3}$.

Note: We changed the two mixed numbers to improper fractions!!!

Using 2(3) = 6 for the denominator, we find the following:

$$
\begin{aligned}
-6\frac{1}{2}-2\frac{2}{3} &= -\frac{13}{2}\cdot\frac{3}{3}-\frac{8}{3}\cdot\frac{2}{2} \\
&= -\frac{39}{6}-\frac{16}{6} \\
&= -\frac{55}{6} \\
&= -\,9\frac{1}{6}
\end{aligned}
$$

Example 8: Subtract Mixed Numbers

Find the difference of $5\frac{1}{4} - 2\frac{3}{4}$. Leave the answer as an improper fraction reduced to lowest terms.

Here we must first change the two mixed numbers to improper fractions.

$$5\frac{1}{4} - 2\frac{3}{4} = \frac{21}{4} - \frac{11}{4}$$

$$= \frac{10}{4}$$ this is NOT reduced to lowest terms!

We notice that 10 and 4 have the factor of 2 in common, and when we divide both by this common factor, we get $\frac{5}{2}$.

Example 9: Subtract Mixed Numbers

Find the difference of $-5\frac{1}{4}-\left(-2\frac{3}{4}\right)$. Leave the answer as an improper fraction reduced to lowest terms.

Here we must first change the two mixed numbers to improper fractions.

$$-5\frac{1}{4}-\left(-2\frac{3}{4}\right)=-\frac{21}{4}-\left(-\frac{11}{4}\right)=-\frac{21}{4}+\frac{11}{4}$$

Remember, **– (–)** is the same as +.

Now, we can combine the two fractions using the rules for fraction addition and combining integers.

$$\frac{-21+11}{4}=\frac{-10}{4}=-\frac{5}{2}$$

Example 10: Multiplying Mixed Numbers

Find the product of $5\frac{1}{4}\left(2\frac{3}{4}\right)$ and express as a mixed number.

Here we must first change the two mixed numbers to improper fractions.

$$5\frac{1}{4}\left(2\frac{3}{4}\right) = \frac{21}{4}\left(\frac{11}{4}\right) = \frac{21 \cdot 11}{4 \cdot 4} = \frac{231}{16}$$

Using the calculator, we find that 16 divides into 231 fourteen times with a remainder of 7.

Therefore, $\frac{231}{16} = 14\frac{7}{16}$.

Example 11: Convert Complex Numbers

Convert $\frac{1}{\frac{1}{14}}$ to a number only containing an integer in the numerator and the denominator.

We see that there is the rational number 1 in the numerator and the rational number $\frac{1}{14}$ in the denominator.

We know that horizontal lines separating two numbers usually indicate division. We also know that if rational numbers are divided, we simply find the product of the first number and the reciprocal of the second number.

Therefore, we will write the following: $\frac{1}{\frac{1}{14}} = 1 \div \frac{1}{14} = 1 \cdot \frac{14}{1} = \frac{14}{1}$

We now have the integers 14 and 1 in the numerator and denominator, respectively. Of course, we can simply express the product as 14.

Example 12: Convert Complex Rational Numbers

Convert $\dfrac{\frac{2}{7}}{\frac{1}{14}}$ to a rational number containing only an integer in the numerator and the denominator.

We see that there is the rational number $\frac{2}{7}$ in the numerator and the rational number $\frac{1}{14}$ in the denominator.

We know that horizontal lines separating two numbers usually indicate division. We also know that if rational numbers are divided, we simply find the product of the first number and the reciprocal of the second number.

Therefore, we will write the following: $\dfrac{\frac{2}{7}}{\frac{1}{14}} = \frac{2}{7} \div \frac{1}{14} = \frac{2}{7} \cdot \frac{14}{1} = \frac{28}{7}$

We now have the integers 28 and 7 in the numerator and denominator, respectively. Of course, we can simply express the product as 4.