Examples
Introduction to Rational Numbers

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Define the rational numbers.
2. Define fractions.
3. Create equivalent fractions.
4. Reduce fractions to lowest terms.
5. Convert improper fractions to mixed numbers.
6. Convert mixed numbers to improper fractions.
7. Express non-terminating/repeating decimal numbers as fractions.
8. Express terminating/non-repeating decimal numbers as fractions.
9. Express fractions as decimal numbers.

Example 1: Create Equivalent Fractions

Create four equivalent fractions for $\frac{5}{13}$ by multiplying the numerator and denominator in turn by 2, 3, 4, and 5.

$$\frac{5}{13} \cdot \frac{2}{2} = \frac{10}{26}$$

$$\frac{5}{13} \cdot \frac{3}{3} = \frac{15}{39}$$

$$\frac{5}{13} \cdot \frac{4}{4} = \frac{20}{52}$$

$$\frac{5}{13} \cdot \frac{5}{5} = \frac{25}{65}$$

Example 2: Reduce Fractions to Lowest Terms

Reduce $\frac{12}{80}$ to lowest terms. Use a calculator as necessary!

We see that both 12 and 80 are divisible by 2 since they are even numbers. We divide the numerator and the denominator by 2 to get $\frac{6}{40}$.

Is this fraction reduced to lowest terms? NO, because we see that both 6 and 40 are also divisible by 2 since they are even numbers.

We divide the numerator and the denominator of the reduced fraction by 2 to get $\frac{3}{20}$. Is this fraction reduced to lowest terms? YES!

It should be obvious that the only factor the numerator and denominator have in common is 1.

Example 3: Convert a Mixed Number to an Improper Fraction

Change $3\frac{1}{2}$ to an improper fraction.

NOTE: $3\frac{1}{2}$ means $3 + \frac{1}{2}$!!!!!!!!!!!!

We multiply the integer of **3** by the denominator of the fractional part which is **2**.

To this product we then add the numerator of the fractional part which is **1**. The denominator stays the same throughout.

In summary, we get $3\frac{1}{2} = \frac{3 \cdot 2 + 1}{2} = \frac{6 + 1}{2} = \frac{7}{2}$.

Example 4: Convert an Improper Fraction to a Mixed Number

Write the improper fraction $\frac{8}{3}$ as a mixed number. Use a calculator as necessary!

We divide 8 by 3. We get a quotient of 2 and a remainder of 2. Note, do the calculation $8 - 2(3)$!

We write the remainder 2 over the original denominator of 3 and place this fraction to the right of the quotient 2.

We find that $\frac{8}{3}$ is equal to the mixed number $2\frac{2}{3}$.

Example 5: Convert an Improper Fraction to a Mixed Number

Write the improper fraction $\frac{235}{17}$ as a mixed number with a fractional part. Use a calculator as necessary!

Using the calculator, we divide 235 by 17. We get a quotient of 13 and a remainder of 14. Note, do the calculation 235 – 13(17)!

We now write the remainder 14 over the original denominator of 17 and place this fraction to the right of the quotient.

We find $\frac{235}{17} = 13\frac{14}{17}$.

Example 6: Express Non-Terminating/Repeating Decimals as Fractions

Express $0.\overline{6}$ as a fraction reduced to lowest terms.

There is one (1) repeating digit. Therefore, the denominator of the fraction becomes 9. The numerator will be the repeating number without the decimal and the bar.

$0.\overline{6} = \dfrac{6}{9}$ This is NOT reduced to lowest terms!

Using the *tests for divisibility* as a guide, we notice that the numerator and denominator are divisible by **3**.

Finally, we can state $0.\overline{6} = \dfrac{2}{3}$ which is reduced to lowest terms.

Example 7: Express Non-Terminating/Repeating Decimals as Fractions

Express $2.\overline{81}$ as an improper fraction reduced to lowest terms.

<u>Let's work on the decimal portion first and then worry about the whole number.</u>

There are two (2) repeating digits in the decimal portion. Therefore, its denominator becomes 99. The numerator will be the repeating number without the decimal and the bar.

$0.\overline{81} = \dfrac{81}{99}$ This is NOT reduced to lowest terms!

Using the *tests for divisibility* as a guide, we notice that the numerator and denominator are divisible by **9**. Therefore, **9** divides both into the numerator and the denominator.

Finally, we can state $\dfrac{81}{99} = \dfrac{9}{11}$ which is reduced to lowest terms.

Example 7: Express Non-Terminating/Repeating Decimals as Fractions

We now know that $2.\overline{81}$ can be written as $2\frac{9}{11}$ and we can change this to an improper fraction.

We multiply the whole number **2** by the denominator of the fractional part which is **11**.

To this product we then add the numerator of the fractional part, which is **9**. The denominator stays the same throughout.

$$2\frac{9}{11} = \frac{2 \cdot 11 + 9}{11} = \frac{31}{11}$$

Example 8: Express Terminating/Non-Repeating Decimals as Fractions

Express **0.7** as a fraction. If necessary, reduce to lowest terms

Write $\dfrac{0.7}{1}$

There is one (1) number after the decimal point, therefore, multiply both the numerator and the denominator by 10. Use a calculator if necessary!

$$\frac{0.7}{1} \cdot \frac{10}{10} = \frac{7}{10}$$ This is reduced to lowest terms!

Example 9: Express Terminating/Non-Repeating Decimals as Fractions

Express **5.49** as a fraction. If necessary, reduce to lowest terms.

Write $\dfrac{5.49}{1}$

There are two (2) numbers after the decimal point, therefore, multiply both the numerator and the denominator by 10·10 = 100. Use a calculator if necessary!

$$\frac{5.49}{1} \cdot \frac{100}{100} = \frac{549}{100}$$ This is reduced to lowest terms!

Express **0.125** as a fraction. If necessary, reduce to lowest terms.

Write $\dfrac{0.125}{1}$

There are three (3) numbers after the decimal point, therefore, multiply both the numerator and the denominator by 10·10·10 = 1000. Use a calculator if necessary!

$$\frac{0.125}{1} \cdot \frac{1000}{1000} = \frac{125}{1000}$$ This is NOT reduced to lowest terms!

Using the *tests for divisibility* as a guide, we notice that the numerator ends in 5 and the denominator in 0. This means that **5** divides into both.

$$\frac{\overset{25}{\cancel{125}}}{\underset{200}{\cancel{1000}}} = \frac{25}{200}$$

Example 10: Express Terminating/Non-Repeating Decimals as Fractions

However, we are not done. We notice that **5** still divides into both the numerator and the denominator.

$$\frac{\overset{5}{\cancel{25}}}{\underset{40}{\cancel{200}}} = \frac{5}{40}$$

However, we are still not done. We again notice that **5** divides into both the numerator and the denominator.

$$\frac{\overset{1}{\cancel{5}}}{\underset{8}{\cancel{40}}} = \frac{1}{8}$$ This is now reduced to lowest terms! The numerator and the denominator no longer have factors in common.

We found that 0.125 is equal to $\frac{1}{8}$.

Example 11: Express Fractions as Decimal Numbers

Express the following fractions as decimal numbers using the calculator. If necessary, round to 3 decimal places.

a. $\dfrac{1}{10}$

Calculator Tip:

1	÷	10	Enter

$\dfrac{1}{10} = 0.1$ There is no need to round!

b. $\dfrac{5}{8}$

$\dfrac{5}{8} = 0.625$ There is no need to round!

Example 13: Express Fractions as Decimal Numbers

Express $\frac{1}{3}$ as a decimal number using the calculator. If necessary, round to 3 decimal places.

Calculator Tip:

1	÷	3	Enter

We notice that the calculator shows us seemingly infinitely many repeating 3's after the decimal point. That is, we encountered a **not terminating and repeating decimal number**.

We are asked to round it to to decimal places to get 0.333. Of course, we could have also written $0.\overline{3}$.