## Examples

## Exponents

## Advanced Order of Operations

Based on power point presentations by Pearson Education, Inc.
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## Learning Objectives

1. Define and evaluate exponential expressions.
2. Use exponent rules.
3. Use the advanced Order of Operations given grouping symbols, exponents, multiplication, division, addition, and subtraction.

## Example 1: Evaluate Exponential Expressions



## Example 2: Evaluate Exponential Expressions

Evaluate $3^{2}$ by hand and using a calculator.
The exponent 2 tell us to use the base 3 twice in a multiplication.
$3^{2}=3(3)=9$
Calculator:
Actually, we can find second powers on the calculator in two different ways. In Example 1, we learned to use the $\mathbf{x}^{2}$ button to evaluate second powers as follows:


However, we can also press the caret ^ button on the calculator followed by the index as follows:

| 3 | $\wedge$ | 2 | Enter |
| :--- | :--- | :--- | :--- |

## Example 3: Evaluate Exponential Expressions

Evaluate $4^{3}$ by hand and using the calculator.
The exponent 3 tell us to use the base 4 three times in a multiplication.

$$
4^{3}=4(4)(4)=64
$$

Calculator:
Here we have no choice but to use the caret ${ }^{\wedge}$ button on the calculator as follows:

| 4 | $\wedge$ | 3 | Enter |
| :---: | :---: | :---: | :---: |

## Example 4: Evaluate Exponential Expressions

Evaluate $100^{1}$.
The exponent 1 tell us to use the base 100 one time in a multiplication.
$100^{1}=100$
NOTE: The value of any number with an exponent of 1 is the number itself. Customarily, the $\mathbf{1}$ is not written!

## Example 5: Evaluate Exponential Expressions



## Example 6: Evaluate Exponential Expressions

Use the Negative Exponent Rule to simplify the following exponential expressions without a calculator:
a. $8^{-2}=\frac{1}{8^{2}}=\frac{1}{8 \cdot 8}=\frac{1}{64}$
b. $5^{-3}=\frac{1}{5^{3}}=\frac{1}{5 \cdot 5 \cdot 5}=\frac{1}{125}$
C. $\quad 7^{-1}=\frac{1}{7^{1}}=\frac{1}{7}$

## Example 7: Evaluate Exponential Expressions

Use the Power Rule for Fractions to simplify the following exponential expressions without a calculator:
a. $\left(\frac{2}{5}\right)^{3}=\frac{2^{3}}{5^{3}}=\frac{2 \cdot 2 \cdot 2}{5 \cdot 5 \cdot 5}=\frac{8}{125}$
b. $\left(\frac{3}{7}\right)^{2} \cdot\left(\frac{1}{3}\right)^{2}=\frac{3^{2}}{7^{2}} \cdot \frac{1^{2}}{3^{2}}$

$$
\begin{aligned}
& =\frac{3 \cdot 3}{7 \cdot 7} \cdot \frac{1 \cdot 1}{3 \cdot 3}=\frac{9}{49} \cdot \frac{1}{9} \\
& =\frac{1}{49}
\end{aligned}
$$

## Example 8: Use the Advanced Order of Operations

Evaluate $7^{2}-20 \div 2^{2} \cdot(2+3)^{3}$.
Given $7^{2}-20 \div 2^{2} \cdot(2+3)^{3}$, we evaluate the parentheses first.
then $7^{2}-20 \div 2^{2} \cdot 5^{3}$. Next, we evaluate both exponents:
$49-20 \div 4 \cdot 125$. Next, we evaluate division:
49-5 125. Next, we evaluate multiplication:
49-625. Next, we evaluate subtraction:

- 576


## Example 9: Use the Advanced Order of Operations

Evaluate $6^{2}-24 \div 2^{2} \cdot 3+1$

Given $6^{2}-24 \div 2^{2} \cdot 3+1$, we evaluate both exponent first. then $36-24 \div 4 \cdot 3+1$. Next, we evaluate division:
$36-6 \cdot 3+1$. Next, we evaluate multiplication:
$36-18+1$. Next, we evaluate subtraction:
$18+1$. Next, we evaluate addition:
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## Example 10: Use the Advanced Order of Operations

Evaluate $(-7)^{0}$ and $-7^{0}$.
We know that $(-7)^{0}$ equals 1 .
In $-7^{0}$ only 7 is raised to the 0 power. Therefore, you must use the Order of Operations rule that states that exponents are evaluated before multiplication.

Now, $-7^{0}$ actually equals $-1\left(7^{0}\right)$. Given that $7^{0}$ equals 1 , we find that $-7^{0}$ must equal -1 (1) or -1 .

## Example 11: Use the Advanced Order of Operations

Evaluate $(-6)^{2}$ and $-6^{2}$.
We know that $(-6)^{2}$ equals $(-6) \cdot(-6)$ which equals 36 .
In $-6^{2}$ only 6 is raised to the 2 nd power. Therefore, you must use the Order of Operations rule that states that exponents are evaluated before multiplication.

Now, $-6^{2}$ actually equals $-1\left(6^{2}\right)$. Given that $6^{2}$ equals 36 , we find that $-6^{2}$ must equal - 1(36) or -36 .

## Example 12: Use the Advanced Order of Operations

Evaluate $(-4)^{3}$ and $-4^{3}$.
$(-4)^{3}=(-4) \cdot(-4) \cdot(-4)=-64$
In $-4^{3}$ only 4 is raised to the 3rd power. Therefore, you must use the Order of Operations rule that states that exponents are evaluated before multiplication.

Now, $-4^{3}$ actually equals $-1\left(4^{3}\right)$. Given that $4^{3}$ equals 64 , we find that $-4^{3}$ must equal -1 (64) or -64 .

