Concepts
Whole and Decimal Numbers

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Use the vocabulary and symbols of arithmetic.
2. Define natural and whole numbers.
3. Round whole numbers.
4. Define decimal numbers.
5. Round decimal numbers.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

What is Arithmetic?

Arithmetic is an elementary branch of mathematics that studies numerical operations like addition, subtraction, multiplication, and division on whole numbers, decimal numbers, fractions, negative numbers, and irrational number. It also includes exponentiation, extraction of roots, and taking logarithms.

Arithmetic operations form the basis of many branches of mathematics. They also play a role in the sciences, like physics and economics, and are present in many aspects of daily life. For example, we use arithmetic to calculate change while shopping, to manage personal finances, to cook and bake, or to build.

The practice of arithmetic is thousands of years old. Ancient civilizations like the Egyptians and the Sumerians invented numeral systems to solve practical arithmetic problems in about 3000 BC. The ancient Indians developed the concept of zero and the decimal system, which Arab mathematicians further refined and spread to the Western world during the medieval period (about 500 to 1500 AD).

1. Some Vocabulary and Symbols of Arithmetic

The Equal Sign (=)

The sign **=** indicates that the values to the right of it are equal to the values to the left of it. Visually, the sign consist of two short horizontal parallel lines.

For example, 8 + 2 = 5 + 5

Addition

Given 8 + 2 = 10, the 8 and the 2 are called **Addend 1 and Addend 2**. The 10 is called **Sum**.

Subtraction

Given 8 – 2 = 6, the 8 is called the **Minuend** and the 2 is called the **Subtrahend**. The 6 is called **Difference**.

Some Vocabulary and Symbols of Arithmetic

Division

$8 \div 2 = 4$. The 8 is called **Dividend** and the 2 is called the **Divisor**. The 4 is called **Quotient**.

Multiplication

$8 \cdot 2$ or 8×2 equals 16. The 8 is called **Multiplicand** and the 2 is called the **Multiplier**. The 16 is called **Product**.

NOTE: In higher mathematics we usually express multiplication using parentheses ( ) and not the multiplication symbols $\cdot$ or $\times$.

That is, instead of $8 \cdot 2$ or 8×2 we write 8(2), which is pronounced "8 times 2".

Some Vocabulary and Symbols of Arithmetic (3 of 3)

"Approximately Equal" Sign ($\cong$)

The sign $\cong$ indicates that the values to the right of it are similar the the values to the left of it but the two values are not exactly equal. Visually, the sign is a "squiggly" over the equal sign. Often, the sign $\approx$ is used instead which means "almost equal."

For example, anyone who has heard of the number π (pi) knows that it is approximately equal to 3.14. That is, $\pi \cong 3.14$.

Factors

Numbers that divide into other numbers without leaving a remainder.

For example, 2 and 3 divide into 6 without a remainder, but 4 and 5 do not. Therefore, only 2 and 3 are factors of 6.

2. Natural and Whole Numbers (1 of 2)

The first set of numbers we will examine is the set of *Natural Numbers* sometimes called counting numbers.

We write this set as *{1, 2, 3, 4, 5, ...}*. Please note that sets of numbers are always enclosed in braces.

The next set is the set of *Whole Numbers*. It includes all the *Natural Numbers* and the number 0.

We write this set as *{0, 1, 2, 3, 4, 5, ...}*.

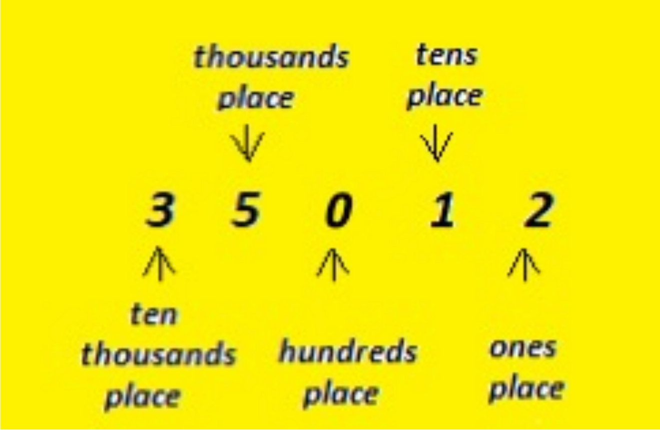
Note: The three dots indicate that this set does not have a final element and that the listing goes on forever. They are called an "ellipsis". The sets of *Natural Numbers* and *Whole Numbers* are infinite sets!

Natural and Whole Numbers (2 of 2)

In our number system, the value of a digit depends on its place, or position, in the number. Each place has a value of 10 times the place to its right. A whole number in standard form is separated into groups of three digits from right to left using commas.

Place Value of Whole Numbers

For example, given the number ***35,012***, we say it has five digits whose respective places or place values are as follows:



3. Whole Numbers and Rounding (1 of 3)

Rounding a number to a certain place value means that we want to find an **approximation** of the number. We will use the ”approximately equal” sign ≅ or the “almost equal” sign ≈.

Rounding is the process for anyone who doesn't care to be exact. That's okay to a point because there are many different uses for numbers. Generally, the size of the number and its use dictates the place value to which it should be rounded.

Rounding Strategy for Whole Numbers

Step 1: Locate the digit that occupies the rounding place.

Example 1:

Round the number 7,473,854,056 to the nearest ten million.

The digit occupying the rounding place is 7. That is, 7,473,854,056.

Whole Numbers and Rounding (2 of 3)

Step 2: Examine the digit to the right of the rounding place and

- if the digit to the right of the rounding place is less than 5, leave the digit in the rounding place unchanged. Fill the remaining places with 0s. This is also called **"rounding down"**.
- if the digit to the right of the rounding place is greater than 5 or equal to 5, add 1 to the digit in the rounding place. Fill the remaining places with 0s. This is also called **"rounding up".**

Example 1 continued:

Given number 7,473,854,056 , the digit to the right of 7 is 3 which is less than 5. We leave the digit in the rounding place unchanged and fill the remaining places with 0s. That is, we are "rounding down".

Finally, we can state 7,473,854,056 ≅ 7,470,000,000

Whole Numbers and Rounding (3 of 3)

Example 2:

Round the number 7,473,854,056 to the nearest million.

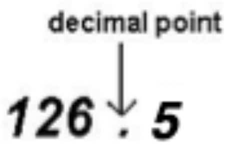
The digit occupying the rounding place is 3. That is, 7,473,854,056.

The digit to the right of 3 is 8 which is greater than 5. We add 1 to the digit in the rounding place and fill the remaining places with 0s. That is, we are "rounding up".

Finally, we can state 7,473,254,056 $\cong$ 7,474,000,000

4. Decimal Numbers (1 of 3)

Our counting system lets us write partial amounts of whole numbers using a clever symbol called the **decimal point**. When a number contains a decimal point, we call it a **decimal number** or simply a **decimal**.



Please note that the 5 to the right of the decimal point, indicates that the number is exactly halfway in between 126 and 127.

Whole numbers can be written as decimal numbers by using zeros in the decimal places. We can add as many decimal places as needed depending on a particular situation.

For example, the number ***45*** can be written as ***45.0*** or ***45.00*** or ***45.000***, and so on.

Decimal Numbers (2 of 3)

Decimal numbers can be (a) terminating; (b) non-terminating and non-repeating; or (c) non-terminating and repeating.

(a) Terminating, Non-Repeating Decimal Number – The number *0.75* is a terminating, non-repeating decimal number.

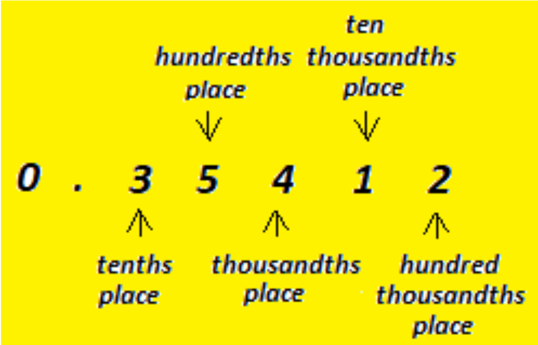
(b) Non-Terminating, Repeating Decimal Number – The number $0.\overline{81}$ is a non-terminating, repeating decimal. It is equal to *0.81 81 81 81 81 81 81...* with *81* repeating infinitely.

(c) Non-Terminating, Non-Repeating Decimal Number – The value of the Greek number Pi, whose symbol is π, is a non-terminating, non-repeating decimal number approximately equal to *3.141592654 ...* or better known as *3.14*. We encounter this number often in circle calculations.

Decimal Numbers (3 of 3)

Place Value of Decimal Numbers

For example, given the number ***0.35412***, we say it has five decimal places whose respective places or place values are as follows:



Please note that when there is no number to the right of the decimal place, we can express this as either ***0.35412*** or simply as ***.35412*** leaving off the ***0*** in the ones place.

5. Decimal Numbers and Rounding (1 of 3)

Just like there is a rounding strategy for whole numbers, there also exists one for decimal numbers.

Rounding Strategy for Decimal Numbers

Step 1: Locate the digit that occupies the rounding place.

Example 3:

Round the number 4.2526 to the nearest tenth (one decimal place).

The digit occupying the rounding place is 2. That is, 4.2526.

Decimal Numbers and Rounding (2 of 3)

Step 2: Examine the digit to the right of the rounding place and

- if the digit to the right of the rounding place is less than 5, leave the digit in the rounding place unchanged and drop the remaining decimal places. This is also called "rounding down".
- if the digit to the right of the rounding place is greater than 5 or equal to 5, add 1 to the digit in the rounding place and drop the remaining decimal places. This is also called "rounding up."

Example 3 continued:

Given 4.2526 , the digit to the right of 2 is 5. We add 1 to the digit in the rounding place and drop the remaining decimal places. That is, we are "rounding up".

Finally, we can state $4.2526 \cong 4.3$

Decimal Numbers and Rounding (3 of 3)

Example 4:

Round the number 4.2526 to the nearest hundredth (two decimal places).

The digit occupying the rounding place is 5. That is, 4.2526.

The digit to the right of 5 is 2 which is less than 5. We leave the digit in the rounding place unchanged and drop the remaining decimal places. That is, we are ”rounding down”.

Finally, we can state $4.2526 \cong 4.25$.