Concepts
Exploring the Right Triangle

Based on power point presentations by Pearson Education, Inc.

Revised by Ingrid Stewart, Ph.D.

Learning Objectives

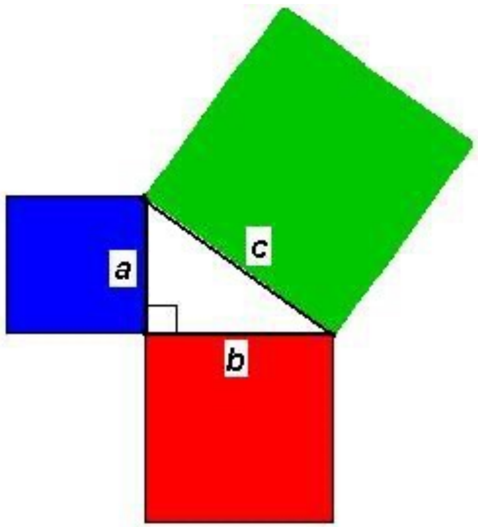
1. Memorize and use the *Pythagorean Theorem*.
2. Memorize trigonometric ratios and evaluate them given sides of right triangles.
3. Evaluate trigonometric ratios given angles in right triangles.
4. Find angle measures given trigonometric ratios.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

1. The Right Triangle and the Pythagorean Theorem (1 of 4)

The right triangle is associated with one of the most famous and useful theorems in mathematics. It is called the **Pythagorean Theorem** named for the Greek mathematician Pythagoras.

The *Pythagorean Theorem* states that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs of the triangle.



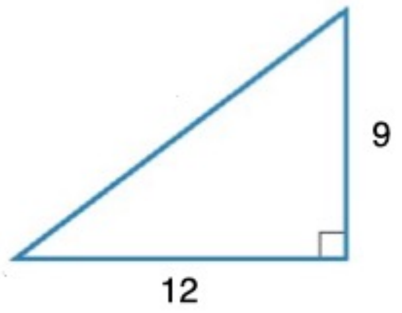
The *Pythagorean Theorem* is usually presented as follows:

Given a right triangle with legs of length a and b and a hypotenuse of length c, then $c^2 = a^2 + b^2$.

The Right Triangle and the Pythagorean Theorem (2 of 4)

Example 1:

Find the length of the hypotenuse c in the following right triangle. Find the answer EXACTLY.



Given legs $a = 9$ and $b = 12$.

Using the *Pythagorean Theorem*, we can write the following:

$9^2 + 12^2 = c^2$

$81 + 144 = c^2$

$c^2 = 225$

The Right Triangle and the Pythagorean Theorem (3 of 4)

Example 1 continued:

Now, we ended up with the squared variable c^2. But what we really need is the value of c. What we will do here is use something called the **Square Root Property** to solve for c. It says the following:

If a squared variable is equal to some number, then the square root of this variable is equal to the positive and negative square root of the number.

Mathematically, this is expressed as follows:

If $x^2 = d$ where d is any number, then x has exactly two solutions, namely $x = \sqrt{d}$ and $x = -\sqrt{d}$.

The Right Triangle and the Pythagorean Theorem

Example 1 continued:

Therefore, by the *Square Root Property*, if $c^2 = 225$ then

$c = \sqrt{225} = 15$ and $c = -\sqrt{225} = -15$.

NOTE: Since we are discussing triangles, and the measure of their sides is never negative, we can rule out any negative solutions.

In conclusion, the length of the hypotenuse c is 15.

2. Definition of Trigonometric Ratios (1 of 5)

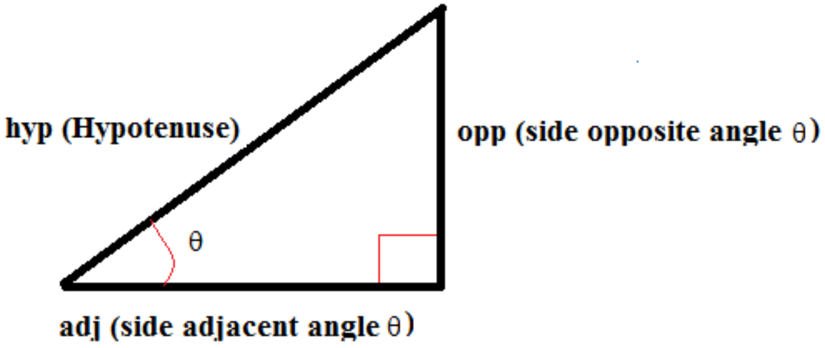
Trigonometric concepts are based on the right triangle. Trigonometry was developed by ancient civilizations. The development of modern trigonometry shifted during the western Age of Enlightenment, beginning with 17th-century mathematicians Isaac Newton and James Stirling and reaching its modern form with Leonhard Euler.

Six trigonometric ratios were eventually established called sine, cosine, tangent, cosecant, secant, and cotangent.

Trigonometric ratios have many real-life applications. They are used to calculate needed angles and lengths in the construction of buildings and in the manufacturing of products such as cars. Various fields like surveying, oceanography, seismology, meteorology, astronomy, acoustics, navigation, electronics, and many more also need to make use of trigonometric ratios.

Definition of Trigonometric Ratios (2 of 5)

Let's draw a right triangle and give the legs and the hypotenuse "special names". We will also name one of the angles using the Greek name **theta** whose symbol is θ (theta)



We will now use this triangle to define the six trigonometric ratios with reference to the angle θ.

Sine Ratio: $\sin\theta = \dfrac{opp}{hyp}$ Cosine Ratio: $\cos\theta = \dfrac{adj}{hyp}$ Tangent Ratio: $\tan\theta = \dfrac{opp}{adj}$

NOTE: Always say "sine theta", "cosine theta", and "tangent theta"!

Definition of Trigonometric Ratios (3 of 5)

The remaining three trigonometric ratios will be stated, but we will not use them much in our course.

Cosecant Ratio: $csc\theta = \frac{hyp}{opp}$ Secant Ratio: $sec\theta = \frac{hyp}{adj}$

Cotangent Ratio: $cot\theta = \frac{adj}{opp}$

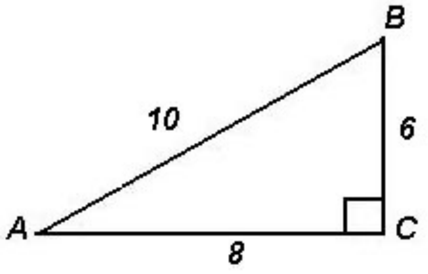
NOTE: Always say "cosecant theta", "secant theta", and "cotangent theta"!

Observe that the cosecant ratio is the reciprocal of the sine ratio; the secant ratio is the reciprocal of the cosine ratio; and the cotangent ratio is the reciprocal of the tangent ratio!

Definition of Trigonometric Ratios

Example 2:

Given the following triangle, find the sine, cosine, and tangent ratios for angles A and B. Express your answers both as a fraction and a decimal.



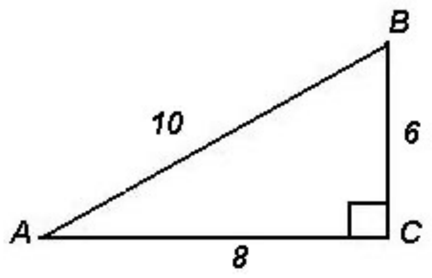
Ratios for Angle A:

$$\sin A = \frac{opp}{hyp} = \frac{6}{10} = \frac{3}{5} = 0.6 \qquad \cos A = \frac{adj}{hyp} = \frac{8}{10} = \frac{4}{5} = 0.8$$

$$\tan A = \frac{opp}{adj} = \frac{6}{8} = \frac{3}{4} = 0.75$$

Definition of Trigonometric Ratios

Example 2 continued:



Ratios for Angle B:

$$\sin B = \frac{opp}{hyp} = \frac{8}{10} = \frac{4}{5} = 0.8 \qquad \cos B = \frac{adj}{hyp} = \frac{6}{10} = \frac{3}{5} = 0.6$$

$$\tan B = \frac{opp}{adj} = \frac{8}{6} = \frac{4}{3} = 1.33$$

Please note that the location of the "side opposite" and the" side adjacent" changes with the location of the angle in the triangle!

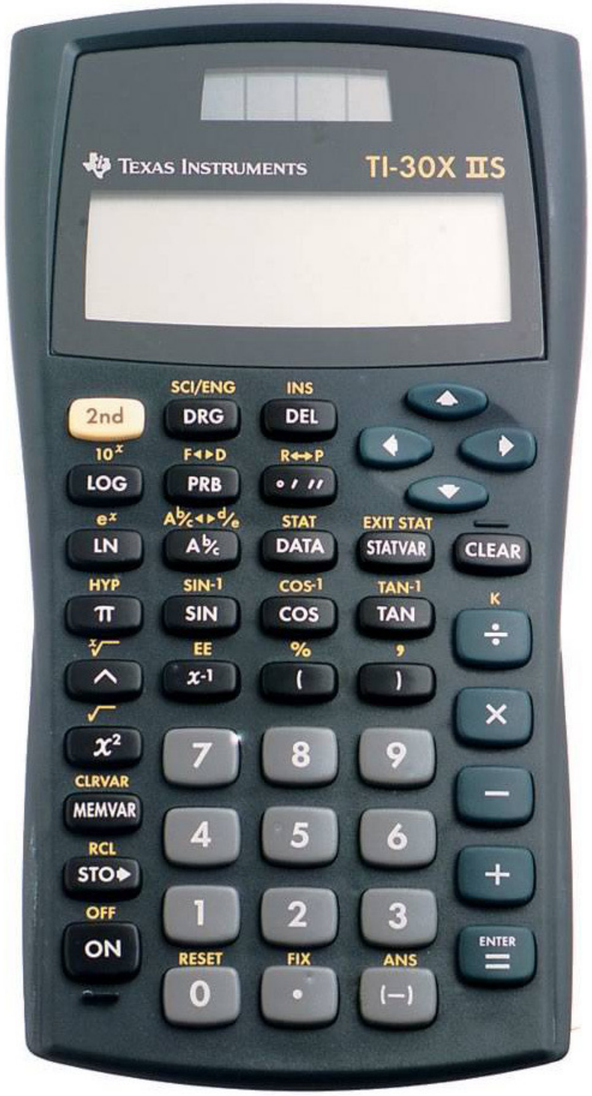
3. Evaluate Trigonometric Ratios Given Angles

Earlier we evaluated trigonometric ratios given the lengths of the sides of a right triangle. Now we will find the values of trigonometric ratios given angle measures.

For this we will use a calculator. How does it know the values? We learn in Calculus how to find these values by hand, ant these concepts are then programmed into it.

WARNING: The calculator evaluates angles in two modes - DEGREE mode and RADIAN mode. We will NOT be discussing radian mode in this course, however, you must be aware of it since you don't want to accidentally be in radian mode when you evaluate trigonometric ratios.

Evaluate Trigonometric Ratios Given Angles (2 of 4)



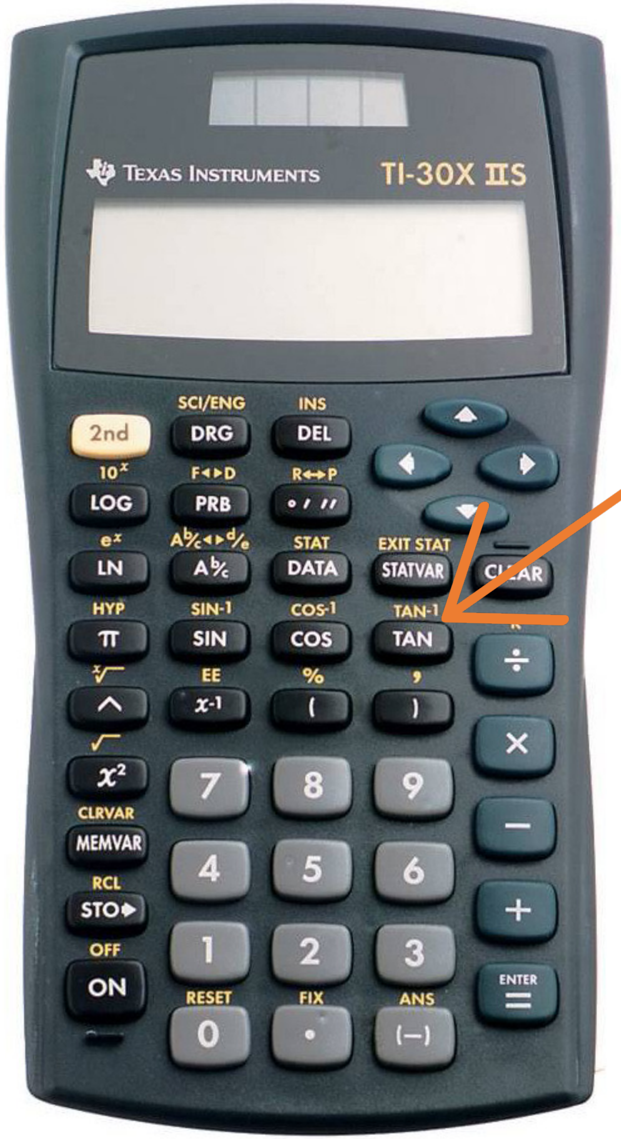
Calculator instructions (based on the TI 30X IIB or IIS) to change from radians to degrees, if necessary.

If you see the word *DEG* in the lower right-hand corner of your calculator window, you are in degree mode. If you see either RAD or GRAD you MUST do the following:

- Press the DRG button on your calculator.
- You will see DEG RAD GRD on the screen with one of them underlined.
- Press the left arrow button until *DEG* is underlined.
- Press Enter. You should now see DEG in the lower right-hand corner of the window.

NOTE: Some calculators do not show what mode they are in. They have a “mode” button. Click it and check what is underlined. Degrees? or Radians? Use the arrows to underline ”Degrees” and press Enter.

Evaluate Trigonometric Ratios Given Angles (3 of 4)



<u>Calculator instructions (based on the TI 30X IIB or IIS) to find the value of trigonometric ratios.</u>

The calculator has a ”sin”, “cos”, and “tan” button.

Press one of them.

Left parenthesis will open when you activate sine, cosine, or tangent. Specifically, you will see sin (, cos (, or tan (.

After you type the degree angle measure, you MUST type the right parenthesis, namely), before you press ENTER.

Evaluate Trigonometric Ratios Given Angles

Example 3:

Find the value of cos 55^{o} rounded to four decimal places, if necessary. Your calculator must be in degree mode!

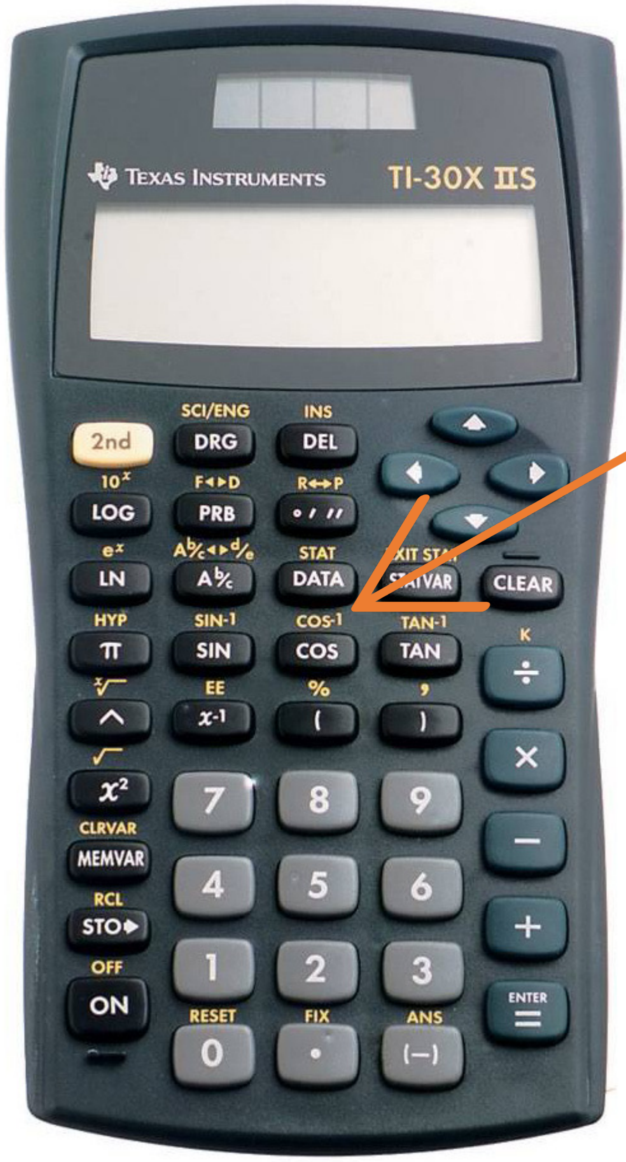
We find that cos 55^{o} is approximately equal to 0.5736. This value is a non-terminal decimal. That is, it is an irrational number.

4. Find Angle Measures (1 of 3)

Earlier we were given angle measures which then allowed us to find the values of the trigonometric ratios. Now we will find angle measures when the values of trigonometric ratios are given.

For this we MUST use a calculator. We are going to use the inverse sine, cosine, and tangent functions. On the calculator you will find $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$. They are called arcsine, arccosine, and arctangent.

Find Angle Measures (2 of 3)



<u>Calculator instructions (based on the TI 30X IIB or IIS) to find an angle measure.</u>

Be sure your calculator is in degree mode!

The inverse trig functions are ABOVE the sin, cos, and tan buttons!

We activate the $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$ function by pressing the 2nd button followed by the “sin”, “cos”, or “tan” button.

Left parenthesis will open when you activate the inverse sine, cosine, or tangent. Specifically, you will see $\sin^{-1}($, $\cos^{-1}($, or $\tan^{-1}($.

After you type the numeric value, you MUST type the right parenthesis, namely), before you press ENTER.

Find Angle Measures (3 of 3)

Example 4:

Use a calculator to find the measure for angle *A* given that $\sin A = 0.86$. Round the angle to two decimal places. Your calculator must be in degree mode!

We write this as $A = \sin^{-1}(0.86)$ and pronounce it as "A equals the arcsine of 0.86".

We find that angle *A* is approximately equal to 59.32^{o}.