



Concepts Radicals and Logarithms

Based on power point presentations by Pearson Education, Inc.
Revised by Ingrid Stewart, Ph.D.

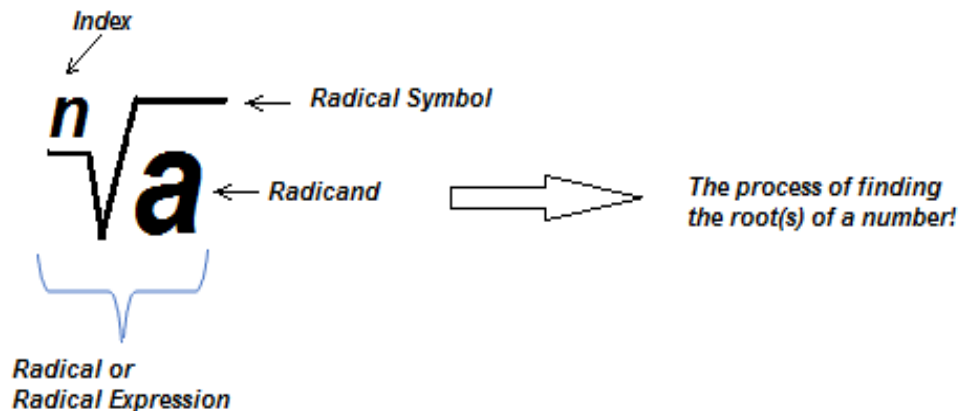
Learning Objectives

1. Define and evaluate some radical expressions.
2. Define and evaluate some logarithmic expressions.
3. Memorize and apply the *Change-of-Base Property* for logarithms.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

1. Definition of Radical Expressions (1 of 5)

Radical expressions or simply “radicals” are related to exponential expressions in that they reverse the operation of raising a number to a power. The radical symbol $\sqrt{\quad}$ indicates this process. Evaluating a radical expression is also called “finding a root.” Please examine the general radical expression carefully!



Some radicals can be evaluated by hand. However, we use the calculator most often. They are programmed with the calculus concepts that allow us to easily evaluate all radicals.

Definition of Radical Expressions (2 of 5)

Radical expressions can have rational, irrational, or imaginary values.

- **Rational values** are of the form $\frac{a}{b}$ where a and b are integers with b not equal to 0. **Integers** are also considered rational values.

Example 1:

- a. Evaluate $\sqrt{9}$ without a calculator. The index is 2 and we often call it a square root!

We know that $3 \cdot 3 = 9$. Therefore, $\sqrt{9} = 3$. Incidentally, $-3 \cdot -3$ also equals 9. However, we only use positive numbers when evaluating a radical.

- b. Evaluate $\sqrt[3]{8}$ without a calculator. The index is 3 and we often call it a cube root!

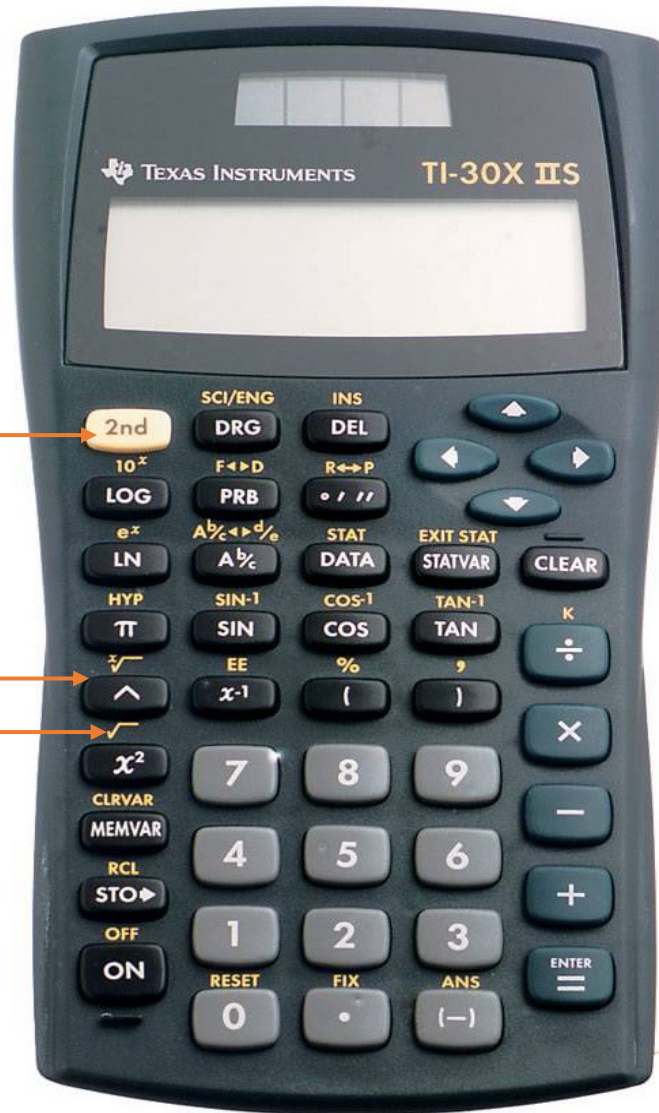
We know that $2 \cdot 2 \cdot 2 = 8$. Therefore, $\sqrt[3]{8} = 2$.

Definition of Radical Expressions (3 of 5)

To invoke the radical functions, press the 2nd button!

\sqrt{x} is used for any index

$\sqrt{\quad}$ is used only for index 2



Example 2:

Evaluate $\sqrt{9}$ with a calculator.

We will use the TI-30X IIS.

- Press the 2nd button and then the x^2 button. You will see $\sqrt{($.
- Type **81** and press the right parentheses **)** button to close the set.
- Press the ENTER button.

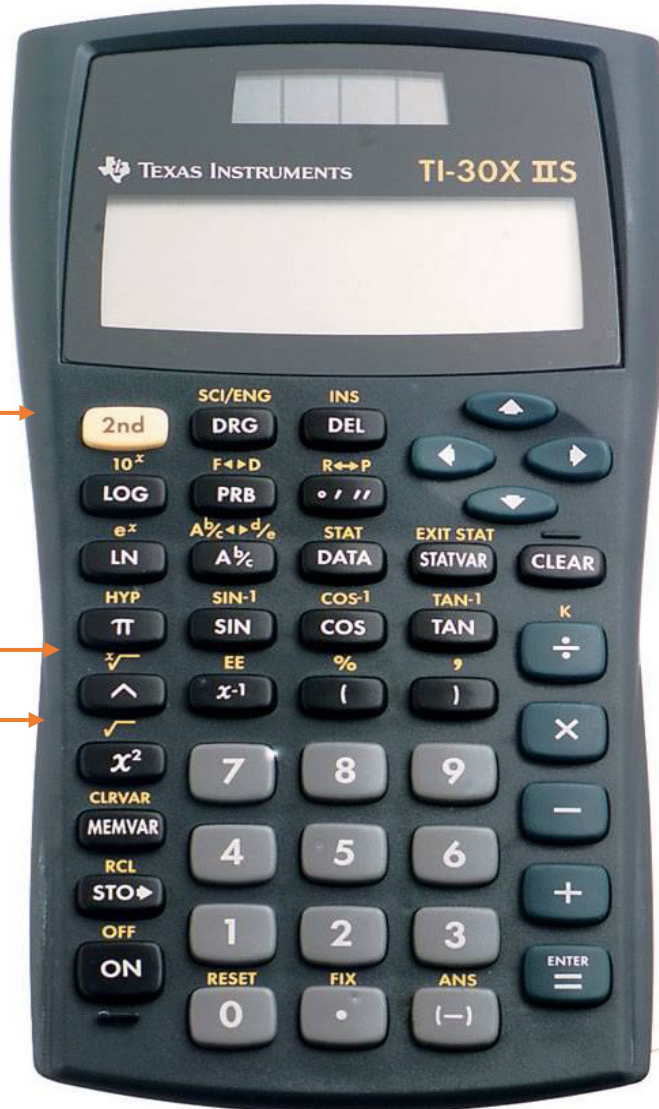
The answer is **3** which is a rational number and more specifically an **integer**.

Definition of Radical Expressions (4 of 5)

To invoke the radical functions, press the 2nd button!

\sqrt{x} is used for any index

$\sqrt{}$ is used only for index 2



Example 3:

Evaluate $\sqrt[3]{8}$ with a calculator.

We will use the TI-30X IIS.

- Type the index **3**.
- Press the 2nd button and then the ^ (caret) button. You will see $3\sqrt{}$.
- Type **8**.
- Press the ENTER button.

The answer is **2** which is a rational number and more specifically an integer.

Definition of Radical Expressions (5 of 5)

- **Irrational values** are those that cannot be converted to the form $\frac{a}{b}$. We will discuss radicals producing irrational values and some examples in the next lesson!
- **Imaginary values** occur when the index is even, and the radicand is negative. We will not discuss imaginary numbers in this course!

For example, in $\sqrt{-9}$ the index is 2 which is even. So, what number times itself equals -9 ? We know $3 \cdot 3 = 9$ and $-3 \cdot -3 = 9$. But there is no number that when multiplied by itself gives us -9 ! Therefore, we say that $\sqrt{-9}$ is imaginary!

2. Definition of Logarithmic Expressions (1 of 4)

Just like radical expressions, logarithmic expressions or simply “logarithms” are also related to exponential expressions. Logarithms were developed by ancient mathematicians who were trying to find unknown exponents.

Please examine the general logarithmic expression carefully!

Logarithmic Expression

The diagram shows the expression $\log_b M$. A horizontal bracket is drawn above the M , with an arrow pointing from the text "The Argument of the Logarithm" to the M . A diagonal arrow points from the text "Logarithm Base (This is a Subscript!)" to the subscript b .

This is expressed as “log base b of M ”.

This logarithmic expression asks us to find the power to which b (base) must be raised to get M (argument)!

Logarithmic expressions can have rational, irrational, or imaginary values.

Definition of Logarithmic Expressions (2 of 4)

The logarithm bases that occur most frequently in applications are **10** and **e**. Please note that **e** is the famous number 2.718281828 ... containing infinitely many decimal places and often rounded to 2.72.

$\log_{10} x$

The 10 is usually left off and we just write **log x**.

$\log_e x$

a logarithm of base **e** is also referred to as the **natural logarithm**. The **e** is usually left off and we write **ln x**.

Please note that the letter "l" in "ln" is a lower case L. It is neither the upper case letter "I" nor the number 1.

Definition of Logarithmic Expressions (3 of 4)

To evaluate most logarithms, we MUST have a calculator. Trying to evaluate logarithms by hand is often extremely cumbersome, but also beyond the scope of this course.

In a calculus course it is shown how to find the values of logarithms by hand and how the calculator is programmed to find values of logarithms.

All calculators have a LOG (base 10) button and an LN (base e) button. Some do have features that allow us to evaluate other bases. However, most of the time we must change other bases to base 10 or base e by using something called the *Change-of-Base Property*.

Definition of Logarithmic Expressions (4 of 4)

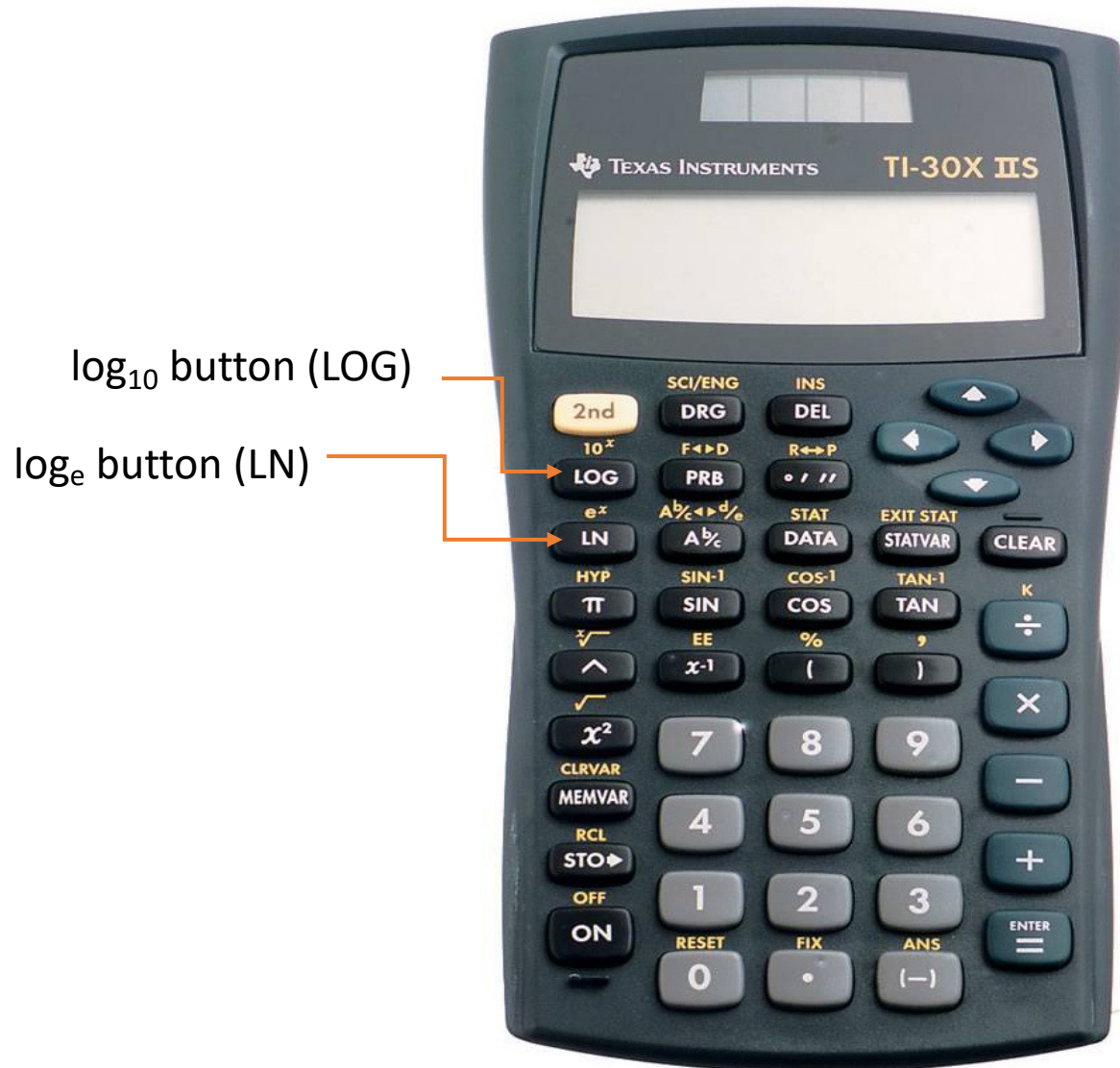
Example 4:

Evaluate $\log_{10} 100$ with a calculator.

We will use the TI-30X IIS.

- Press the LOG button. You will see **log (.**
- Type **100.**
- Press the right parenthesis button **)** to “close” the set.
- Press the ENTER button.

The answer is **2** which is a rational number and more specifically an **integer**.



3. The Change-of-Base Property (1 of 2)

The *Change-of-Base Property* gives us a way to change any logarithm to base 10 or base e so that we can then use the calculator to evaluate it.

It does not matter which one of the following versions of the property we use. Here, we will assume that base b is neither equal to 10 nor to e .

Using base 10

$$\log_b M = \frac{\log_{10} M}{\log_{10} b}$$

Using base e

$$\log_b M = \frac{\log_e M}{\log_e b}$$

The Change-of-Base Property (2 of 2)

Example 5:

Evaluate $\log_7 2506$. Round the answer to two decimal places.

Let's use both versions of the *Change-of-Base Property* to illustrate that it does not matter which one we use. In either case, we must use a calculator.

$$\log_7 2506 = \frac{\log_{10} 2506}{\log_{10} 7} \cong 4.02$$

$$\log_7 2506 = \frac{\log_e 2506}{\log_e 7} \cong 4.02$$