Concepts
Radical and Logarithmic Expressions

Based on power point presentations by Pearson Education, Inc.

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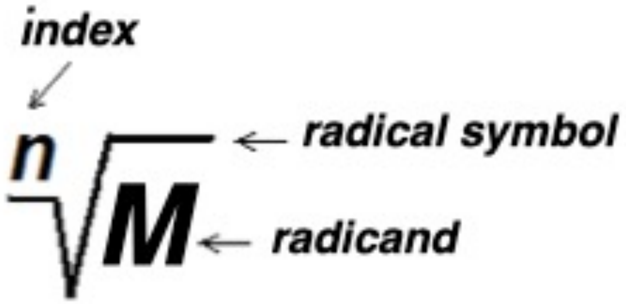
Learning Objectives

1. Evaluate radical expressions.
2. Evaluate logarithmic expressions.
3. Memorize and apply the *Change-of-Base Property* for logarithms.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

1. Evaluate Radical Expressions (1 of 7)

Radical expressions or simply "radicals" are related to exponential expressions in that they reverse the operation of raising a number to a power. The radical symbol $\sqrt{\quad}$ indicates this process. Evaluating a radical expression is also called "finding a root." Please examine the general radical expression carefully!



This expression asks us to find "the *nth* root of *M*" where *M* is a real number or a mathematical expression and *n* is a positive integer.

Some radicals can be evaluated by hand. However, we use calculators most often. They are programmed with the calculus concepts that allow us to approximate the value of all radical expressions.

Evaluate Radical Expressions (2 of 7)

Example 1:

Explain what $\sqrt{16}$ means.

Here we do not see an index. We must automatically know that it is **2**. The 2 is usually not written.

This exponential expression asks us to find "the 2nd root of 16". HOWEVER, since the index 2 appears most often in mathematics and the sciences, we overwhelmingly prefer to state "**find the square root of 16.**"

Evaluate Radical Expressions (3 of 7)

Radical expressions can have rational, irrational, or imaginary values.

- **Rational values** are of the form $\frac{\boldsymbol{a}}{\boldsymbol{b}}$ where ***a*** and ***b*** are integers with ***b*** not equal to 0. Don't forget that integers can also be written in this form!

Example 2:

Evaluate $\sqrt{16}$ ("the square root of 16") without a calculator.

Here we do not see an index. We must automatically know that it is **2**. The 2 is usually not written.

In order to evaluate a radical with index **2**, we must find a number that, when multiplied **two times**, equals 16. We know that $4 \cdot 4 = 16$. Therefore, $\sqrt{16}$ equals 4.

Incidentally, $-4 \cdot -4$ also equals 16. However, we always use positive numbers when evaluating radicals.

Evaluate Radical Expressions (4 of 7)

Example 3:

a. Evaluate $\sqrt[4]{16}$ (“the fourth root of 16”) without a calculator. The index is **4**.

In order to evaluate a radical with index **4**, we must find a number that, when multiplied **four times**, equals 16. We know that (2) · (2) · (2) · (2) = 16. Therefore, $\sqrt[4]{16}$ equals 2.

b. Evaluate $\sqrt[3]{-8}$ (”the third root or cube root of negative 8”) without a calculator.

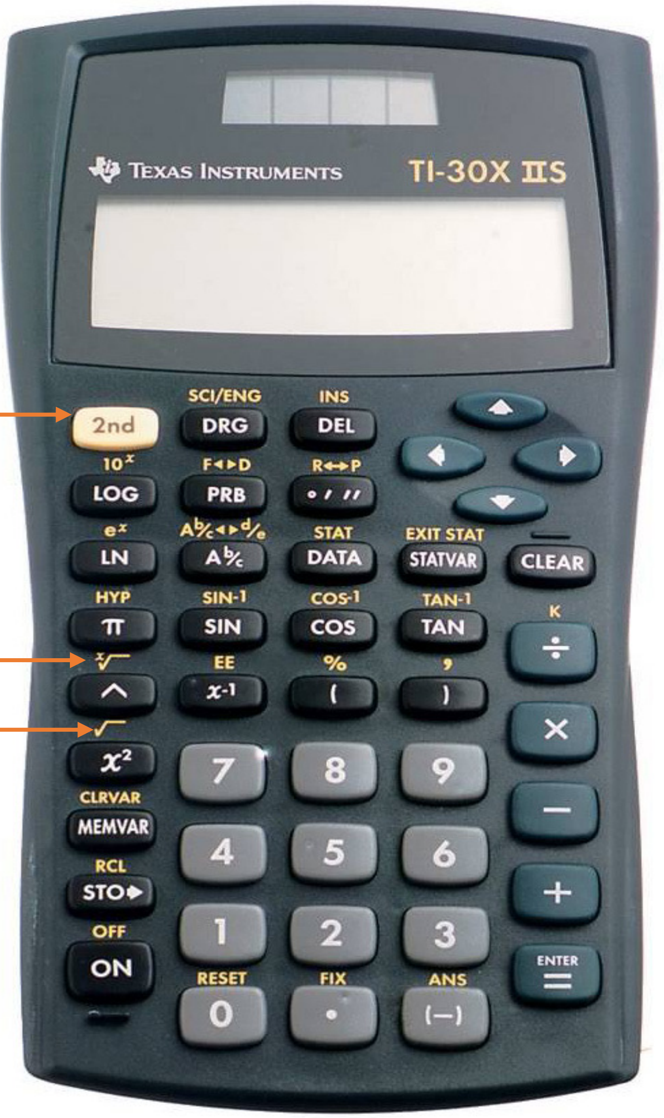
In order to evaluate a radical with index **3**, we must find a number that, when multiplied **three times**, equals – 8. We know that (– 2) · (– 2) · (– 2) = – 8. Therefore, $\sqrt[3]{-8}$ equals – 2.

Evaluate Radical Expressions (5 of 7)

To invoke the radical functions, press the 2nd button!

$\sqrt[x]{}$ is used for any index

$\sqrt{}$ is used only for index 2



Example 4:

Evaluate $\sqrt{9}$ with a calculator.

We will use the TI-30X IIS:

- Press the 2nd button and then the x^2 button. You will see $\sqrt{}($.
- Type **9** and press the right parentheses **)** button to close the parentheses.
- Press the ENTER button.

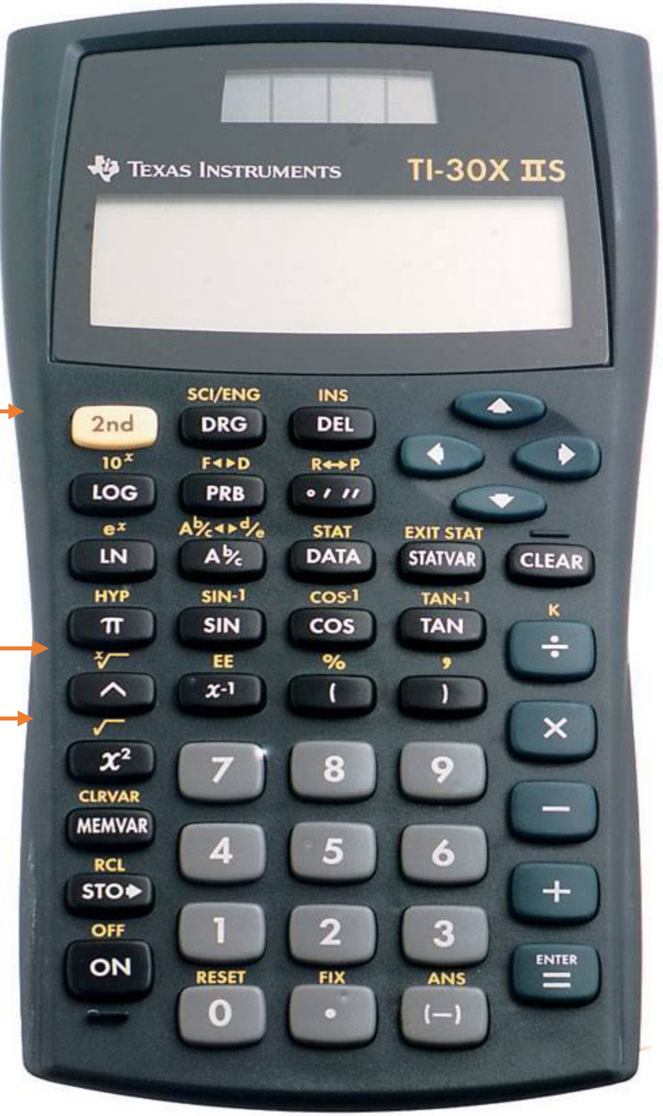
The answer displayed is **3** which is a rational number but more specifically an **integer**.

Evaluate Radical Expressions (6 of 7)

To invoke the radical functions, press the 2nd button!

$\sqrt[x]{\ }$ is used for any index

$\sqrt{\ }$ is used only for index 2



Example 5:

Evaluate $\sqrt[3]{8}$ with a calculator.

We will use the TI-30X IIS:

- Type the index **3**.
- Press the 2nd button and then the ^ (caret) button. You will see $3\sqrt[x]{\ }$.
- Type **8**.
- Press the ENTER button.

The answer displayed is **2** which is a rational number but more specifically an integer.

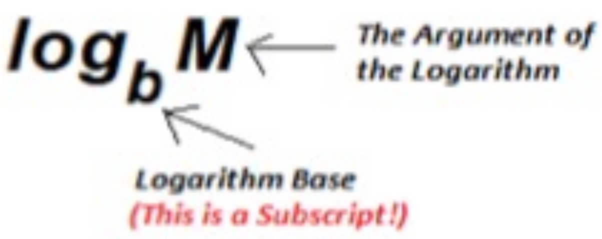
Definition of Radical Expressions (7 of 7)

- **Irrational values** are those that cannot be converted to the form $\frac{a}{b}$. We will discuss radical expressions producing irrational values and some examples in the next lesson!
- **Imaginary values** occur when the index is even, and the radicand is negative. We will not discuss imaginary numbers in this course!

For example, in $\sqrt{-9}$ the index is 2 which is even. So, what number times itself equals – 9? We know 3 · 3 = 9 and – 3· – 3 = 9. But there is no number that when multiplied by itself gives us – 9! Therefore, we say that $\sqrt{-9}$ is imaginary!

2. Evaluate Logarithmic Expressions (1 of 5)

Just like radical expressions, logarithmic expressions or simply "logarithms" are also related to exponential expressions. Logarithms were developed by ancient mathematicians who were trying to find unknown exponents. Please examine the general logarithmic expression carefully!



This is expressed as "log base *b* of *M*" where *M* can be a real number or a mathematical expression and *b* is greater than 0.

This logarithmic expression asks us to find the power to which *b* (base) must be raised to get *M* (argument)!

Logarithmic expressions can have rational, irrational, or imaginary values.

Evaluate Logarithmic Expressions

Example 6:

Explain what $\log_4 16$ means.

This logarithmic expression asks us to find the power to which **4** (base) must be raised to get **16** (argument)!

Can you come up with the answer without not yet knowing much about logarithms? The answer must be **2**, right? Because $4^2 = 16$.

Evaluate Logarithmic Expressions (3 of 5)

The logarithm bases that occur most frequently in applications are **10** and ***e***. Please note that ***e*** is the famous number 2.718281828 …. containing infinitely many decimal places and often rounded to 2.72.

$\log_{10} x$

The 10 is usually left off and we just write **log *x*.**

$\log_e x$

a logarithm of base ***e*** is also referred to as the **natural logarithm**. The ***e*** is usually left off and we write **ln *x***.

Please note that the letter "l" in "ln" is a lower case L. It is neither the upper case letter "i" nor the number 1.

Evaluate Logarithmic Expressions

To evaluate most logarithms, we MUST have a calculator. Trying to evaluate logarithms by hand is often extremely cumbersome, but also beyond the scope of this course.

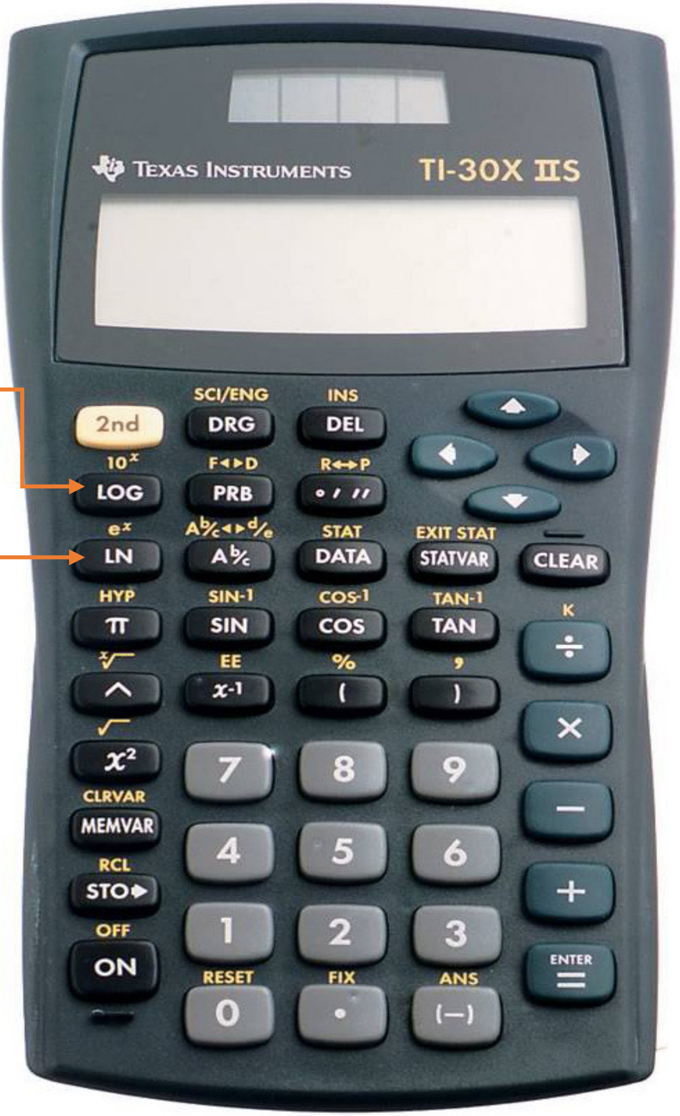
In a calculus course it is shown how to find the values of logarithms by hand and how the calculator is programmed to find values of logarithms.

All calculators have a LOG (base 10) button and an LN (base ***e***) button. Some do have features that allow us to evaluate other bases. However, most of the time we must change other bases to base 10 or base ***e*** by using something called the *Change-of-Base Property*.

Evaluate Logarithmic Expressions (5 of 5)

$\log_{10}$ button (LOG)

$\log_e$ button (LN)



Example 7:

Evaluate $\log_{10}$ **100** with a calculator.

We will use the TI-30X IIS.

- Press the LOG button. You will see **log (**.
- Type **100**.
- Press the right parenthesis button **)** to "close" the set.
- Press the ENTER button.

The answer is **2** which is a rational number and more specifically an **integer**.

3. The Change-of-Base Property (1 of 2)

The *Change-of-Base Property* gives us a way to change any logarithm to base 10 or base ***e*** so that we can then use the calculator to evaluate it.

It does not matter which one of the following versions of the property we use. Here, we will assume that base **b** is neither equal to 10 nor to ***e***.

Using base 10

$$\log_b M = \frac{\log_{10} M}{\log_{10} b}$$

Using base ***e***

$$\log_b M = \frac{\log_e M}{\log_e b}$$

The Change-of-Base Property (2 of 2)

Example 8:

Evaluate log_7 2506. Round the answer to two decimal places.

Let's use both versions of the *Change-of-Base Property* to illustrate that it does not matter which one we use. In either case, we must use a calculator.

$$log_7 2506 = \frac{log_{10} 2506}{log_{10} 7} \cong 4.02$$

$$log_7 2506 = \frac{log_e 2506}{log_e 7} \cong 4.02$$