



Concepts

Operations on Rational Numbers

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Multiply rational numbers.
2. Divide rational numbers.
3. Add and subtract rational numbers with like denominators.
4. Add and subtract rational numbers with unlike denominators.
5. Perform operations on mixed numbers.
6. Change complex fractions to simple form.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

1. Multiply Rational Numbers

Two rational numbers of the form $\frac{a}{b}$ are **multiplied** by finding the product of their numerators and the products of their denominators.

For example,
$$\left(-\frac{2}{3}\right)\left(-\frac{9}{4}\right) = \frac{(-2)(-9)}{(3)(4)} = \frac{18}{12} = \frac{3}{2} .$$

NOTE: We always reduce the result to lowest terms.

2. Divide Rational Numbers

Two rational numbers of the form $\frac{a}{b}$ are **divided** by finding the product of the first number (called *dividend*) and the **reciprocal**** of the second number (called *divisor*).

The **reciprocal**** of a number is a new number in which the numerator and denominator of the given number is exchanged.

For example, $-\frac{3}{5} \div \frac{7}{11} = -\frac{3}{5} \cdot \frac{11}{7} = -\frac{3 \cdot 11}{5 \cdot 7} = -\frac{33}{35}$.

Note: The reciprocal of $\frac{7}{11}$ is $\frac{11}{7}$.

3. Add/Subtract Rational Numbers with Like Denominators

The sum or difference of two or more rational numbers with **like or common denominators** is the sum or difference of their numerators over the like denominator.

For example, $\frac{3}{7} - \frac{2}{7} = \frac{3-2}{7} = \frac{1}{7}$

or $\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7}$.

4. Add/Subtract Rational Numbers with Unlike Denominators

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The sum or difference of two or more rational numbers with **unlike denominators** is more cumbersome to calculate.

We must first change all rational numbers in an addition or subtraction so that they have the same denominators. That is, we will create special equivalent fractions.

For example, we can write $\frac{3}{4} + \frac{1}{6}$ as $\frac{9}{12} + \frac{2}{12}$.

NOTE: We created equivalent fractions! The first fraction in the sum was multiplied by $\frac{3}{3}$ and the second fraction by $\frac{2}{2}$.

Add/Subtract Rational Numbers with Unlike Denominators

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Strategy for Adding/Subtracting Rational Numbers with Unlike Denominators:

Step 1: Find a number divisible by all denominators. Although it makes no difference in the end, we usually prefer to find the smallest such number because the subsequent calculations may not be as cumbersome.

Example 1:

Find the sum of $\frac{3}{4} + \frac{1}{6}$.

Let's find a number divisible by all denominators. Please understand that we can always find such a number by calculating the product of all denominators. In our case, that would be $4(6) = 24$.

However, we might notice that **12** is also a number divisible by 4 and 6. It certainly is smaller than 24 and therefore easier to work with. Actually, it is the smallest number divisible by all denominators.

Add/Subtract Rational Numbers with Unlike Denominators

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Step 2: Change all denominators to the number found in Step 1. Then carry out the addition or subtraction. If necessary, simplify any resulting fractions.

Example 1 continued:

Given $\frac{3}{4} + \frac{1}{6}$, let's change all denominators to **12** by multiplying the existing denominators by an appropriate number. Of course, we have to multiply the numerators by this number as well.

$$\frac{3}{4} + \frac{1}{6} = \frac{3 \cdot 3}{4 \cdot 3} + \frac{1 \cdot 2}{6 \cdot 2}$$

NOTE: We created equivalent fractions! The first fraction in the sum was multiplied by $\frac{3}{3}$ and the second fraction by $\frac{2}{2}$.

Add/Subtract Rational Numbers with Unlike Denominators

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After multiplication, we end up with $\frac{9}{12} + \frac{2}{12}$.

All that's left to do now is add the numerators and put this sum over the denominator to get $\frac{11}{12}$ which is reduced to lowest terms.

5. Operations on Mixed Numbers

To add, subtract, multiply, and divide mixed numbers, you must first change them to improper fractions. Then you can proceed as per the rules above.

For example, $5\frac{1}{4} - 2\frac{3}{4} = \frac{21}{4} - \frac{11}{4}$ or $5\frac{1}{4} \left(2\frac{3}{4}\right) = \frac{21}{4} \left(\frac{11}{4}\right)$

6. Complex Fractions

Fractions with numerators and/or denominators that are not integers are called complex fractions.

For example, $\frac{\frac{2}{7}}{\frac{1}{14}}$.

In mathematics, we are required to change these complex fractions so that only integers are in their numerators and denominators. We use the fact that a fraction bar indicates division!

For example, $\frac{\frac{2}{7}}{\frac{1}{14}} = \frac{2}{7} \div \frac{1}{14} = \frac{2}{7} \cdot \frac{14}{1}$.