Concepts
Irrational and Real Numbers

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Define irrational numbers.
2. Evaluate irrational numbers derived from radicals.
3. Evaluate irrational numbers derived from logarithms.
4. Evaluate irrational numbers derived from nature.
5. Define real numbers and recognize their subsets.

1. Definition of Irrational Numbers

So far, we discussed integers and rational numbers. **Integers** are positive and negative natural numbers and the number 0. **Rational numbers** are of the form $\frac{a}{b}$ where ***a*** and ***b*** are integers and ***b*** is not equal to 0.

Given this definition, we find that integers can be considered rational numbers. For example, 8 can equal $\frac{8}{1}$ or $\frac{16}{2}$, etc.

Irrational numbers cannot be written in the form $\frac{a}{b}$. They are non-terminating, non-repeating decimal numbers.

Irrational numbers come from many sources, such as radicals and logarithms. They also occur in nature.

2. Irrational Numbers derived from Radicals (1 of 2)

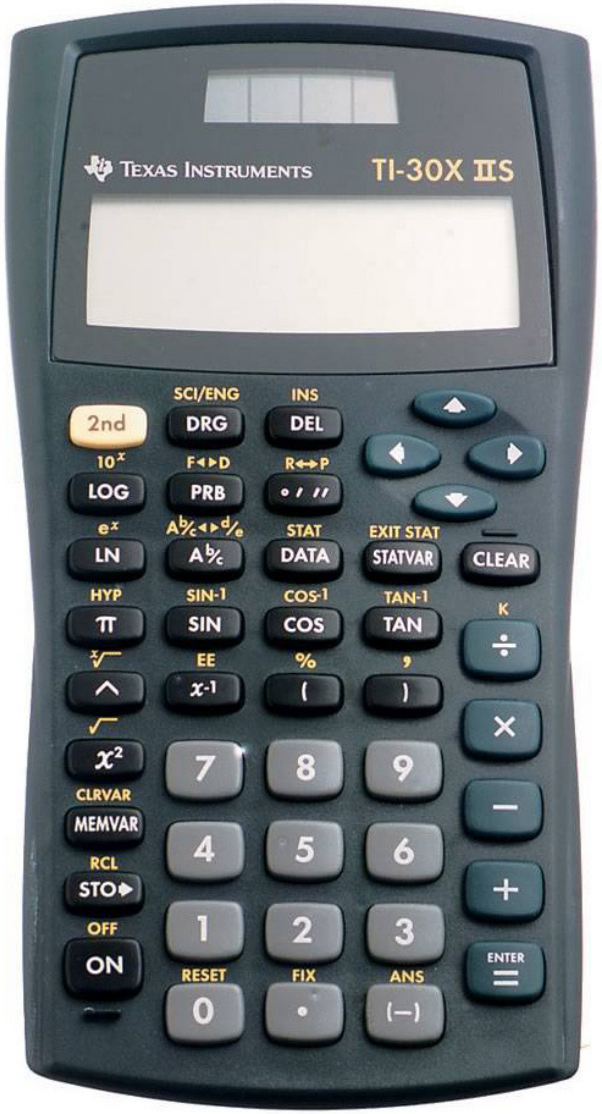
We already learned that certain radicals are rational numbers (or integers).

For example, $\sqrt{81}$, because there is a rational number that produces 81 when multiplied by itself, namely 9.

However, there are infinitely many radicals which are NOT rational numbers (or integers).

For example, $\sqrt{10}$, is an irrational number because there is NO rational number that produces 10 when multiplied by itself.

Irrational Numbers derived from Radicals (2 of 2)



Example 1:

Evaluate $\sqrt{40}$ with a calculator.

We will use the TI-30X IIS.

- Press the 2nd button. Then the x^2 button. We will see $\sqrt{(}$.
- Type **40** and press the right parenthesis **)** button to close the set.
- Press the ENTER button.

The answer is **6.32455532 …** which has infinitely many decimal places. This makes it an irrational number.

NOTE: The calculator does not tell us that we are dealing with an irrational number. It simply fills up all available slots on its screen with decimal places. YOU must know that it is an irrational number with infinitely many decimal places.

3. Irrational Numbers derived from Logarithms (1 of 2)

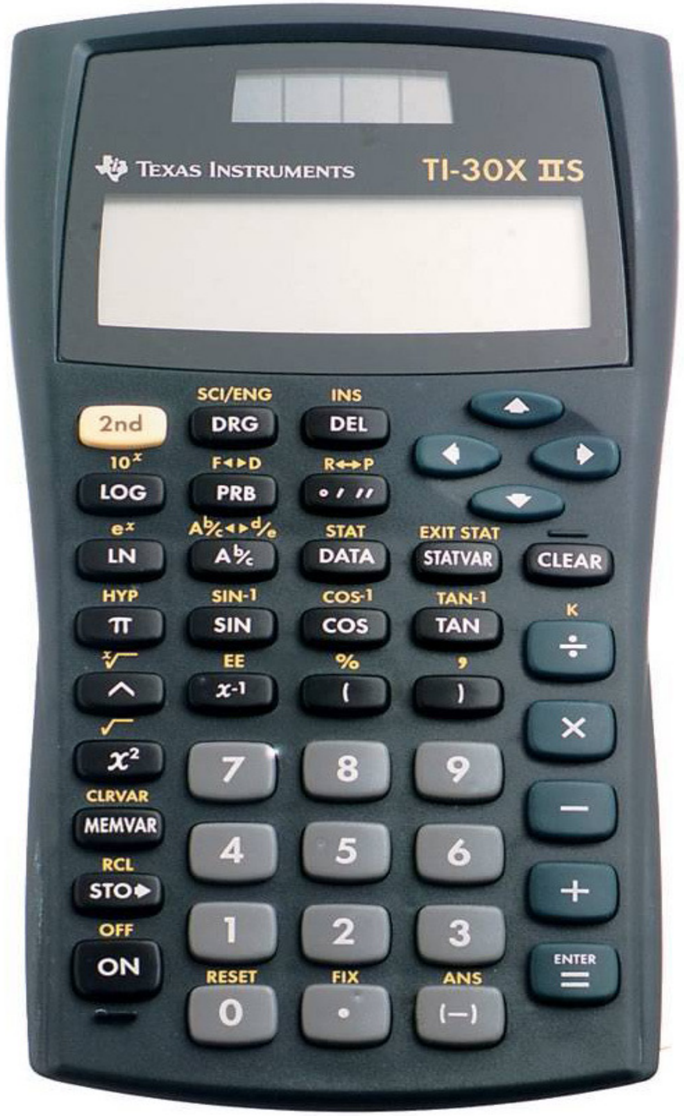
We already learned that certain logarithms are rational numbers (or integers).

For example, $\log_{10} 1000 = 3$.

However, there are infinitely many logarithms which are NOT rational numbers (or integers).

For example, $\log_{10} 99$, is an irrational number because there is NO rational number to which we can raise 10 to produce 99.

Irrational Numbers derived from Logarithms (2 of 2)



Example 2:

Evaluate **log 99** with a calculator.

We will use the TI-30X IIS.

- Press the LOG button because we are dealing with a log base 10. You will see **log (**.
- Type **99**.
- Press the right parenthesis button **)** to “close” the set.
- Press the ENTER button.

The answer is **1.995635195 …** which has infinitely many decimal places. This makes it an irrational number.

4. Irrational Numbers Derived from Nature (1 of 4)

The Number π

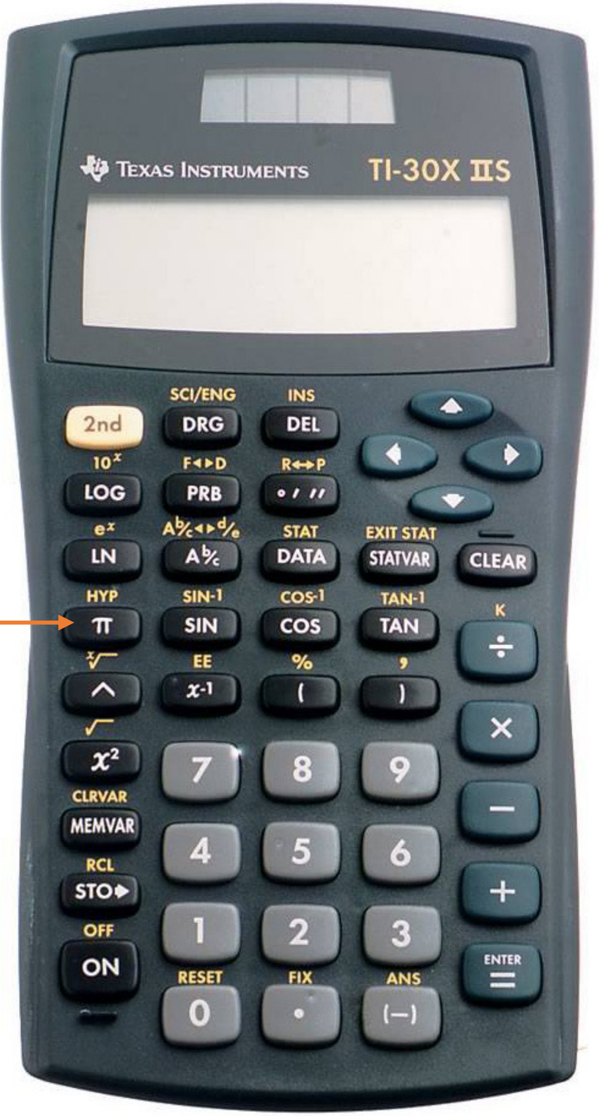
In high school, we learned that the circle is a geometric figure with a special relationship between its distance around the circle (circumference) and its diameter.

That is, if we divide the circumference of any circle by its diameter, the quotient is always the same number. The name of this number is the Greek letter Pi (pronounced like pie) and its symbol is π.

The number π is the irrational number 3.141592654… with infinitely many decimal places. We often state that it is approximately equal to 3.14.

Irrational Numbers Derived from Nature (2 of 4)

π



The calculator has a π button. Press it and then press ***ENTER***.

We find that π = 3.141592654… . Note that the decimal portion is never-ending!

NOTE: The calculator does not tell us that π is an irrational number. It simply fills up all available slots on its screen with decimal places. YOU must know that it is an irrational number with infinitely many decimal places.

Irrational Numbers Derived from Nature (4 of 4)

The Number *e*

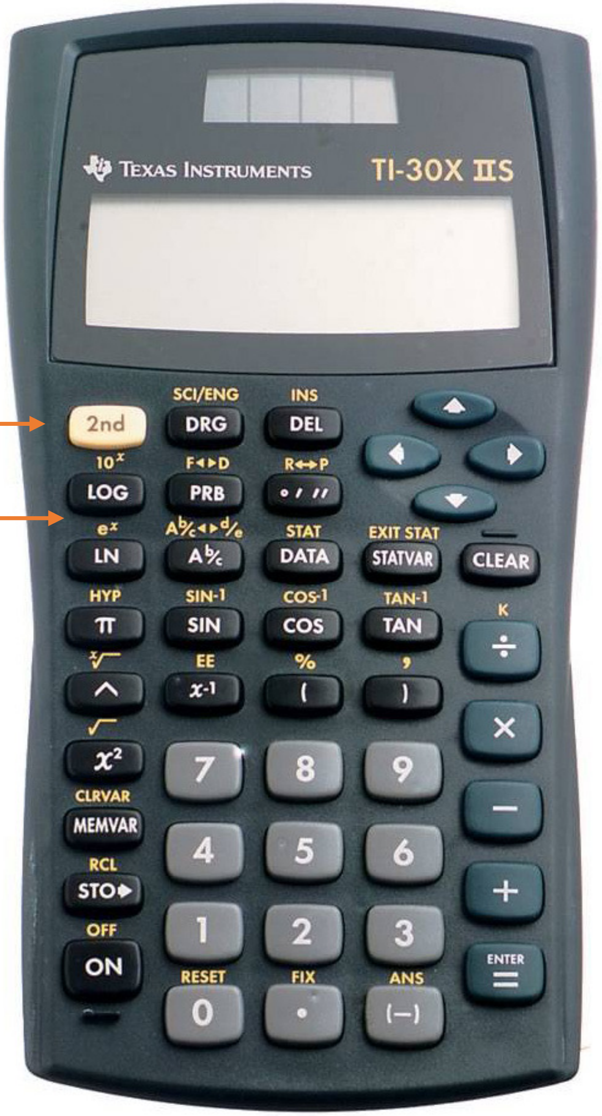
The number ***e*** was discovered by the Swiss mathematician Jacob Bernoulli in 1683 while studying compounded interest. The the first appearance of ***e*** was in the publication "Mechanica" in 1736 by the Swiss mathematician Leonard Euler. The number ***e*** is also known as **Euler's number**.

The number ***e*** is the irrational number 2.718281828 … with infinitely many decimal places. We often state that it is approximately equal to 2.72.

Irrational Numbers Derived from Nature (4 of 4)

To invoke ***e*** press the 2^{nd} button.

$\mathbf{e^x}$ - press the LN button to invoke ***e***.



The calculator does NOT have an ***e*** button. We must press the 2^{nd} button and then the LN button. The calculator window shows **e^ (**.

We input 1, press the right parenthesis button, and then press ***ENTER***.

We find that ***e*** = 2.718281828 … . Note that the decimal portion is never-ending!

NOTE: The calculator does not tell us that ***e*** is an irrational number. It simply fills up all available slots on its screen with decimal places. YOU must know that it is an irrational number with infinitely many decimal places.

5. Definition of the Real Numbers

The **Real Numbers** consist of the following sets of numbers:

- The set of **Natural Numbers:** $\{1, 2, 3, 4, 5, \ldots\}$
- The set of **Whole Numbers:** $\{0, 1, 2, 3, 4, 5, \ldots\}$
- The set of **Integers:** $\{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$
- The set of **Rational Numbers:** numbers that can be written as the quotient of two integers with the denominator not equal to 0.
- The set of **Irrational Numbers:** numbers that cannot be expressed as a quotient of integers.