Concepts
Introduction to Rational Numbers

Based on power point presentations by Pearson Education, Inc.

Revised by Ingrid Stewart, Ph.D.

Learning Objectives

1. Define the rational numbers.
2. Define fractions.
3. Create equivalent fractions.
4. Reduce fractions to lowest terms.
5. Convert improper fractions to mixed numbers.
6. Convert mixed numbers to improper fractions.
7. Express non-terminating/repeating decimal numbers as fractions.
8. Express terminating/non-repeating decimal numbers as fractions.
9. Express fractions as decimal numbers.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

1. Definition of Rational Numbers 1 of 3

We know that the set of *Natural Numbers* consists of the following numbers:

{1, 2, 3, 4, 5, 6, …}

The set of *Integers* is defined as the *natural numbers*, plus the number *0*, plus the *negatives of the natural numbers*.

That is, {…, – 3, – 2, – 1, 0, 1, 2, 3, …}.

Now, we will examine the set of ***Rational Numbers***. It consists of all numbers which can be expressed in the form $\frac{a}{b}$, where *a* and *b* are integers and *b* is not equal to *0*.

Definition of Rational Numbers (2 of 3)

The horizontal line between *a* and *b* indicates division. That is, $\frac{a}{b}$ can be written as $a \div b$.

The integer *a* in $\frac{a}{b}$ is called the ***numerator***.

The integer *b* is called the ***denominator***.

Sometimes a slant line / replaces the horizontal line, and the rational number is written as *a/b*. However, this is a less common way of expressing a rational number.

Definition of Rational Numbers

Please note that integers are considered rational numbers because they can be written as $\frac{a}{b}$ with b = 1.

For example, the number 8 can be written as a rational number as $\frac{8}{1}$.

Also, many decimal numbers can be expressed as rational numbers. We will show later that there is a way to express certain decimal numbers in the form $\frac{a}{b}$.

2. Definition of a Fraction (1 of 2)

In everyday life, we we usually call every number $\frac{a}{b}$ a fraction, whether it is positive or negative. However, that's actually not quite correct.

By definition, only those rational numbers in which ***a*** and ***b*** are *natural numbers {1, 2, 3, 4, …}* should be called fractions.

For example, $\frac{-3}{4}$ is a rational number but not technically a fraction because its numerator is not a natural number.

Definition of a Fraction (2 of 2)

Proper Fractions

Fractions in which the numerator is less than the denominator are called *proper fractions*.

For example, $\frac{3}{4}$.

Improper Fractions

Fractions in which the numerator is greater than the denominator are called *improper fractions*.

For example, $\frac{8}{3}$.

Definition of a Fraction (2 of 2)

Mixed Numbers

Numbers that have a whole part and a fractional part that is a proper fraction are called *mixed numbers*.

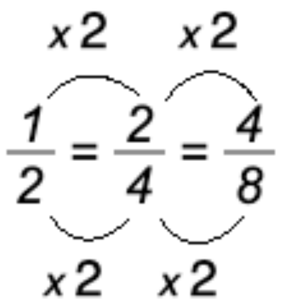
For example, $2\frac{2}{3}$. Please note that $2\frac{2}{3}$ means $2+\frac{2}{3}$. On the other hand, $-6\frac{1}{2}$ means $-6-\frac{1}{2}$.

3. Equivalent Fractions

Equivalent fractions have the same value, even though they may look different.

For example, $\frac{1}{2}$ and $\frac{2}{4}$ and $\frac{4}{8}$ have the same value.

Why? Because when you multiply **both** the numerator and the denominator by the same number, the fractions keeps their value. All three of them are half of a whole.



4. Reduce Fractions to Lowest Terms (1 of 3)

We just discussed equivalent fractions. So, which one should we use in our work?

For example, do we work with $\frac{1}{2}$ or $\frac{2}{4}$ or $\frac{4}{8}$?

In mathematics, we always want to use the fraction whose numerator and denominator do not have any factors in common! That is, we want to work with fractions that are **reduced to lowest terms.**

For example, given $\frac{2}{4}$, the numerator and the denominator have the factor 2 in common while the numerator and denominator in $\frac{4}{8}$ have the factor in common.

Reduce Fractions to Lowest Terms (2 of 3)

Strategy for Reducing Fractions to Lowest Terms:

Step 1 - Use your best guess as to what factors the numerator and the denominator might have in common. You might want to use some divisibility rules such as

- a number is divisible by 2 if the last digit is an even number (0, 2, 4, 6, 8, …)
- a number is divisible by 3 if the sum of its digits is divisible by 3
- a number is divisible by 5 if the last digit is 0 or 5
- a number is divisible by 9 if the sum of its digits is divisible by 9

Example 1:

Reduce $\frac{18}{270}$ to lowest terms.

We see that both 18 and 270 are divisible by 2 since they are even numbers. We also find that the sums of the digits in 18 and 270 are 9 and therefore the numbers are divisible by 9. This means that both numbers are divisible by $2 \cdot 9 = 18$.

Reduce Fractions to Lowest Terms (3 of 3)

Step 2 – Divide the numerator and the denominator by the number found in Step 1. Then reevaluate the reduced fraction.

- If the numerator and denominator in the reduced fraction only have a factor of 1 in common, the fraction is reduced to lowest terms.
- If the the numerator and denominator in the reduced fraction still have factors other than 1 in common, repeat Step 1 and 2.

Example 1 continued:

Given $\frac{18}{270}$, we divide the numerator and the denominator by 18 to get $\frac{1}{15}$. Is this fraction reduced to lowest terms? YES!

It should be obvious that the only factor the numerator and denominator have in common is 1.

5. Convert Improper Fractions to Mixed Numbers (1 of 2)

We can convert improper fractions to mixed numbers as follows:

Step 1 - Divide the denominator into the numerator to find the quotient and the remainder. Please note, the quotient will be an integer! If necessary, use a calculator to do this.

Example 2:

Convert $\frac{13}{3}$ to a mixed number. Use a calculator as necessary!

We divide 13 by 3. We get a quotient of 4 and a remainder of 1. Note, do the calculation 13 – 4(3)!

5. Convert Improper Fractions to Mixed Numbers (2 of 2)

Step 2 - Place the remainder over the given denominator and place it to the right of the quotient.

Example 2 continued:

We write the remainder 1 over the original denominator of 3 and place this fraction to the right of the quotient 4.

We find that $\frac{13}{3}$ is equal to the mixed number $4\frac{1}{3}$.

6. Convert Mixed Numbers to Improper Fractions (1 of 2)

We can convert mixed numbers to improper fractions as follows:

Step 1 - Multiply the integer part by denominator of the fractional part.

Example 3:

Convert $4\frac{1}{3}$ to an improper fraction.

We multiply the integer of 4 by the denominator of the fractional part which is 3.

We get $4 \cdot 3 = 12$.

Convert Mixed Numbers to Improper Fractions (2 of 2)

Step 2 - Add the numerator of the fractional part to the product in Step 1.

Example 3 continued:

We get 12 + 1 = 13.

Step 3 - Place the sum from Step 2 over the given denominator.

Example 3 continued:

We get $\frac{13}{3}$.

7. Express Non-Terminating/Repeating Decimal Numbers as Fractions (1 of 3)

Examples of non-terminating, repeating decimals are

0.6666666666 … or ***0.818181818181818181 …*** . Both can be written as follows:

$0.\overline{6}$ and $0.\overline{81}$ with horizontal bars over the number or numbers that repeat indefinitely.

Normally, we would have to go through a lengthy (and boring) process to find its equivalent fraction. We are not going to do this here. Instead, we will be discussing a shortcut method.

Express Non-Terminating/Repeating Decimal Numbers as Fractions (2 of 3)

Shortcut to change a non-terminating/repeating decimal number to a fraction:

1. The denominator of the fraction will have as many 9's as the number of repeating digits. The numerator will be the repeating number without the decimal or the bar.
2. Reduce the fraction to lowest terms, if necessary.

Express Non-Terminating/Repeating Decimal Numbers as Fractions (3 of 3)

Example 4:

Change $0.\overline{81}$ to a fraction.

There are two repeating digits. Therefore, the denominator of the equivalent fraction will consist of two nines, or 99.

That is, $0.\overline{81} = \frac{81}{99}$ with the numerator the repeating number without the decimal or the bar.

All that's left to do is reduce the fraction to lowest terms.

$$\frac{81}{99} = \frac{9}{11}$$

8. Express Terminating/Non-Repeating Decimal Numbers as Fractions (1 of 2)

Examples of terminating, non-repeating decimals are ***0.6***, ***0.81***, ***0.356***, etc.

Following is a strategy for changing terminating/non-repeating decimal numbers into fractions:

Following is a strategy for changing terminating/non-repeating decimal numbers into fractions:

1. Write down the decimal divided by 1.
2. Multiply the both numerator and denominator by 10 multiplied by itself a “certain number” of times. The ”certain number” depends on the number of decimal places!
3. Reduce the fraction to lowest terms, if necessary

Express Terminating/Non-Repeating Decimal Numbers as Fractions (2 of 2)

Example 5:

Change 0.06 to a fraction.

First, we write $\frac{0.06}{1}$.

Second, we notice that in ***0.06*** there are two (2) numbers after the decimal place. Therefore, we multiply both the numerator and denominator by 10 multiplied by itself two (2) times, that is, $10 \cdot 10 = 100$

$$\frac{0.06}{1} \cdot \frac{100}{100} = \frac{6}{100}$$

Finally, we reduce the fraction to lowest terms.

$$\frac{6}{100} = \frac{3}{50}$$

9. Express Fractions as Decimal Numbers

Any fraction can be expressed as a decimal number by dividing the denominator into the numerator.

The easiest way to change a fraction into a decimal number is by using a calculator. You simply divide the numerator by the denominator.

For example, given $\frac{3}{4}$, we use the calculator to divide 3 by 4 to get 0.75.