



# Concepts The Integers Simple Order of Operations

Based on power point presentations by Pearson Education, Inc.  
Revised by Ingrid Stewart, Ph.D.

# Learning Objectives

1. Define the integers.
2. Find the absolute value of an integer.
3. Add integers.
4. Subtract integers.
5. Multiply integers.
6. Divide integers.
7. Use the calculator to perform operations on integers.
8. Use the Simple Order of Operations.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

# 1. Definition of Integers (1 of 2)

We learned that the ***Natural Numbers*** consists of the following set of numbers:  
 $\{1, 2, 3, 4, 5, 6, \dots\}$ .

The set of ***Integers*** is defined as the *Natural Numbers*, plus the number 0, plus the *negatives of the natural numbers*. We use the minus symbol – to illustrate negativity.

That is, the Integers consist of the following set of numbers:  
 $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ .

We call the negatives of the natural numbers “negative integers” and the other ones “positive integers”.

# Definition of Integers (2 of 2)

Negative integers are smaller than positive integers. The further to the left we go, the smaller the numbers. For example,  $-3$  is less than  $-2$ ,  $-1$ ,  $0$ ,  $1$  etc.

The positive integers can be written in two ways:

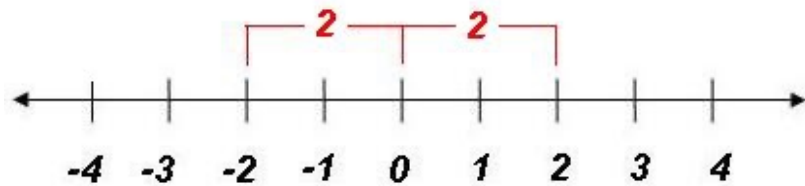
1. Do not write the sign. For example,  $3$  means “positive three”. This is the way we commonly see it in mathematics.
2. Use a “+” sign. For example,  $+3$  is also “positive three”. The  $+$  sign is usually not written.

The negative integers obviously need to show the sign  $-$ . For example,  $-3$ . We say, “negative three”. We DO NOT say “minus three.”

## 2. Definition of Absolute Value

The **absolute value** is defined to be a distance from the point 0 to some other point. The point can be positive or negative. Absolute values have many uses. In this lesson, we will illustrate how to add integers using absolute value.

For example, let's look at a distance of 2 units starting at the point 0.



We can see that the distance between 0 and 2 is **2**. BUT the distance between 0 and  $-2$  is also **2**. We can express this as follows:

$|2| = 2$  means that the distance between 0 and 2 is 2. It is pronounced “the absolute value of 2 equals 2.”

$|-2| = 2$  means that the distance between 0 and  $-2$  is 2. It is pronounced “the absolute value of  $-2$  equals 2.”

### 3. Integer Addition (1 if 4)

The positive and negative sign in front of a number is called a **directional sign!**

When we add + (or subtract –) numbers with directional signs, we call the addition sign + and the subtraction sign – **operational signs.**

“Official” Rules for Adding Integers:

1. To add two numbers with the same directional sign, add their absolute values. The sum has the same direction sign as the addends.
2. To add two numbers with different directional signs, find their absolute value and subtract the lesser absolute value from the greater. The sum has the same directional sign as the addend with the greater absolute value.

This is a difficult rule to remember! Therefore, we usually don't use it. Instead, we use the idea of gambling gains (+) and gambling losses (–).

# Integer Addition (2 of 4)

Let's now add integers using gambling gains (+) and gambling losses (-).

1. Adding a gambling loss to another gambling loss is still a gambling loss. Therefore,  $(-) + (-) = (-)$ .

For example,  $-3 + (-2) = -5$

**Note we separate the operational sign and the directional sign with parentheses.**

2. Adding a gambling gain to another gambling gain is still a gambling gain. Therefore,  $(+) + (+) = (+)$ .

For example,  $+6 + (+4) = +10$ .

**Note we usually do not write positive directional signs. That is, we usually just state  $6 + 4 = 10$ .**



# Integer Addition (3 of 4)

3. Adding a gambling gain to a gambling loss that is larger than the gain, we still have a gambling loss. Therefore, (a small +) + (a large -) = (-).

For example,  $-7 + (+1) = -6$ .

4. Adding a gambling gain to a gambling loss that is smaller than the gain, we still have a gambling gain. Therefore, (a large +) + (a small -) = (+).

For example,  $-3 + (+10) = +7$  or simply 7.

# Integer Addition (4 of 4)

Notice, on the previous slides, we wrote operational signs (add, subtract, multiply, divide) and directional signs (positive, negative number) next to each other. This can be confusing.

In general, we omit one of the signs. Specifically, when two signs are written next to each other, we can simplify this as follows:

$+ (+)$ becomes $+$	$+ (-)$ becomes $-$
$- (-)$ becomes $+$	$- (+)$ becomes $-$

For example,  $-2 + (-9) = -2 - 9$ . Then we say, “negative 2 **plus** a negative 9”. The “plus” is inserted into the wording in place of the implied operational sign  $+$ .

## 4. Integer Subtraction

Rules for Subtracting Integers:

1. Change the **operational sign** from minus to plus and at the same time change the **directional sign** of the subtrahend (the second number) to the opposite sign.
2. Apply the appropriate rule for adding integers.

For example,  $-3 - (+7)$  will be written as  $-3 + (-7)$  and ultimately as  $-3 - 7$  which is “negative 3 **plus** a negative 7” and equals  $-10$  (we have a gambling loss and to it we add another gambling loss).

## 5. Integer Multiplication (1 of 2)

Rules for Multiplying Integers:

1. If the integers have the same directional signs, their product is positive.

$$(-)(-) = (+)$$

$$(+)(+) = (+)$$

For example,  $-7(-2) = +14$  or simply 14. Furthermore,  $+7(+2) = +14$  or simply 14. Note we usually do not write positive directional signs. That is, we usually just state  $7(2) = 14$ .

# Integer Multiplication (2 of 2)

2. If the integers have different directional signs, their product is negative.

$$(-)(+) = (-)$$

$$(+)(-) = (-)$$

For example,  $-7(+2) = -14$  or  $+7(-2) = -14$ .

## 6. Integer Division (1 of 2)

Rules for Dividing Integers:

1. If two integers have the same directional signs, their quotient is positive.

$$- \div - = +$$

$$+ \div + = +$$

For example,  $-12 \div (-3) = +4$  or simply 4. Furthermore,  $+12 \div (+3) = +4$  or simply 4. In both cases, we usually express the quotient as 4 without the + sign.

## Integer Division (2 of 2)

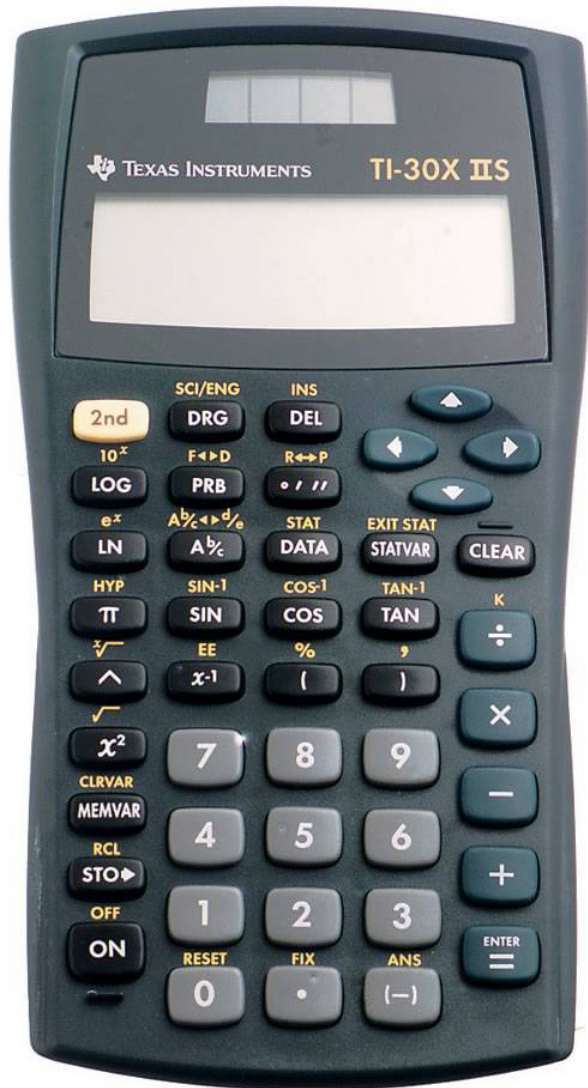
2. If two integers having different directional signs, their quotient is negative.

$$+ \div - = -$$

$$- \div + = -$$

For example,  $-12 \div (+3) = -4$  or  $+12 \div (-3) = -4$ .

## 7. Use the Calculator to Perform Operations on Integers



The calculator has a **minus** button  $-$ , which carries out the operation of subtraction. You can find it in the column of dark gray buttons (on the very right) on your calculator.

The calculator also has a **negative** button  $(-)$  to input negative numbers. You can find it in the last column of the light gray buttons.



### 3. Simple Order of Operations (1 of 2)

Say, you are asked to compute  $48 - 20 \div 2 \cdot 5 + 9$ . Would you start by subtracting 20 from 48 and then divide this result by 2 and so on? Actually, that's not what you would do.

In mathematics, we have an **Order of Operations**. It consists of rules that state the sequence in which multiple operations should be solved in a mathematical expression.

In this lesson, we will discuss the order given addition, subtraction, multiplication, and division.

1. Multiplication and Division are done first, specifically in the order in which they occur, working from left to right.
2. Addition and Subtraction are done next, again in the order in which they occur, working from left to right.

# Simple Order of Operations (2 of 2)

Example:

Evaluate  $48 - 20 \div 2 \cdot 5 + 9$ .

There is addition, subtraction, multiplication, and division. Therefore, we begin by evaluating the division. Remember, multiplication and division are done before addition and subtraction working from left to right.

$$48 - 20 \div 2 \cdot 5 + 9$$

$$48 - 10 \cdot 5 + 9 \quad (\text{evaluated the division})$$

$$48 - 50 + 9 \quad (\text{evaluated the multiplication})$$

$$-2 + 9 \quad (\text{evaluated the subtraction})$$

$$7 \quad (\text{evaluated the addition})$$