Concepts
Linear Inequalities in One Variable

Based on power point presentations by Pearson Education, Inc.

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Learning Objectives

1. Recognize inequality signs and interpret their meaning.
2. Solve simple linear inequalities in one variable.
3. Solve linear compound inequalities in one variable.

NOTE: This lesson contains some examples. You can find more examples in the "Examples" document also located in the appropriate MOM Learning Materials folder.

1. Introduction to Inequalities (1 of 6)

The following four symbols

$$< \qquad \leq \qquad > \qquad \geq$$

are called *inequality symbols* or *inequality signs.*

$<$ is expressed as “less than”

$\leq$ is expressed as “less than or equal to”

$>$ is expressed as “greater than”

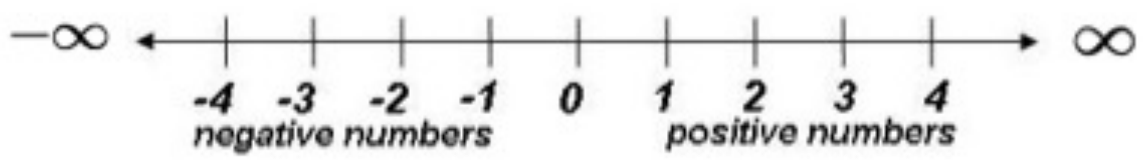
$\geq$ is expressed as “greater than or equal to”

The inequality signs have many uses. For example, they can be used to compare the ”size” of two numbers or they can be used to express a set of numbers.

Solve Simple Linear Inequalities in One Variable (2 of 6)

Before we go on, let's discuss the concept of number line since we will use it extensively in subsequent discussions.

In mathematics, we often like to see a visual representation of sets of numbers. We do this with the help of a **number line**. We draw a straight line with arrows at both ends. The integers are often shown as specially marked points evenly space on the line. In between the integers lie infinitely many other numbers.



We place the symbol ∞ on the right end of the line and $-\infty$ on the left end. The symbols indicate positive and negative infinity, respectively. Please note that we usually don't write + to express positive infinity.

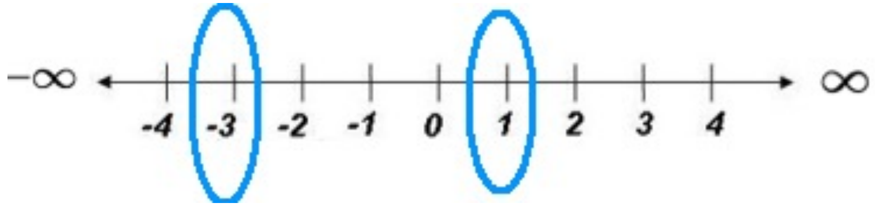
Introduction to Inequalities (3 of 6)

Comparing the "size" of two numbers:

Example 1:

Read the following statement: $-3 < 1$

We read the mathematical statement from left to right. That is, -3 is less than 1.



Read the following statement: $-1 > -4$

Again, we read the mathematical statement from left to right. That is, -1 is greater than -4.

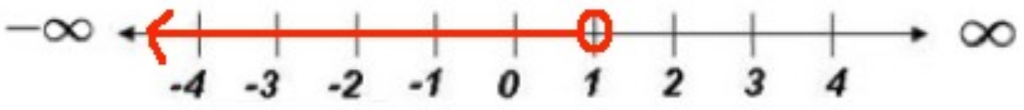
Introduction to Inequalities (4 of 6)

Express a set of numbers:

Example 2:

Explain the statement $x < 1$.

This statement expresses a set of numbers where x represents ALL numbers that are **less than** 1. This set DOES NOT include 1!



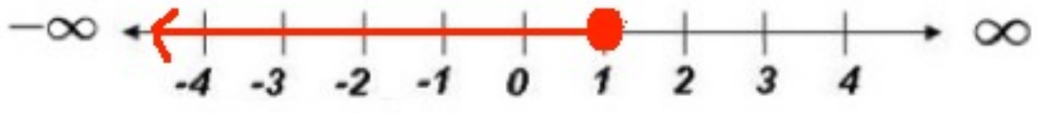
The red line indicates all the numbers included. The arrow indicates that there are infinitely many numbers beyond 1 in the set. There is a CIRCLE at 1 which indicates that the number IS NOT included in the set!

Introduction to Inequalities (5 of 6)

Example 3:

Explain the statement $x \leq 1$.

This statement expresses a set of numbers where x represents ALL numbers that are **less than or equal to** 1. This set includes 1!



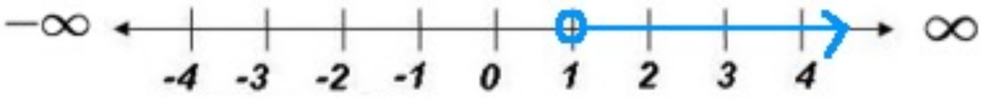
The red line indicates all the numbers included. The arrow indicates that there are infinitely many numbers beyond 1 in the set. There is a DOT at 1 which indicates that the number IS included in the set!

Introduction to Inequalities (6 of 6)

Example 4:

a. Explain the statement $x > 1$.

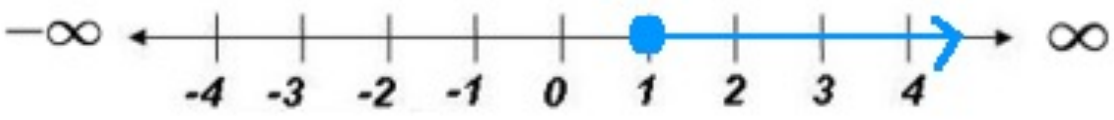
This statement expresses a set of numbers where x represents ALL numbers that **are greater than** 1. This set DOES NOT include 1!



The circle indicates that 1 is NOT included!

b. Explain the statement $x \geq 1$.

This statement expresses a set of numbers where x represents ALL numbers that are **greater than or equal to** 1. This set includes 1!



The dot indicates that 1 is included!

2. Solve Simple Linear Inequalities in One Variable

Linear inequalities in one variable look much like linear equalities. They differ from linear equalities because they contain an inequality sign and not an equal sign.

For example, $3 - 5x = 13$ is a linear equation (not in general form). Below are examples of four different linear inequalities:

$3 - 5x \geq 13$ $\quad$ $3 - 5x > 13$ $\quad$ $3 - 5x \leq 13$ $\quad$ $3 - 5x < 13$

We solve linear inequalities almost exactly like linear equalities, except for one very important difference:

> When both sides of an inequality are multiplied or divided by a negative number, the inequality symbol changes direction.

Solve Simple Linear Inequalities in One Variable (2 of 4)

Linear equalities have one solution. For example, $x = -2$.

On the other hand, linear inequalities have infinitely many solutions. We call this a **solution set**, and it includes one of the inequality signs.

For example, $x \geq -2$ (all numbers greater than or equal to – 2 with – 2 included)

$x > -2$ (all numbers greater than – 2 with – 2 NOT included)

$x \leq -2$ (all numbers less than or equal to – 2 with – 2 included)

$x < -2$ (all numbers less than – 2 with – 2 NOT included)

Solve Simple Linear Inequalities in One Variable (3 of 4)

Example 5:

Solve the linear inequality $3 - 5x \geq 13$, then represent the solution set graphically.

$3 - 5x \geq 13$

$-5x \geq 10$ Subtracted 3 from both sides!

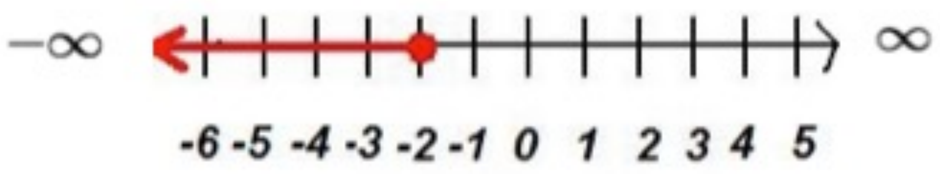
$x \leq -2$ We divided both sides of the inequality by -5. Note that the inequality sign changed direction!

$x \leq -2$ is the **solution set**! It contains all numbers less than or equal to -2 with -2 included.

Solve Simple Linear Inequalities in One Variable (4 of 4)

Example 5 continued:

Following is its graphical representation of the solution set $x \leq -2$ on a number line:



The red line indicates all the numbers included in the solution set. The arrow indicates that there are infinitely many numbers beyond – 6. There is a DOT at – 2 which indicates that the number IS INCLUDED in the solution set!

NOTE: A CIRCLE at – 2 would indicate that the number IS NOT INCLUDED in the solution set.

3. Solve Linear Compound Inequalities (1 of 3)

Compound inequalities contain two inequality signs with the variable in between them. They have infinitely many solutions.

For example, $-1 < x + 4 < 1$ is a linear compound inequality. We express this as "$x + 4$ is greater than -1 **AND** $x + 4$ is less than 1".

When solving for the variable, we isolate the variable by applying the addition, subtraction, multiplication, and division axioms as needed to the right and left side of the inequality, as well as to the middle.

Solve Compound Inequalities (2 of 3)

Example 6:

Solve the linear compound inequality $-3 < 2x + 1 \leq 3$, then represent the solution set graphically.

Our goal is to isolate the variable *x* in the middle.

$-3 < 2x + 1 \leq 3$

$-4 < 2x \leq 2$ We subtracted 1 from all three parts of the inequality.

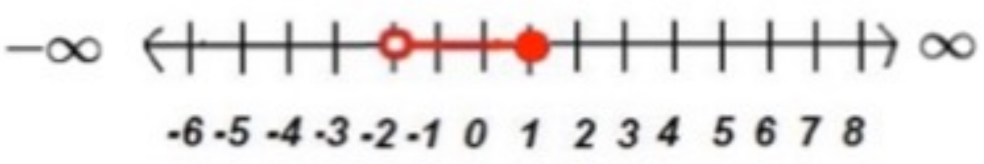
$-2 < x \leq 1$ We divided all three parts of the inequality by 2.

$-2 < x \leq 1$ (*x* is greater than -2 **AND** *x* is less than or equal to 1) is the **solution set**! It contains all numbers between -2 and 1 with -2 NOT included but 1 included.

Solve Compound Inequalities (3 of 3)

Example 6 continued:

Following is its graphical representation of the solution set $-2 < x \leq 1$ on a number line:



The red line indicates all numbers included in the solution set. There is a CIRCLE at – 2 and a DOT at 1 to indicate that – 2 is NOT included but 1 is included in the solution set!