

Concepts Exponents Advanced Order of Operations

Based on power point presentations by Pearson Education, Inc.
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Learning Objectives

1. Define and evaluate exponential expressions.
2. Use exponent rules.
3. Use the Order of Operations given grouping symbols, exponents, multiplication, division, addition, and subtraction.

NOTE: This lesson contains some examples. You can find more examples in the “Examples” document also located in the appropriate MOM Learning Materials folder.

1. Definition of Exponential Expressions (1 of 4)

Exponential expressions are of the form b^x , where b is called a **base** and x **exponent** or **power**. The exponent states how many times to multiply base b by itself.

For example, given 5^2 , the base is **5** and the exponent (or power) is **2**.

The exponent states to use **5** two times in a multiplication, so that 5^2 equals $5 \cdot 5 = 25$.

NOTE: 5^2 is read as “five to the second power” or “five raised to the second power” or “five squared”.

Definition of Exponential Expressions (2 of 4)

Example 1:

Evaluate the following exponential expressions by hand:

a. 4^3

The exponent states to use **4** three times in a multiplication, so that 4^3 equals $4 \cdot 4 \cdot 4 = 64$.

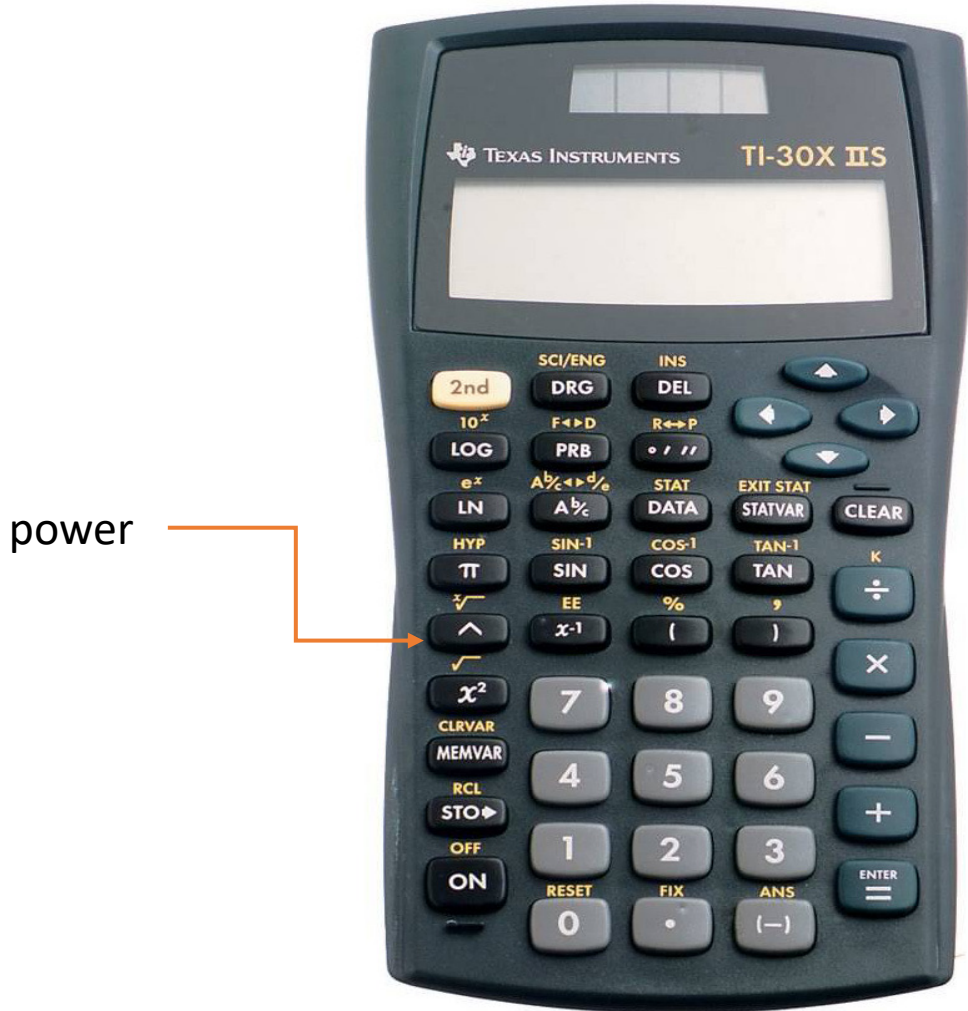
NOTE: 4^3 is read as “four to the third power” or “four raised to the third power” or “four cubed”.

b. 2^4

The exponent states to use **2** four times in a multiplication, so that 2^4 equals $2 \cdot 2 \cdot 2 \cdot 2 = 16$.

NOTE: 2^4 is read as “two to the fourth power” or “two raised to the fourth power”.

Definition of Exponential Expressions (3 of 4)



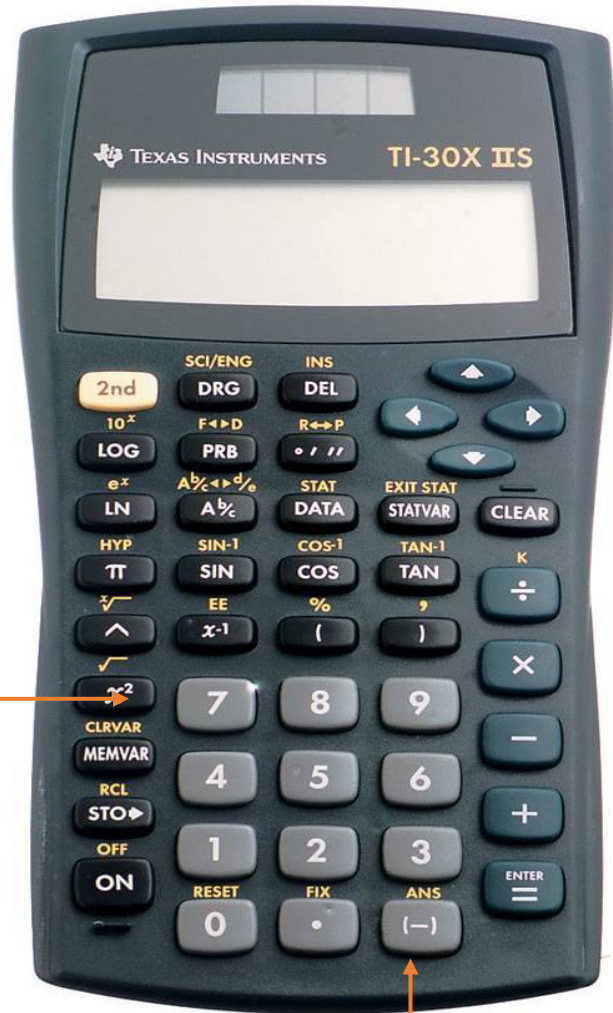
Example 2:

Evaluate 4^3 on the calculator.

1. Type 4.
2. Press the caret ^ button. This indicates “raising to a power.”
3. Type 3.
4. Press the ENTER button.

The answer is **64**.

Definition of Exponential Expressions (4 of 4)



power

negative sign – don't confuse with subtraction sign!

Example 3:

Evaluate $(-5)^2$ by hand and using a calculator.

$$(-5)^2 = (-5)(-5) = 25$$

Using the TI-30X IIS Calculator:

Press the left parenthesis button (.

Press the negative sign button (-). Do not use the subtraction button!

Type **3**.

Press the right parenthesis button).

Press the caret ^ button on the calculator.

Type **2** and press the ENTER button.

The answer is **25**.

2. The Rules of Exponents (1 of 5)

The *Rules of Exponents*, also called *Laws of Exponents* or *Properties of Exponents*, make the process of simplifying expressions involving exponents easier. Many arithmetic operations like addition, subtraction, multiplication, and division can be conveniently performed in quick steps using the Rules of Exponents.

Specifically, we will discuss the Negative-Exponent Rule, the Zero-Exponent Rule, and the Power-of-a-Quotient Rule.

The Rules of Exponents (2 of 5)

The Negative-Exponent Rule

If b is any real number other than 0 and n is a natural number, then

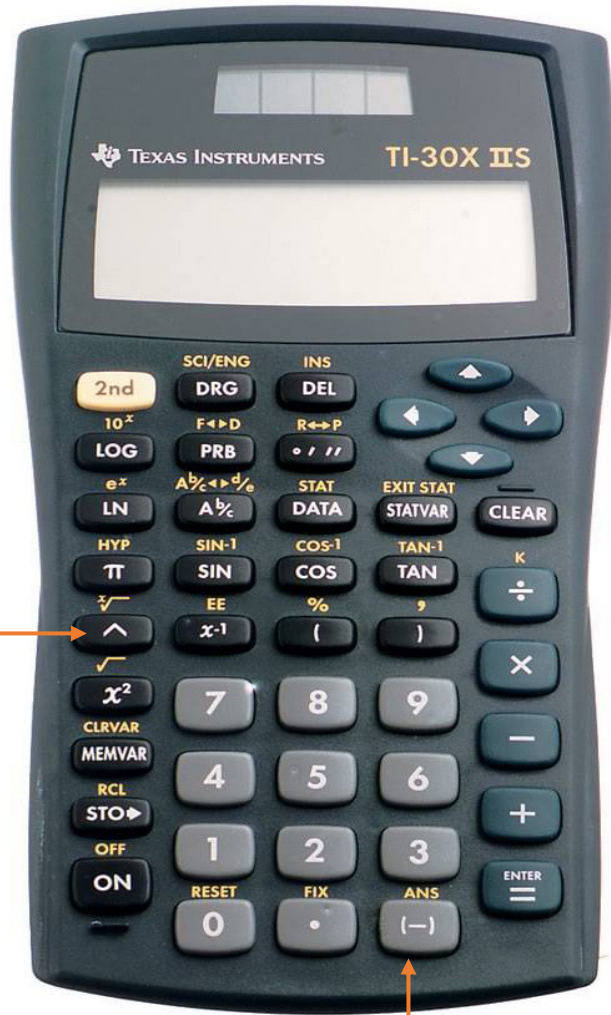
$$b^{-n} = \frac{1}{b^n}$$

This means when a number is raised to a negative power, we write a fraction with a 1 in the numerator and place into the denominator the same number raised to the positive power.

$$\text{For example, } 5^{-3} = \frac{1}{5^3} = \frac{1}{125} \text{ or } 2^{-1} = \frac{1}{2^1} = \frac{1}{2} .$$

Please note that the power of 1 is usually NOT written. For instance, we hardly ever state 2^1 . It is always just 2.

The Rules of Exponents (3 of 9)



power

negative sign – don't confuse with subtraction sign!

Example 4:

Evaluate 4^{-3} on the TI-30X IIS calculator.

1. Type 4.
2. Press the caret ^ button.
3. Press the negative sign button (-). Do not use the subtraction button!
4. Type 3.
5. Press the ENTER button.

The answer is $0.015625 = \frac{1}{64}$.

Note: The TI-30X IIS does not give you a fraction!

The Rules of Exponents (4 of 5)

The Zero-Exponent Rule

If b is any real number other than 0, then $b^0 = 1$.

This means when a number is raised to the zero power, the result is ALWAYS 1.

For example, $(-4)^0 = 1$

or $7^0 = 1$

or $1000^0 = 1$

The Rules of Exponents (5 of 5)

The Power-of-a-Quotient Rule

If a and b are real numbers, $b \neq 0$, or algebraic expressions, and n is an integer, then

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

This means, when a fraction is raised to a power, raise the numerator AND the denominator to that power.

For example, $\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3} = \frac{2 \cdot 2 \cdot 2}{5 \cdot 5 \cdot 5} = \frac{8}{125}$.

3. Advanced Order of Operations (1 of 6)

In an earlier lesson, we discussed the simple *Order of Operations* given only addition, subtraction, multiplication, and division.

In this lesson, we will add **grouping symbols** and exponents to mathematical expressions.

Grouping symbols are used to indicate that a particular collection of numbers and mathematical operations are to be grouped together and considered as one number. The most common grouping symbols used in algebra are parentheses (), brackets [], braces { }, and the fraction bar. Incidentally, the singular of parentheses is one parenthesis.

Advanced Order of Operations (2 of 6)

The *Order of Operations* including addition, subtraction, multiplication, division, grouping symbols, and exponents is as follows:

1. Grouping symbols are evaluated first.
2. Exponents are evaluated next.
3. Multiplications and divisions are done next, in the order in which they occur, working from left to right.
4. Addition and subtraction are done last, in the order in which they occur, working from left to right.

Advanced Order of Operations (3 of 6)

Example 5:

Evaluate $7 - 20 \div 2 \cdot (2 + 1)^3 + 93$.

Given $7 - 20 \div 2 \cdot (2 + 1)^3 + 93$, we evaluate the parentheses first.

then $7 - 20 \div 2 \cdot 3^3 + 93$. Next, we evaluate the exponent:

$7 - 20 \div 2 \cdot 27 + 93$. Next, we evaluate division:

$7 - 10 \cdot 27 + 93$. Next, we evaluate multiplication:

$7 - 270 + 93$. Next, we evaluate subtraction:

$-263 + 93$. Next, we evaluate addition:

-170

Advanced Order of Operations (4 of 6)

Example 6:

Evaluate $6^2 - 24 \div 2^2 \cdot 3 + 1$.

Given $6^2 - 24 \div 2^2 \cdot 3 + 1$, we evaluate both exponents first.

then $36 - 24 \div 4 \cdot 3 + 1$. Next, we evaluate division:

$36 - 6 \cdot 3 + 1$. Next, we evaluate multiplication:

$36 - 18 + 1$. Next, we evaluate subtraction:

$18 + 1$. Next, we evaluate addition:

19

Advanced Order of Operations (5 of 6)

Example 7:

Evaluate $(-5)^0$ and -5^0 .

We know that $(-5)^0$ equals 1.

In -5^0 only 5 is raised to the 0 power. Therefore, you must use the *Order of Operations* rule that states that exponents are evaluated before multiplication.

Now, -5^0 actually equals $-1(5^0)$. Given that 5^0 equals 1, we find that -5^0 must equal $-1(1)$ or -1 .

Advanced Order of Operations (6 of 6)

Example 8:

Evaluate $(-4)^2$ and -4^2 .

We know that $(-4)^2$ equals $(-4) \cdot (-4)$ which equals 16.

In -4^2 only 4 is raised to the 2nd power. Therefore, you must use the *Order of Operations* rule that states that exponents are evaluated before multiplication.

Now, -4^2 actually equals $-1(4^2)$. Given that 4^2 equals 16, we find that -4^2 must equal $-1(16)$ or -16 .