DETAILED SOLUTIONS AND CONCEPTS - SOLVING TRIGONOMETRIC EQUATIONS - PART 2

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Please Send Questions and Comments to ingrid.stewart@csn.edu. Thank you!

PLEASE NOTE THAT YOU CANNOT ALWAYS USE A CALCULATOR ON THE ACCUPLACER - COLLEGE-LEVEL MATHEMATICS TEST! YOU MUST BE ABLE TO DO SOME PROBLEMS WITHOUT A CALCULATOR!

In the previous lecture we restricted the solutions of our trigonometric equations to a particular interval. In this lecture we will look at examples that ask us to find ALL solutions given that trigonometric equations have infinitely many solutions! Additionally, we will be solving more complicated equations requiring the use of the *Pythagorean Identities* and/or factoring !!!

Lastly, we will look at trigonometric equations of the following form and its solution strategy:

$$\sin(ax) = C$$
$$\cos(ax) = C$$
$$\tan(ax) = C, \text{ where } a \neq 1$$

Solution Strategy:

Step 1:

Find the solution interval for angle ax.

Step 2:

If necessary, isolate the trigonometric expression on one side of the equation. **This lets you know the sign of the numeric value (positive or negative).** Then use the *Inverse Trigonometric Function* concept to find a solution for the angle ax with the calculator.

HINT: It is easier to find this angle in degrees! Even if you are asked for solutions in radians, you might want to find their degree equivalent first, then change the final answer to radians!!!

Step 3:

Find the Reference Angle α for the angle ax, if it exists. Remember that Quadrantal Angles do NOT have Reference Angles.

Step 4:

Find the solutions for angle ax using **ITS** solution interval and **All Students Take Calculus**.

You must be able to explain the solution process (see examples below).

Step 5:

Solve for angle x.

Problem 1:

Solve $2\cos x = -\sqrt{3}$ for x finding **ALL** possible solutions. Express your answer in **EXACT** radians AND in **EXACT** degrees.

Note: We are asked to find ALL solutions! In this case, we ALWAYS find the solutions in the interval $[0°, 360°)$ or $[0, 2\pi)$ first. Then we generalize to the "ALL" case.

Step 1:

Isolate the trigonometric expression on one side:

$$2\cos x = -\sqrt{3}$$

$$\cos x = \frac{-\sqrt{3}}{2}$$ **The numeric value is negative !**

and using the calculator, we find

$$x = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$x = 150°$$

NOTE: It's easier to use degrees in the solution process! Then, for your final answer, change the solutions to radians!

Step 2:

Reference Angle for angle $x = 150^o$ is $\alpha = 30^o$.

Please note that the range of the arccosine is limited to the interval $[0°, 180°]$. That is why the calculator gives us 150^o and not -30^o.

Step 3:

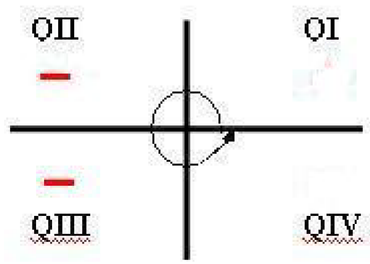
Here we are going to use the fact that once the trigonometric expression is isolated on one side, the numeric value is **negative**. See Step 1 above.

You must explain the solution process. You can either use a sentence ...

For example:

The numeric value of cosine is **negative** for angles in QII and QIII (see Step 1). Given a reference angle of 30^o, the solutions for x on the interval $[0°,360°)$ are as follows.

or you can show a picture showing the solution interval and the quadrants in which we have to find the angles ...



In any case, the solutions are

$$x_1 = 150° \equiv \frac{5\pi}{6}$$

$$x_2 = 180° + 30° = 210° \equiv \frac{7\pi}{6}$$

To show all solutions, we add $360°k$ to the degree solutions or $2\pi k$ to the radian solutions, where k is defined as any integer.

All solutions in radians:

$$\frac{5\pi}{6} + 2\pi k, \quad \frac{7\pi}{6} + 2\pi k$$, where k is any integer

All solutions in degrees:

$$150° + 360°k, \quad 210° + 360°k$$, where k is any integer

NOTE: You can think of k as the number of round trips from and to the terminal side of the angles found in Step 4. A negative k indicates a clockwise movement (negative angles) and a positive k indicates a counter-clockwise movement (positive angles).

For example,

when $k = -1$ we get the solutions on the interval $[-2\pi, 0)$

when $k = 0$ we get the solutions on the interval $[0, 2\pi)$

when $k = 1$ we get the solutions on the interval $[2\pi, 4\pi)$

etc.

Problem 2:

Solve $\tan x = -12.08$ for x finding ALL possible solutions. Express your answers in degrees.

Please note that -12.08 is NOT a numeric value associated with our "special" angles. In this case I CANNOT ask you to find EXACT solutions! Therefore, let's agree to round ALL calculations to 2 decimal places.

Step 1:

$\tan x = -12.08$ **The numeric value is negative !**

and using the calculator, we find

$$x = \tan^{-1}(-12.08)$$
$$x \approx -85.27°$$

Step 2:

Reference Angle for angle $x \approx -85.27°$ is $\alpha \approx 85.27°$.

Step 3:

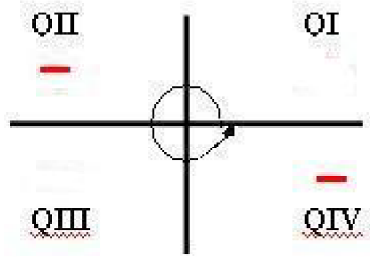
Here we are going to use the fact that once the trigonometric expression is isolated on one side, the numeric value is **negative**. See Step 1 above.

You must explain the solution process. You can either use a sentence ...

For example:

The numeric value of tangent is **negative** for angles in QII and QIV (see Step 1). Given a reference angle of $85.27°$, the solutions for x on the interval $[0°, 360°)$ are as follows.

or you can show a picture showing the solution interval and the quadrants in which we have to find the angles ...



In any case, the solutions are

$$x_1 = 180^o - 85.27^o = 94.73^o$$

$$x_2 = 360^o - 85.27^o = 274.73^o$$

Then all solutions are:

$94.73^\circ + 360^\circ k,\ \ 274.73^\circ + 360^\circ k$, where k is any integer

NOTE:

Since we are dealing with a tangent, which is the only trigonometric ratio that has positive (negative) numeric values in opposite quadrants, you can also write the solutions as $94.73^\circ + 180^\circ k$.

Problem 3:

Solve $2\sin^2 x = 3\sin x - 1$ for x on the interval $[0^\circ, 360^\circ)$. Express your answers in degrees.

Notice that we are dealing with an equation that is "like a quadratic." You have dealt with them before in Intermediate Algebra. Remember that quadratic equations are of degree 2.

Quadratic equations can be solved by using either the *Zero Product Principle* and factoring, the *Square Root Method*, or the *Quadratic Formula*.

For the given equation, we will factor and then use the *Zero Product Principle*. Therefore, we first have to shift all terms to one side as follows.

$$2\sin^2 x - 3\sin x + 1 = 0$$

Please note that we have a quadratic equation of the form

$$2a^2 - 3a + 1 = 0$$

which we can factor as follows

$$(2a-1)(a-1)=0$$

We can factor the given trigonometric equation as follows:

$$(2\sin x-1)(\sin x-1)=0$$

which means that $\sin x-1=0$ and/or $2\sin x-1=0$ by the *Zero Product Principle*.

a. Solve $\sin x-1=0$ for x on the interval $[0°,360°)$. Express your answers in degrees.

Step 1:

$$\sin x=1$$

and using the calculator, we find

$$x=\sin^{-1}(1)$$
$$x=90°$$

Step 2:

$90°$ is a Quadrantal Angle, which has no Reference Angle!

Step 3:

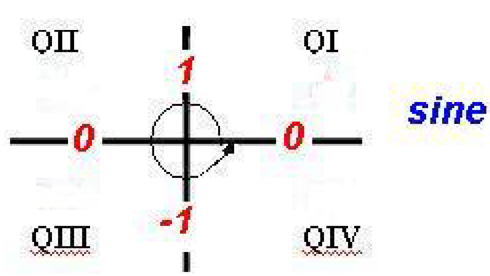
Here we are going to use the fact that once the trigonometric expression is isolated on one side, the numeric value is 1. See Step 1 above.

You must explain the solution process. You can either use a sentence ...

For example:

The numeric value of sine is 1 only for $90°$ in the interval $[0°,360°)$. Therefore, the solutions for x are as follows.

or you can show a picture showing the solution interval and the quadrants in which we have to find the angles ...



In any case, the solution is

$x = 90^o$

b. Solve $2\sin x - 1 = 0$ for x on the interval $[0^\circ, 360^\circ)$. Express your answers in degrees.

Step 1:

We are going to use the concept of *Inverse Trigonometric Functions* to solve for the angle x using the calculator.

$\sin x = \frac{1}{2}$ **The numeric value is positive !**

$x = \arcsin \frac{1}{2}$

$x = 30^o$

NOTE: It's easier to use degrees in the solution process! Then, for your final answer, change the solutions to radians!

Step 2:

Reference Angle for angle $x = 30^o$ is $\alpha = 30^o$.

Step 3:

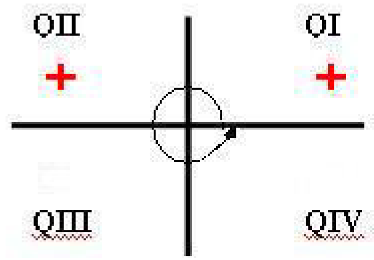
Here we are going to use the fact that once the trigonometric expression is isolated on one side, the numeric value is **positive**. See Step 1 above.

You must explain the solution process. You can either use a sentence ...

For example:

The numeric value of sine is **positive** for angles in QI and QII (see Step 1). Given a reference angle of 30^o, the solutions for x on the interval $[0^\circ, 360^\circ)$ are as follows.

or you can show a picture showing the solution interval and the quadrants in which we have to find the angles ...



In any case, the solutions are

$x_1 = 30^o$

$x_2 = 180^o - 30^o = 150^o$

Then all solutions are 30^o, 90^o, and 150^o.

Problem 4:

Solve $2\cos^2 x + 3\sin x = 0$ for x on the interval $[0°, 360°)$. Express your answers in degrees.

This equation contains a $\cos^2 x$ and a $\sin x$. We first must change the equation so that it contains either all cosines or all sines. Remembering your fundamental identities, let's change $\cos^2 x$ to $1 - \sin^2 x$.

$$2(1 - \sin^2 x) + 3\sin x = 0$$

$$2 - 2\sin^2 x + 3\sin x = 0$$

$$2\sin^2 x - 3\sin x - 2 = 0$$

Then we will again factor to get

$$(2\sin x + 1)(\sin x - 2) = 0$$

which means that we have to solve the equations $2\sin x + 1 = 0$ and $\sin x - 2 = 0$ when applying the *Zero Product Principle*.

a. Solve $2\sin x + 1 = 0$ for x on the interval $[0°, 360°)$. Express your answers in degrees.

Step 1:

We are going to use the concept of *Inverse Trigonometric Functions* to solve for the angle x using the calculator.

$\sin x = -\frac{1}{2}$ **The numeric value is negative !**

x = arcsin $-\frac{1}{2}$

x = -30º

NOTE: It's easier to use degrees in the solution process! Then, for your final answer, change the solutions to radians!

Step 2:

Reference Angle for angle ***x = -30º*** is α ***= 30º***.

Step 3:

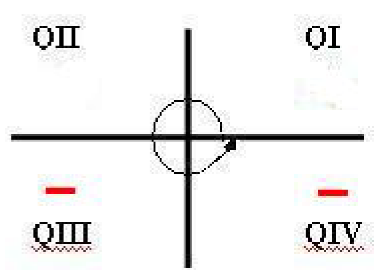
Here we are going to use the fact that once the trigonometric expression is isolated on one side, the numeric value is **negative**. See Step 1 above.

You must explain the solution process. You can either use a sentence ...

For example:

The numeric value of sine is **negative** for angles in QIII and QIV (see Step 1). Given a reference angle of ***30º***, the solutions for ***x*** on the interval $[0°, 360°)$ are as follows.

or you can show a picture showing the solution interval and the quadrants in which we have to find the angles ...



In any case, the solutions are

$x_1 = 180° + 30° = 210°$

$x_2 = 360° - 30° = 330°$

b. However, there is no solution for $\sin x - 2 = 0$, which is $\sin x = 2$.

Your calculator will indicated "Error." That is, because $\sin x$ can only take on values that are between and including -1 and 1.

Then all solutions are 210^o and 330^o.

Problem 5:

Solve $2\cos x + 2\sin x\cos x = 0$ for x finding ALL solutions. Express your answers in **EXACT** radians.

This equation contains sines and cosines. However, in this case we can factor as follows

$$2\cos x(1+\sin x) = 0$$

which means that we have to solve the equations $\cos x = 0$ and $1 + \sin x = 0$ by the *Zero Product Principle*.

a. Solve $\cos x = 0$ for x on the interval $[0, 2\pi)$ first. Express your answers in **EXACT** radians.

Step 1:

$$\cos x = 0$$

and using the calculator, we find

$$x = \cos^{-1} 0$$

$$x = 90^\circ$$

NOTE: It's easier to use degrees in the solution process! Then, for your final answer, change the solutions to radians!

Step 2:

90° is a Quadrantal Angles, which has no Reference Angle!

Step 3:

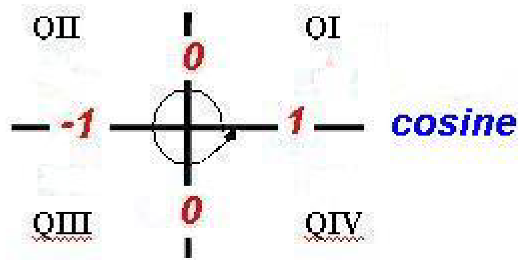
Here we are going to use the fact that once the trigonometric expression is isolated on one side, the numeric value is 0. See Step 1 above.

You must explain the solution process. You can either use a sentence ...

For example:

The numeric value of cosine is 0 for $90°$ and $270°$ in the interval $[0°,360°)$. Therefore, the solutions for x on the interval $[0°,360°)$ are as follows.

or you can show a picture showing the solution interval and the quadrants in which we have to find the angles ...



In any case, the solutions are

$$x_1 = 90° \equiv \frac{\pi}{2}$$

$$x_2 = 270° \equiv \frac{3\pi}{2}$$

b. Solve $1 + \sin x = 0$ for x on the interval $[0,2\pi)$. Express your answers in **EXACT** radians.

Step 1:

$\sin x = -1$

and using the calculator, we find

$x = \arcsin(-1)$

$x = -90^o$

NOTE: It's easier to use degrees in the solution process! Then, for your final answer, change the solutions to radians!

Step 2:

-90^o is a Quadrantal Angle, which has no Reference Angle!

Step 3:

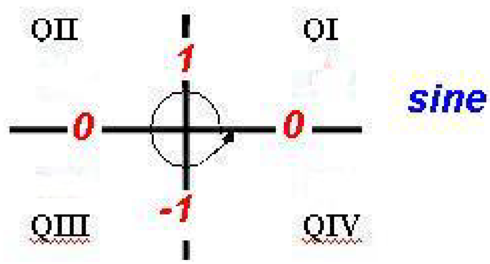
Here we are going to use the fact that once the trigonometric expression is isolated on one side, the numeric value is -*1*. See Step 1 above.

You must explain the solution process. You can either use a sentence ...

For example:

The numeric value of sine is ***-1*** only for ***270º*** in the interval $[0°, 360°)$.
Therefore, the solutions for x are as follows.

or you can show a picture showing the solution interval and the quadrants in which we have to find the angles ...



In any case, the solution is

$$x = 270° \equiv \frac{3\pi}{2}$$

Then all solutions are $\frac{\pi}{2} + 2\pi k$ and $\frac{3\pi}{2} + 2\pi k$.

Problem 6:

Solve $\tan^2 x = 3$ for x on the interval $[0, 2\pi)$. Express your answers in **EXACT** radians.

Since the equation is "like a quadratic equation" of the form $u^2 = d$, we will use the *Square Root Property* to help us solve for x.

That is, $\tan x = \pm\sqrt{3}$

This indicates that we must find solutions for x, given $\tan x = \sqrt{3}$ and $\tan x = -\sqrt{3}$.

Solve $\tan x = \sqrt{3}$ for x on the interval $[0, 2\pi)$. Express your answers in **EXACT** radians.

Step 1:

$\tan x = \sqrt{3}$ **The numeric value is positive !**

and using the calculator, we find

$$x = \tan^{-1}\left(\sqrt{3}\right)$$
$$x = 60°$$

NOTE: It's easier to use degrees in the solution process! Then, for your final answer, change the solutions to radians!

Step 2:

Reference Angle for angle $x = 60^o$ is $\alpha = 60^o$.

Step 3:

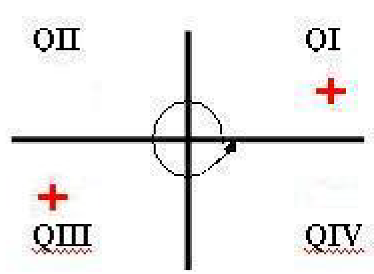
Here we are going to use the fact that once the trigonometric expression is isolated on one side, the numeric value is **positive**. See Step 1 above.

You must explain the solution process. You can either use a sentence ...

For example:

The numeric value of tangent is **positive** for angles in QI and QIII (see Step 1). Given a reference angle of 60^o, the solutions for x on the interval $[0°, 360°)$ are as follows.

or you can show a picture showing the solution interval and the quadrants in which we have to find the angles ...



In any case, the solutions are

$$x_1 = 60^\circ \equiv \frac{\pi}{3}$$

$$x_2 = 240^\circ \equiv \frac{4\pi}{3}$$

Solve $\tan x = -\sqrt{3}$ for x on the interval $[0, 2\pi)$. Express your answers in **EXACT** radians.

Step 1:

$\tan x = -\sqrt{3}$ **The numeric value is negative !**

and using the calculator, we find

$$x = \tan^{-1}\left(-\sqrt{3}\right)$$
$$x = -60^\circ$$

NOTE: It's easier to use degrees in the solution process! Then, for your final answer, change the solutions to radians!

Step 2:

Reference Angle for angle $x = -60^o$ is $\alpha = 60^o$.

Please note that the range of the arctangent is limited to the interval $(-90^\circ, 90^\circ)$. That is why the calculator gives us -60^o and not 60^o.

Step 3:

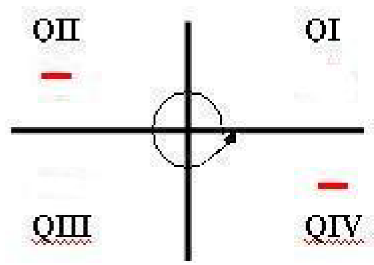
Here we are going to use the fact that once the trigonometric expression is isolated on one side, the numeric value is **negative**. See Step 1 above.

You must explain the solution process. You can either use a sentence ...

For example:

The numeric value of tangent is **negative** for angles in QII and QIV (see Step 1). Given a reference angle of 60^o, the solutions for x on the interval $[0,2\pi)$ are as follows.

or you can show a picture showing the solution interval and the quadrants in which we have to find the angles ...



In any case, the solutions are

$$x_1 = 120^o \equiv \frac{2\pi}{3}$$

$$x_2 = 300^o \equiv \frac{5\pi}{3}$$

In total, there are four solutions which are

$$x_1 = 60^\circ \equiv \frac{\pi}{3}$$

$$x_2 = 240^\circ \equiv \frac{4\pi}{3}$$

$$x_3 = 120^o \equiv \frac{2\pi}{3}$$

$$x_4 = 300^o \equiv \frac{5\pi}{3}$$

Problem 7:

Solve $1-\sqrt{2}\cos(\frac{x}{2})=0$ for x on the interval $[0^\circ, 720^\circ)$. Express your answers in degrees.

Step 1:

The solution interval for $\frac{x}{2}$ is $0^o \le \frac{x}{2} < 360^o$.

Note that the solution interval for angle x is $0^o \le x < 720^o$!!!

Step 2:

Isolate the trigonometric expression on one side:

$$1-\sqrt{2}\cos(\tfrac{x}{2})=0$$

$$\cos\left(\frac{x}{2}\right)=\frac{1}{\sqrt{2}}$$ **The numeric value is positive !**

and using the calculator, we find

$$\frac{x}{2}=\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\frac{x}{2}=45^\circ$$

Step 3:

Reference Angle for angle $\frac{x}{2}$ $= 45^o$ is $\alpha = 45^o$.

Step 4:

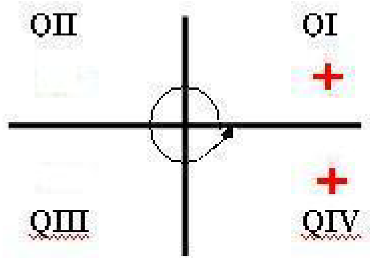
Here we are going to use the fact that once the trigonometric expression is isolated on one side, the numeric value is **positive**. See Step 2 above.

You must explain the solution process. You can either use a sentence ...

For example:

The numeric value of cosine is **positive** for angles in QI and QIV (see Step 1). Given a reference angle of 45^o, the solutions for $\frac{x}{2}$ on the interval $[0^\circ, 360^\circ)$ are as follows.

or you can show a picture ...



In any case, the solutions are

$$\frac{x_1}{2} = 45^\circ$$

$$\frac{x_2}{2} = 315^\circ$$

Step 5:

Since we want to solve for x, we have to multiply both sides of the above equations by 2 to get the final solutions

$$x_1 = 90^\circ$$

$$x_2 = 630^\circ$$

Problem 8:

Solve $2\cos 2x = -1$ for x on the interval $[0^\circ, 180^\circ)$. Express your answers in degrees.

Step 1:

The solution interval for the angle $2x$ is $0^\circ \leq 2x < 360^\circ$.

Note that the solution interval for angle x is $0^\circ \leq x < 180^\circ$!!!

Step 2:

Isolate the trigonometric expression on one side:

$$2\cos 2x = -1$$

$\cos 2x = -\frac{1}{2}$ **The numeric value is negative !**

Using the calculator, we find

$$2x = \cos^{-1}\left(-\tfrac{1}{2}\right)$$
$$2x = 120^\circ$$

Step 3:

Reference Angle of angle $2x = 120^o$ is $\alpha = 60^o$.

Step 4:

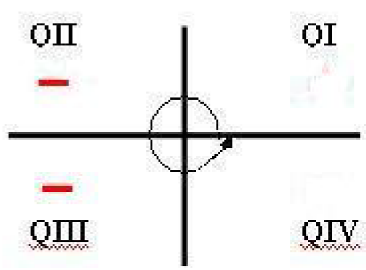
Here we are going to use the fact that once the trigonometric expression is isolated on one side, the numeric value is **negative**. See Step 2 above.

You must explain the solution process. You can either use a sentence ...

For example:

The numeric value of cosine is **negative** for angles in QII and QIII (see Step 1). Given a reference angle of 60^o, the solutions for $2x$ on the interval $[0^\circ, 360^\circ)$ are as follows.

or you can show a picture ...



In any case, the solutions are

$$2x_1 = 120^\circ$$

$$2x_2 = 240^\circ$$

Step 5:

Since we want to solve for x, we have to divide both sides of the above equations by 2 to get the final solutions

$$x_1 = 60°$$

$$x_2 = 120°$$

Problem 9:

Solve $2\sin 4x = 1$ for x on the interval $[0, \pi)$. Express your answers in radians.

Step 1:

The solution interval for angle $4x$ is $0 \le 4x < 4\pi$.

Note that the solution interval for angle x is $0 \le x < \pi$!
!!

Step 2:

Isolate the trigonometric expression on one side:

$$2\sin 4x = 1$$

$\sin 4x = \frac{1}{2}$ **The numeric value is positive !**

and using the calculator, we find

$$4x = \sin^{-1}\left(\tfrac{1}{2}\right)$$
$$4x = 30°$$

NOTE: It's easier to use degrees in the solution process! Then, for your final answer, change the solutions to radians!

Step 3:

Reference Angle of angle $4x = 30°$ is $\alpha = 30°$.

Step 4:

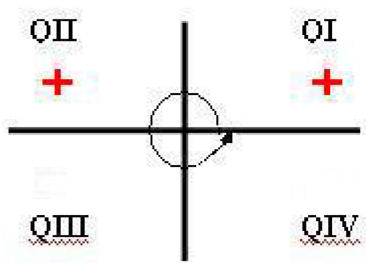
Here we are going to use the fact that once the trigonometric expression is isolated on one side, the numeric value is **positive**. See Step 2 above.

You must explain the solution process. You can either use a sentence ...

For example:

The numeric value of sine is **positive** for angles in QI and QII (see Step 1). Given a reference angle of 30^o, the solutions for $4x$ on the interval $[0, 4\pi)$ are as follows.

or you can show a picture ...



In any case, the solutions are

$$4x_1 = 30^\circ \equiv \frac{\pi}{6}$$

$$4x_2 = 150^\circ \equiv \frac{5\pi}{6}$$

$$4x_3 = 30^\circ + 360^\circ = 390^\circ \equiv \frac{13\pi}{6}$$

$$4x_4 = 150^\circ + 360^\circ = 510^\circ \equiv \frac{17\pi}{6}$$

Step 5:

Since we want to solve for x, we have to divide both sides of the above equations by 4 to get the final solutions

$$x_1 = \frac{\pi}{24} \qquad x_2 = \frac{5\pi}{24} \qquad x_3 = \frac{13\pi}{24}$$

$$x_4 = \frac{17\pi}{24}$$

Problem 10:

Solve $2\tan\frac{x}{3} = -2$ for x on the interval $[0,6\pi)$. Express your answers in radians.

Step 1:

The solution interval for angle $\frac{x}{3}$ is $0 \le \frac{x}{3} < 2\pi$.

Note that the solution interval for angle x is $0 \le x < 6\pi$!!!

Step 2:

Isolate the trigonometric expression on one side:

$$2\tan\frac{x}{3} = -2$$

$\tan\frac{x}{3} = -1$ **The numeric value is negative !**

and using the calculator, we find

$$\frac{x}{3} = \tan^{-1}(-1)$$
$$\frac{x}{3} = -45^\circ$$

NOTE: It's easier to use degrees in the solution process! Then, for your final answer, change the solutions to radians!

Step 3:

Reference Angle of angle $\frac{x}{3} = -45^o$ is $\alpha = 45^o$.

Step 4:

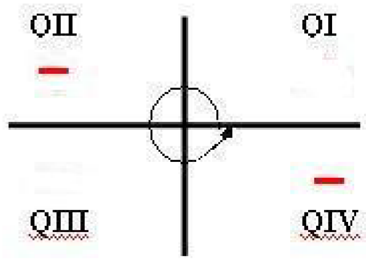
Here we are going to use the fact that once the trigonometric expression is isolated on one side, the numeric value is **negative**. See Step 2 above.

You must explain the solution process. You can either use a sentence ...

For example:

The numeric value of tangent is **negative** for angles in QII and QIV (see Step 1). Given a reference angle of 45^o, the solutions for $\frac{x}{3}$ on the interval $[0,2\pi)$ are as follows.

or you can show a picture ...



In any case, the solutions are

$$\frac{x_1}{3} = 135° \equiv \frac{3\pi}{4}$$

$$\frac{x_2}{3} = 315° \equiv \frac{7\pi}{4}$$

Step 5:

Since we want to solve for x, we have to multiply both sides of the above equations by 3 to get the final solutions

$$x_1 = \frac{9\pi}{4}$$

$$x_2 = \frac{21\pi}{4}$$

Problem 11:

Solve $\sin 3x = -1$ for x on the interval $\left[0, \frac{2\pi}{3}\right)$. Express your answers in radians.

Step 1:

The solution interval for angle $3x$ is $0 \leq 3x < 2\pi$.

Note that the solution interval for angle x is $0 \leq x < \frac{2\pi}{3}$!!!

Step 2:

$$\sin 3x = -1$$

and using the calculator, we find

$$3x = \sin^{-1}(-1)$$
$$3x = -90°$$

NOTE: It's easier to use degrees in the solution process! Then, for your final answer, change the solutions to radians!

Step 3:

-90^o is a Quadrantal Angle, which has no Reference Angle!

Step 4:

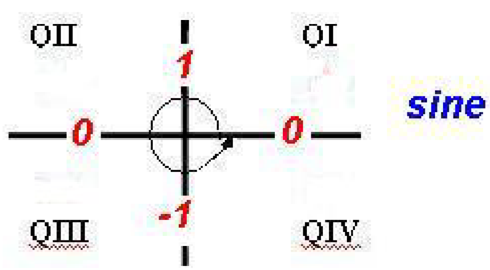
Here we are going to use the fact that once the trigonometric expression is isolated on one side, the numeric value is **negative**. See Step 2 above.

You must explain the solution process. You can either use a sentence ...

For example:

The numeric value of sine is -1 only for $270°$ in the interval $[0°, 360°)$. Therefore, the solutions for $3x$ on the interval $[0°, 360°)$ are as follows.

or you can show a picture ...



In any case, there is only one solution which is

$$3x = 270° \equiv \frac{3\pi}{2}$$

Step 5:

Since we want to solve for x, we have to divide both sides of the above equation by 3 to get the final solution

$$x = \frac{\pi}{2}$$

Problem 12:

Solve $\cos 4x = 0$ for x on the interval $\left[0, \frac{\pi}{2}\right)$. Express your answers in radians.

Step 1:

The solution interval for angle $4x$ is $0 \leq 4x < 2\pi$.

Note that the solution interval for angle x is $0 \leq x < \frac{\pi}{2}$!!!

Step 2:

$$4x = \cos^{-1}(0)$$

$$4x = 90^\circ$$

NOTE: It's easier to use degrees in the solution process! Then, for your final answer, change the solutions to radians!

Step 3:

90^o is a Quadrantal Angle, which has no Reference Angle!

Step 4:

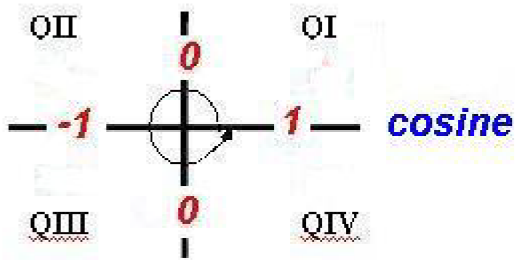
Here we are going to use the fact that once the trigonometric expression is isolated on one side, the numeric value is **zero**.

You must explain the solution process. You can either use a sentence ...

For example:

The numeric value of cosine is 0 only for 90° and 270° in the interval $[0^\circ, 360^\circ)$. Therefore, the solutions for $4x$ on the interval $[0^\circ, 360^\circ)$ are as follows.

or you can show a picture ...



In any case, there are two solutions which are

$$4x_1 = 90° \equiv \frac{\pi}{2}$$

$$4x_2 = 270° \equiv \frac{3\pi}{2}$$

Step 5:

Since we want to solve for x, we have to divide both sides of the above equations by 4 to get the final solutions

$$x_1 = \frac{\pi}{8}$$

$$x_2 = \frac{3\pi}{8}$$