



DETAILED SOLUTIONS AND CONCEPTS - SEQUENCES AND SERIES
Prepared by Ingrid Stewart, Ph.D., College of Southern Nevada
Please Send Questions and Comments to ingrid.stewart@csn.edu. Thank you!

PLEASE NOTE THAT YOU CANNOT ALWAYS USE A CALCULATOR ON THE ACCUPLACER - COLLEGE-LEVEL MATHEMATICS TEST! YOU MUST BE ABLE TO DO SOME PROBLEMS WITHOUT A CALCULATOR!

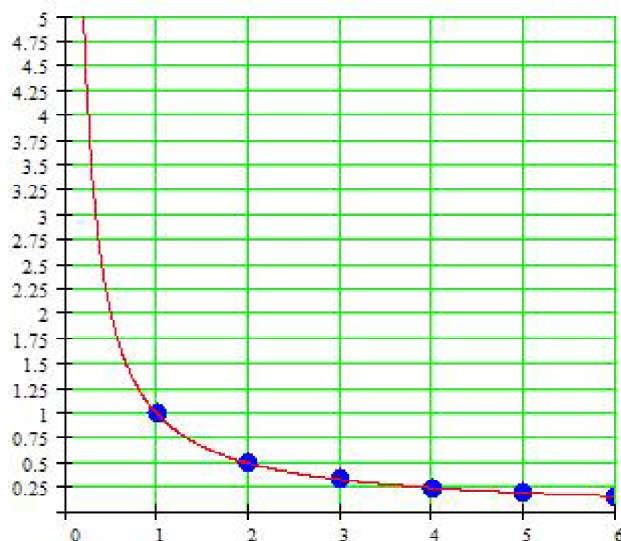
Sequence

A sequence is a function whose domain is the set of positive integers. It is usually represented by a subscripted letter in braces. In other words, we do not see the traditional function notation for sequences.

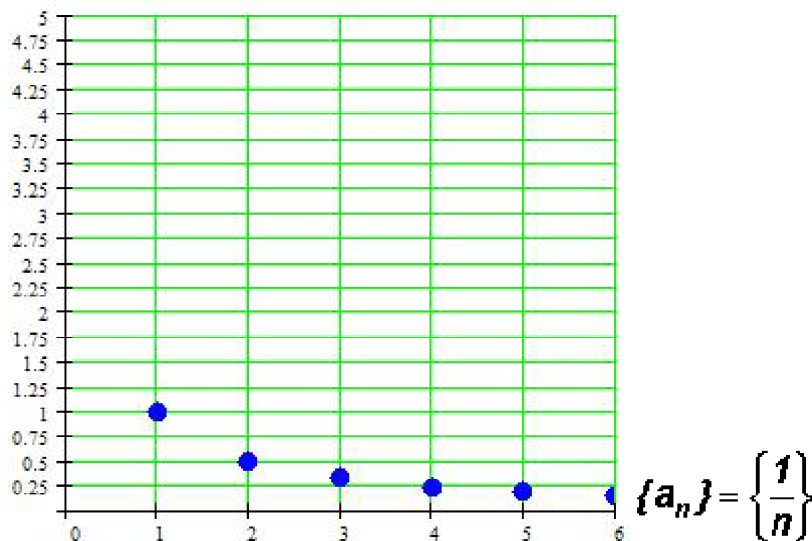
For example, let $\{a_n\}$ be a sequence, where a_1 represents the first term, a_2 represents the second term, ..., and a_n represents the n th term.

Let's illustrate the concept with an example. Below you find the graph of the function

$f(x) = \frac{1}{x}$, where $x > 0$ and the graph of the sequence $\{a_n\} = \left\{ \frac{1}{n} \right\}$, where n is a positive integer.



$$f(x) = \frac{1}{x}$$



The difference between the graph of the rational function $f(x) = \frac{1}{x}$ and the sequence

$\{a_n\} = \left\{ \frac{1}{n} \right\}$ is that on the graph of the sequence you will only see points whose x-coordinates are positive integers.

The terms of the sequence $\{a_n\} = \left\{ \frac{1}{n} \right\}$ are the y-coordinates of the points in the graph above, that is, $a_1 = 1$, $a_2 = \frac{1}{2}$, $a_3 = \frac{1}{3}$, $a_4 = \frac{1}{4}$, etc.

Arithmetic Sequence

When the difference between successive terms of a sequence is always the same nonzero number, the sequence is called arithmetic.

For example, 4, 6, 8, 10, 12 ... is an arithmetic sequence since the difference of successive terms is always 2.

Geometric Sequence

When the ratio of successive terms of a sequence is always the same nonzero number, the sequence is called geometric.

For example, 2, 6, 18, 54, 162, ... is a geometric sequence since the ratio of successive terms is always 3.

Summation

It is often important to be able to find the sum of the first n terms of a sequence $\{a_n\}$, that is, $a_1 + a_2 + a_3 + \dots + a_n$. This can become quite cumbersome. Therefore, the upper case Greek letter sigma Σ is used as follows to indicate this summation.

$\sum_{k=1}^n a_k$, which is pronounced as the *summation of a_k for $k = 1$ to n* !

That is, $\sum_{k=1}^n a_k = a_1 + a_2 + a_3 + \dots + a_n$, where k is the **index of summation** and the **upper and lower bounds of summation** are n and 1 .

NOTE: The index of summation is also often denoted as i or j .

Series

The summation of $\sum_{k=1}^n a_k = a_1 + a_2 + a_3 + \dots + a_n$ is called a *series*. The summation

$\sum_{k=1}^{\infty} a_k = a_1 + a_2 + a_3 + \dots + a_n + \dots$ is called an *infinite series*. The numbers a_1, a_2, a_3 are called **terms** of the series.

For some series it is convenient to begin the index at $n = 0$ or some other integer. Note that a series does not necessarily have to represent the sum of a sequence.

Factorial

A factorial is the product of a given integer and all smaller **positive integers**. The factorial of n is written $n!$ using an exclamation mark and is pronounced "n factorial."

For example, $n! = n(n-1)(n-2)(n-3) \dots \cdot 3 \cdot 2 \cdot 1$ or $n! = n(n-1)!$

Exception: $0! = 1$

Problem 1:

$$\{a_n\} = \left\{ \frac{n-1}{n} \right\}$$

Write the first four terms of the sequence

$$a_1 = \frac{1-1}{1} = 0, \quad a_2 = \frac{2-1}{2} = \frac{1}{2}, \quad a_3 = \frac{3-1}{3} = \frac{2}{3}, \quad a_4 = \frac{4-1}{4} = \frac{3}{4}$$

Problem 2:

Write the first three terms of the sequence $\{a_n\} = \{n^2(n+1)\}$.

$$a_1 = 1^2(1+1) = 2, \quad a_2 = 2^2(2+1) = 12, \quad a_3 = 3^2(3+1) = 36$$

Problem 3:

Show that the sequence $\{s_n\} = \{3n + 5\}$ is arithmetic.

This is what we do. We'll write out the first 5 terms as well as the last two terms $\{3(n-1) + 5\}$ and $\{3n + 5\}$.

$$8, 11, 14, 17, 20, \dots, 3(n-1) + 5, 3n + 5$$

We can immediately see that the difference between the first five terms is **3**. All that's left to do is make sure that the difference between the last two terms is also 3.

That is, $(3n + 5) - [3(n-1) + 5] =$

$$3n + 5 - (3n - 3 + 5) =$$

$$3n + 5 - 3n + 3 - 5 =$$

$$3$$

This establishes the proof that the difference between successive terms is 3 and we do have an arithmetic sequence!

Problem 4:

Show that the sequence $\{s_n\} = \{3^n\}$ is geometric.

Again, we'll write out the first 5 terms as well as the last two terms $\{3^{n-1}\}$ and $\{3^n\}$.

$$3, 9, 27, 81, 243, \dots, 3^{n-1}, 3^n$$

Looking at the first five terms, we can immediately see that the ratio of successive terms is **3**. All that's left to do is make sure that the ration of the last two terms is also 3.

That is,

$$\frac{3^n}{3^{n-1}} = 3^{n-(n-1)} = 3$$

This establishes the proof that the ratio of successive terms is 3 and we do have a geometric sequence!

Problem 5:

Write out the series $\sum_{j=1}^3 (2^j + 1)$ and find the sum.

$$\sum_{j=1}^3 (2^j + 1) = (2^1 + 1) + (2^2 + 1) + (2^3 + 1) = 3 + 5 + 9 = 17$$

Problem 6:

Write out the series $\sum_{i=1}^3 6i$ and find the sum.

$$\sum_{i=1}^3 6i = 6(1) + 6(2) + 6(3) = 6 + 12 + 18 = 36$$

Problem 7:

Write out the series $\sum_{i=1}^3 6$ and find the sum.

$$\sum_{i=1}^3 6 = 6 + 6 + 6 = 18$$

Problem 8:

Evaluate $5!$.

$$5(4)(3)(2)(1) = 120$$

Problem 8:

Evaluate $5!$.

$$5(4)(3)(2)(1) = 120$$

Problem 9:

Write $11!$ as $n(n - 1)!$

$$11 \cdot 10!$$

Problem 10:

Evaluate $\frac{9!}{8!}$.

Lets write out the products and reduce as follows:

$$\frac{9 \cdot 8!}{8!} = 9$$

Problem 11:

Evaluate $\frac{3!7!}{4!}$.

Lets write out the products and reduce as follows:

$$\frac{3!(7)(6)(5) \cdot 4!}{4!} = 3(2)(1)(7)(6)(5) = 1260$$

Problem 12:

Evaluate $6! - 5!$.

Lets write out the products

$$6 \cdot 5! - 5!$$

Notice that both terms have the factor $5!$ in common! Let's factor it out!

$$5!(6 - 1)$$

Please note that if you factor $5!$ out of the first term you are left with 6 and if you factor it out of the second term you are left with -1 .

$$(5)(5)(4)(3)(2)(1) = 600$$