DETAILED SOLUTIONS AND CONCEPTS - POLYNOMIAL AND RATIONAL INEQUALITIES

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Please Send Questions and Comments to ingrid.stewart@csn.edu. Thank you!

PLEASE NOTE THAT YOU CANNOT ALWAYS USE A CALCULATOR ON THE ACCUPLACER - COLLEGE-LEVEL MATHEMATICS TEST! YOU MUST BE ABLE TO DO SOME PROBLEMS WITHOUT A CALCULATOR!

Strategy for Solving Polynomial Inequalities

- **Step 1:** Rewrite the polynomial inequality so that ***0*** is on the right side.
- **Step 2:** Replace the inequality sign with an equal sign and solve the equation.
- **Step 3:** Use the numbers found in Step 2 to divide the number line into intervals.
- **Step 4:** Select any number from each interval and evaluate the final inequality from Step 1. If the test number produces a true statement, then all numbers in that interval belong to the solution set. The solution set is the union of all such test intervals.
- **Step 5:** Check for any single numbers that may be included in the solution set in addition to the numbers belonging to a certain interval. This happens most often given a "greater than or equal to 0" situation when 0 by itself might be included in the solution set.

Strategy for Solving Rational Inequalities

- **Step 1:** Rewrite the rational inequality so that ***0*** is on the right side and a single fraction on the left side.
- **Step 2:** Set the numerator AND the denominator of the left side equal to ***0*** and solve both equations.
- **Step 3:** Use the numbers found in Step 2 to divide the number line into intervals.
- **Step 4:** Select any number from each interval and evaluate the final inequality from Step 1. If the test number produces a true statement, then all numbers in that interval belong to the solution set. The solution set is the union of all such test intervals.
- **Step 5:** Check for any single numbers that may be included in the solution set in addition to the numbers belonging to a certain interval. This happens most often given a "greater than or equal to 0" situation when 0 by itself might be included in the solution set.

Problem 1:

Find the solution set for $x^2 - 2x > 8$ in Interval Notation.

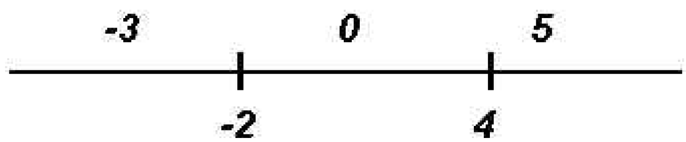
Step 1: $x^2 - 2x - 8 > 0$

Step 2: $x^2 - 2x - 8 = 0$

$$(x-4)(x+2) = 0$$

$x = 4$ or $x = -2$

Step 3:



Step 4:

$$(-3)^2 - 2(-3) - 8 \overset{?}{>} 0$$
$$9 + 6 - 8 > 0 \text{ True}$$

$$(0)^2 - 2(0) - 8 \overset{?}{>} 0$$
$$-8 > 0 \text{ False}$$

$$5^2 - 2(5) - 8 \overset{?}{>} 0$$
$$25 - 10 - 8 > 0 \text{ True}$$

The solution set is $(-\infty, -2) \cup (4, \infty)$.

Please note that the values -2 and 4 are NOT included in the set because we strictly have a "greater than" condition.

Problem 2:

Find the solution set for $x^2 - 9 < 0$ in Interval Notation.

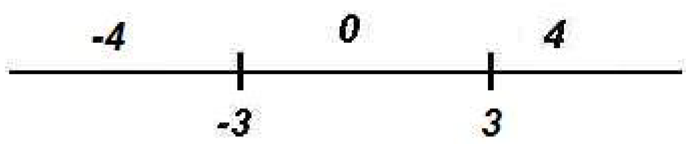
Step 1: Done

Step 2:

$$x^2 - 9 = 0$$
$$(x-3)(x+3) = 0$$

$x = 3$ or $x = -3$

Step 3:



Step 4:

$$(-4)^2 - 9 \overset{?}{<} 0$$
$$16 - 9 < 0 \text{ False}$$

$$(4)^2 - 9 \overset{?}{<} 0$$
$$16 - 9 < 0 \text{ False}$$

$$(0)^2 - 9 \overset{?}{<} 0$$
$$0 - 9 < 0 \text{ True}$$

The solution set is $(-3, 3)$.

Please note that the values -3 and 3 are NOT included in the set because we strictly have a "less than" condition.

Problem 3:

Find the solution set for $2x^3 \geq -16x^2 - 30x$ in Interval Notation.

Step 1: $2x^3 + 16x^2 + 30x \geq 0$

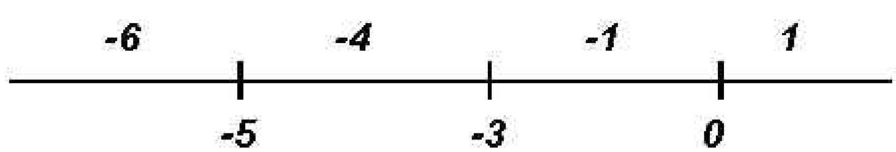
Step 2: $2x^3 + 16x^2 + 30x = 0$

$$2x(x^2 + 8x + 15) = 0$$

$$2x(x+3)(x+5) = 0$$

$x = 0$ or $x = -3$ or $x = -5$

Step 3:



Step 4:

$$2(-6)^3 + 16(-6)^2 + 30(-6) \overset{?}{\geq} 0$$
$$-36 \geq 0 \text{ False}$$

$$2(-4)^3 + 16(-4)^2 + 30(-4) \overset{?}{\geq} 0$$
$$8 \geq 0 \text{ True}$$

$$2(-1)^3 + 16(-1)^2 + 30(-1) \overset{?}{\geq} 0$$
$$-16 \geq 0 \text{ False}$$

$$2(1)^3 + 16(1)^2 + 30(1) \overset{?}{\geq} 0$$
$$48 \geq 0 \text{ True}$$

The solution set is $[-5,-3] \cup [0,\infty)$.

Please note that the values -5, -3, and 0 are included in the set because we have a "greater than or equal to" condition.

Problem 4:

Find the domain of the function $y = x^2\sqrt{9 - x^2}$ in Interval Notation.

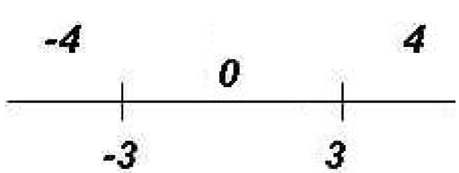
Please note that we will only use the radical for the domain calculation since IT is the only part of the function that could create imaginary y-values.

We know that the domain consists of all numbers so that the radicand $9 - x^2 \geq 0$. That is, we have to solve a polynomial inequality.

Step 1:

$$9 - x^2 = 0$$
$$x^2 = 9$$
$$x = \pm\sqrt{9} = \pm 3$$

Step 2:



Step 3:

$$9-(-4)^2 \overset{?}{\geq} 0 \text{ False} \qquad 9-0^2 \overset{?}{\geq} 0 \text{ True} \qquad 9-4^2 \overset{?}{\geq} 0 \text{ False}$$

The the domain is $[-3,3]$.

Problem 5:

Find the solution set for $\frac{5}{x-2} < \frac{17-x}{2x-4}$ in Interval Notation.

Step 1:

$$\frac{5}{x-2} - \frac{17-x}{2x-4} < 0$$

$$\frac{5}{x-2} - \frac{17-x}{2(x-2)} < 0$$

The LCD is $2(x - 2)$

$$\frac{5(2)}{2(x-2)} - \frac{17-x}{2(x-2)} < 0$$

$$\frac{10-(17-x)}{2(x-2)} < 0$$

$$\frac{-7+x}{2(x-2)} < 0$$

Step 2:

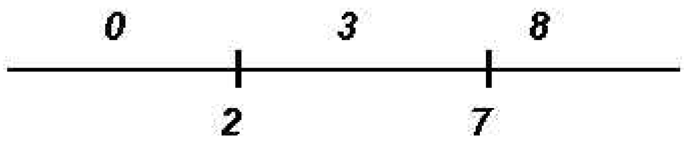
Set the numerator equal to 0

$$-7+x=0$$
$$x=7$$

and set the denominator equal to 0

$$2(x-2)=0$$
$$x=2$$

Step 3:



Step 4:

$$\frac{-7+0}{2(0-2)} \overset{?}{<} 0 \qquad \frac{-7+3}{2(3-2)} \overset{?}{<} 0$$

$$\frac{-7}{-4} < 0 \textit{ False} \qquad \frac{-4}{2} < 0 \textit{ True}$$

$$\frac{-7+8}{2(8-2)} \overset{?}{<} 0$$

$$\frac{1}{12} < 0 \textit{ False}$$

The solution set is $(2,7)$.

Please note that the number 7 is NOT included in the solution set because we strictly have a "less than" condition. However, the number 2 is NOT included because it makes the denominator equal to 0 which would create an undefined condition.

Problem 6:

Find the solution set for $\frac{x}{x+3} \geq 0$ in Interval Notation.

Step 1: Already done!

Step 2:

Set the numerator equal to 0

$x = 0$

and set the denominator equal to 0

$$x + 3 = 0$$
$$x = -3$$

Step 3:

Step 4:

$$\frac{-4}{-4+3} \overset{?}{\geq} 0 \qquad \frac{-1}{-1+3} \overset{?}{\geq} 0 \qquad \frac{1}{1+3} \overset{?}{\geq} 0$$

$$4 \geq 0 \text{ True} \qquad -0.5 \geq 0 \text{ False} \qquad 0.25 \geq 0 \text{ True}$$

The solution set is $(-\infty, -3) \cup [0, \infty)$. Please note that the number 0 is included in the solution set because we have a "greater than or equal to" condition. However, the number -3 is NOT included because it makes the denominator equal to 0 which would create an undefined condition.