PROBLEMS AND SOLUTIONS - POLYNOMIAL FUNCTIONS
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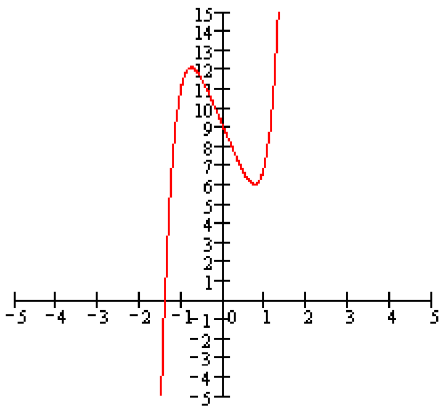
Please Send Questions and Comments to ingrid.stewart@csn.edu. Thank you!

PLEASE NOTE THAT YOU CANNOT ALWAYS USE A CALCULATOR ON THE ACCUPLACER - COLLEGE-LEVEL MATHEMATICS TEST! YOU MUST BE ABLE TO DO SOME PROBLEMS WITHOUT A CALCULATOR!

Problem 1:

Given $f(x) = 3x^5 - 5x + 9$ and its graph

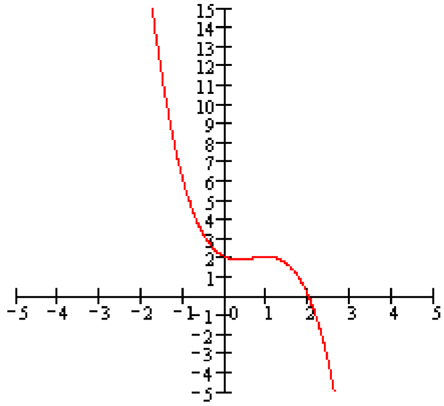


find the following:

a. Leading coefficient and degree of the polynomial
b. Number of real *Zero*s and their approximate values using the graph
c. Number of imaginary *Zeros*

Problem 2:

Given $g(x) = -x^3 + 2x^2 - x + 2$ and its graph

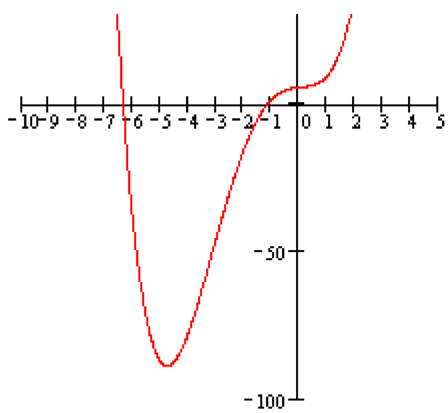


find the following:

a. Leading coefficient and degree of the polynomial
b. Number of real *Zero*s and their approximate values using the graph
c. Number of imaginary *Zeros*

Problem 3:

Given $h(x) = \frac{1}{2}x^4 + 3x^3 - x^2 + x + 5$ and its graph

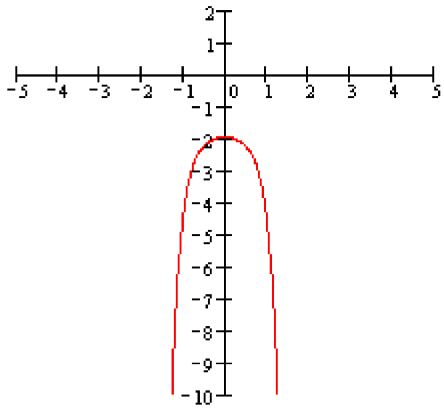


find the following:

a. Leading coefficient and degree of the polynomial
b. Number of real *Zero*s and their approximate values using the graph
c. Number of imaginary *Zeros*

Problem 4:

Given $k(x) = -\frac{3}{2}x^6 - x^2 - 2$ and its graph

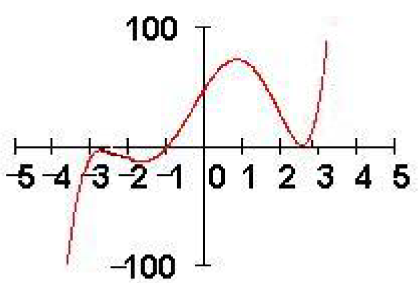


find the following:

a. Leading coefficient and degree of the polynomial
b. Number of real *Zero*s and their approximate values using the graph
c. Number of imaginary *Zeros*

Problem 5:

Given $g(x) = x^5 + x^4 - 14x^3 - 14x^2 + 49x + 49$ and its graph

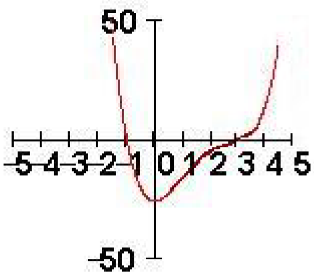


find the following:

a. Leading coefficient and degree of the polynomial
b. Number of real *Zero*s and their approximate values using the graph
c. Number of imaginary *Zeros*

Problem 6:

Given $m(x) = x^4 - 8x^3 + 18x^2 - 27$ and its graph



find the following:

a. Leading coefficient and degree of the polynomial
b. Number of real *Zero*s and their approximate values using the graph
c. Number of imaginary *Zeros*

Problem 7:

Show that $h(x) = \frac{1}{2}x^4 + 3x^3 - x^2 + x + 5$ has a real *Zero* between -2 and -1.

Problem 8:

Show that $g(x) = -x^3 + 2x^2 - x + 2$ has a real *Zero* between 1 and 3.

Problem 9:

If the polynomial $f(x) = 3x^5 - 5x + 9$ has exactly one real *Zero* r between -4 and 0, which of the following is true?

-4 < r < -3

-3 < r < -2

-2 < r < -1

-1 < r < 0

Problem 10:

Find the factored form of a polynomial function with leading coefficient ***6*** and the following *Zeros*:

$-\frac{1}{3}$, $\frac{5}{2}$, and ***3***

Problem 11:

Find the factored form of a polynomial function with leading coefficient $\boldsymbol{2}$ and the following *Zeros*:

$1-\sqrt{3}$, $1+\sqrt{3}$, and $\frac{2}{3}$ (multiplicity 2)

Problem 12:

Find the factored form of a polynomial function with leading coefficient $\boldsymbol{2}$ and the following *Zeros*:

$1-2i$, $1+2i$, and ***-5*** (multiplicity 3)

Problem 13:

Find the factored form of a polynomial function with leading coefficient $\boldsymbol{1}$ and the following *Zeros*:

-5 (multiplicity 3), $-2-i\sqrt{7}$, and $-2+i\sqrt{7}$

Problem 14:

Find the factored form of three polynomial functions with the following *Zeros* and leading coefficients ***1***, ***5***, and ***-10***.

0, ***2***, and ***-3*** (multiplicity 2)

Problem 15:

If ***i*** is a *Zero*, what other imaginary number MUST also be a *Zero* according to the *Conjugate Pairs Theorem*?

Problem 16:

If ***3 - 2i*** is a *Zero*, what other imaginary number MUST also be a *Zero* according to the *Conjugate Pairs Theorem*?

Problem 17:

If ***1 + 6i*** is a *Zero*, what other imaginary number MUST also be a *Zero* according to the *Conjugate Pairs Theorem*?

Problem 18:

If ***-5 - 6i*** is a *Zero*, what other imaginary number MUST also be a *Zero* according to the *Conjugate Pairs Theorem*?

SOLUTIONS

You can find detailed solutions below the link for this problem set!

Problem 1:

The leading coefficient, ***3***, is positive and the degree of the polynomial is five, which is odd.

We know that the function must have five *Zeros* because it is of degree five. Looking at the graph, we notice that the function has one real *Zero*, which must have an approximate value of ***-1.5*** (the x-intercept).

Since there is only one real *Zero*, the function must have four imaginary *Zeros*. Remember that imaginary *Zeros* occur in conjugate pairs!

Problem 2:

The leading coefficient, ***-1***, is negative and the degree of the polynomial is three, which is odd.

We know that the function must have three *Zeros* because it is of degree three. Looking at the graph, we notice that the function has one real *Zero*, which must have an approximate value of ***2*** (the x-intercept).

Since there is only one real *Zero*, the function must have two imaginary *Zeros*. Remember that imaginary *Zeros* occur in conjugate pairs!

Problem 3:

The leading coefficient, $\frac{1}{2}$, is positive and the degree of the polynomial is four, which is even.

We know that the function must have four *Zeros* because it is of degree four. Looking at the graph, we notice that the function has two real *Zeros*, which must have an approximate value of ***-6.25*** and ***-1*** (the x-intercepts).

Since there are two real *Zero*, the function must have two imaginary *Zeros*. Remember that imaginary *Zeros* occur in conjugate pairs!

Problem 4:

The leading coefficient, $-\frac{3}{2}$, is negative and the degree of the polynomial is six, which is even.

We know that the function must have six *Zeros* because it is of degree six. Looking at the graph, we notice that the function has no real *Zeros* since it has no x-intercepts.

Since there are no real *Zeros*, the function must have six imaginary *Zeros*. Remember that imaginary *Zeros* occur in conjugate pairs!

Problem 5:

The leading coefficient, ***1,*** is positive and the degree of the polynomial is five, which is odd.

We know that the function must have five *Zeros* because it is of degree five. Looking at the graph, we notice that the function has three real *Zeros*, which must have an approximate value of ***-2.75*** and ***-1*** and ***2.75*** (the x-intercepts). However, since the graph is parabolic in shape at the intercept ***-2.75*** and ***2.75***, they must represent *Zeros* of even multiplicity.

Given a degree of five, we concede that this polynomial must have *double Zeros* at ***-2.75*** and ***2.75*** and a single *Zero* at ***-1***.

Since there are five real *Zeros*, the function has no imaginary *Zeros*.

Problem 6:

The leading coefficient, ***1*** is positive and the degree of the polynomial is four, which is even.

We know that the function must have four *Zeros* because it is of degree four. Looking at the graph, we notice that the function has two real *Zeros*, which must have an approximate value of ***-1*** and ***3*** (the x-intercepts). However, since the graph mimics the picture of a cubic function at the intercept ***3***, it must represent a *Zero* of odd multiplicity greater than 1.

Given a degree of four, we concede that this polynomial must have a *triple Zeros* at ***3*** and a single *Zero* at ***-1***.

Since there are four real *Zeros*, the function has no imaginary *Zeros*.

Problem 7:

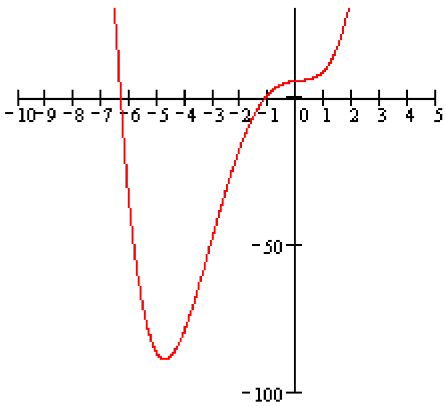
We will use the *Intermediate Value Theorem* to find whether the values of ***h(-1)*** and ***h(-2)*** are of opposite sign. If this is the case, then by the *Intermediate Value Theorem* we can conclude that there must be an x-intercept, and hence a real *Zero*.

$$h(-1) = \tfrac{1}{2}(-1)^4 + 3(-1)^3 - (-1)^2 + (-1) + 5 = \tfrac{3}{2}$$

$$h(-2) = \tfrac{1}{2}(-2)^4 + 3(-2)^3 - (-2)^2 + (-2) + 5 = -22$$

Since the values of ***h(-1)*** and ***h(-2)*** are of opposite sign, we can conclude that there is an x-intercept between -2 and -1, and hence a real *Zero*.

Following is the graph of the polynomial function to illustrate our reasoning.



Problem 8:

Show that $g(x) = -x^3 + 2x^2 - x + 2$ has a real *Zero* between 1 and 3.

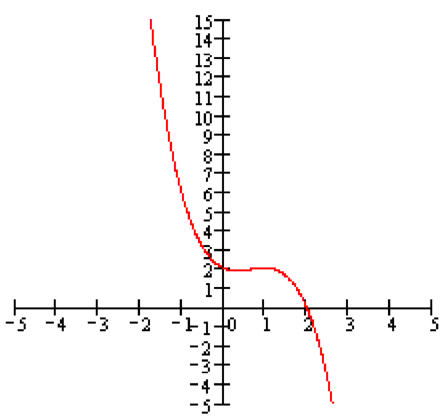
We will use the *Intermediate Value Theorem* to find whether the values of $g(1)$ and $g(3)$ are of opposite sign. If this is the case, then by the *Intermediate Value Theorem* we can conclude that there must be an x-intercept, and hence a real *Zero*.

$$g(1) = -(1)^3 + 2(1)^2 - 1 + 2 = 2$$

$$g(3) = -(3)^3 + 2(3)^2 - 3 + 2 = -7$$

Since the values of $g(1)$ and $g(3)$ are of opposite sign, we can conclude that there is an x-intercept between 1 and 3, and hence a real *Zero*.

Following is the graph of the polynomial function to illustrate our reasoning.



Problem 9:

We will use the *Intermediate Value Theorem* to find whether the values of $f(-4)$ and $f(-3)$ are of opposite sign. If they are not, we will find whether the values of $f(-3)$ and $h(-2)$ are of opposite sign, and so on.

$$f(-4) = 3(-4)^5 - 5(-4) + 9 = -3043$$

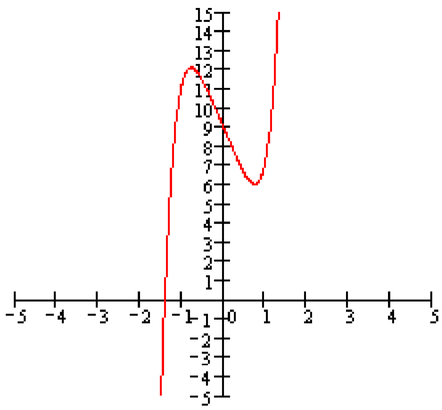
$$f(-3) = 3(-3)^5 - 5(-3) + 9 = -705$$

$$f(-2) = 3(-2)^5 - 5(-2) + 9 = -77$$

$$f(-1) = 3(-1)^5 - 5(-1) + 9 = 11$$

Since the values of $f(-2)$ and $f(-1)$ are of opposite sign, we can conclude that there is an x-intercept between -2 and -1, and hence a real *Zero*.

Following is the graph of the polynomial funciton to illustrate our reasoning.



Problem 10:

$$y = 6[x - (-\tfrac{1}{3})](x - \tfrac{5}{2})(x - 3)$$
$$y = 6(x + \tfrac{1}{3})(x - \tfrac{5}{2})(x - 3)$$

or $y = (3x + 1)(2x - 5)(x - 3)$

Problem 11:

$$y = 2[x - (1 - \sqrt{3})][x - (1 + \sqrt{3})](x - \tfrac{2}{3})^2$$
$$y = 2(x - 1 + \sqrt{3})(x - 1 - \sqrt{3})(x - \tfrac{2}{3})^2$$

Problem 12:

$$y = 2[x-(1-2i)][x-(1+2i)][x-(-5)]^3$$
$$y = 2(x-1+2i)(x-1-2i)(x+5)^3$$

Problem 13:

$$p(x) = (x+5)^3(x+2+i\sqrt{7})(x+2-i\sqrt{7})$$

Problem 14:

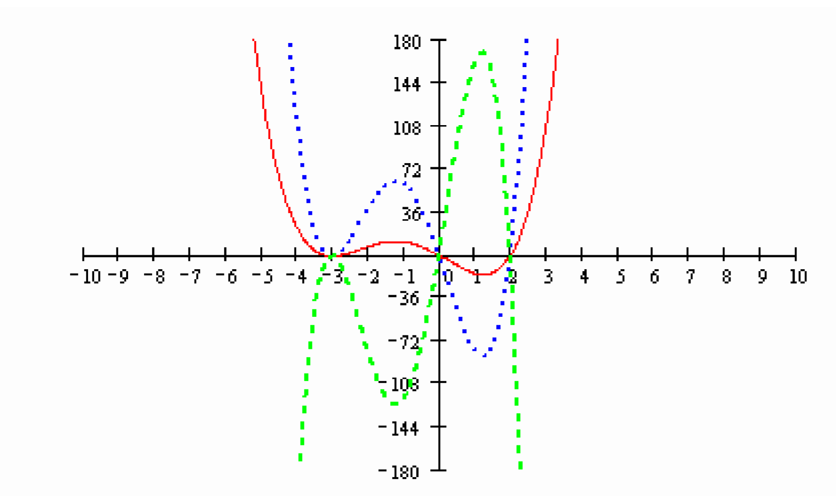
$$p(x) = x(x-2)(x+3)^2$$

$$f(x) = 5x(x-2)(x+3)^2$$

$$g(x) = -10x(x-2)(x+3)^2$$

Following is a graph showing all three functions. Note that these functions have the same *Zeros*. The different leading coefficients affect the location of the peaks and valleys.

The solid graph (red) shows the function ***p***; the dotted graph (blue) shows the function ***f***; and the dashed graph (green) shows the function ***g***.



Problem 15:

Answer: ***-i*** because imaginary *Zeros* occur in conjugate pairs.

Problem 16:

Answer: ***3 + 2i*** because imaginary *Zeros* occur in conjugate pairs.

Problem 17:

Answer: ***1 - 6i*** because imaginary *Zeros* occur in conjugate pairs.

Problem 18:

Answer: ***-5 + 6i*** because imaginary *Zeros* occur in conjugate pairs.